Resolving Nondeterminism by Chance

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IARCS Verification Seminar Series
16 Sep, 2025

Joint work with
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UNIVERSITY OF LIVERPOOL

Resolving Nondeterminism

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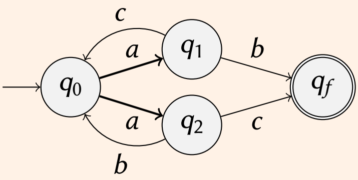

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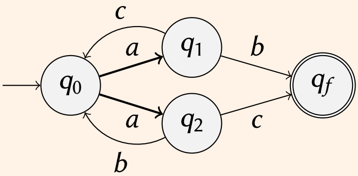

Resolver : strategy to choose next transition based on history

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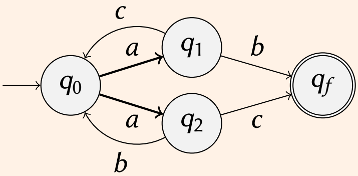

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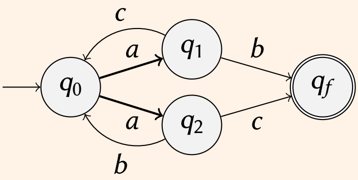

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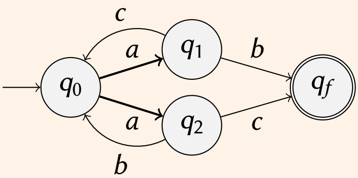

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Run on *acabab*

$$q_0 \xrightarrow{a} q_1 \xrightarrow{c} q_0 \xrightarrow{a} q_2 \xrightarrow{b} q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_f$$

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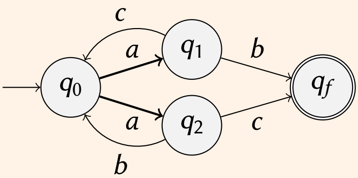

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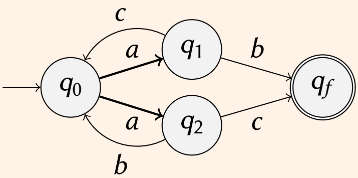

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No uniform strategy for accepting all words

Resolvability

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Resolvable : If a resolver accepts all words in language

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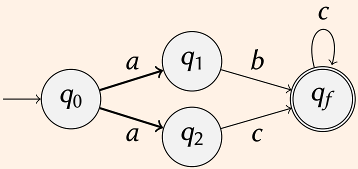
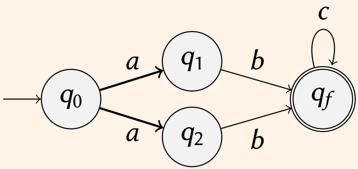


$\mathcal{L} = abc^* + ac^+$



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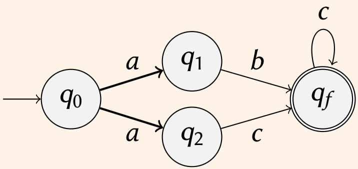

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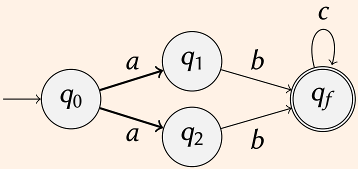

$\mathcal{L} = abc^*$

Commonly known as History Deterministic (HD) or Good for Games (GFG) automata

What are they good for?

Good for games automata studied for reactive synthesis
[Henzinger, Piterman'06]

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Has been studied for several models

- ω-regular automata
- Pushdown systems
- Timed automata
- VASS, etc

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This work : Generalise resolver strategies

Generalising Resolver Strategy

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Stochastic Resolver

Resolve using randomised strategy

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Stochastic Resolver

Resolve using randomised strategy

Produces probabilistic finite automaton (PFA) from NFA

Generalising Resolver Strategy

Stochastic Resolver

Resolve using randomised strategy

Produces probabilistic finite automaton (PFA) from NFA

Accept all words in language with some threshold probability

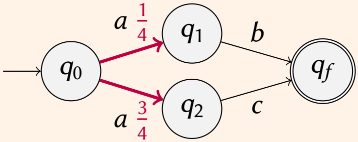
Overview

- Stochastic resolvers
- Classification of resolvable automata
- Complexity of recognising stochastic resolvability

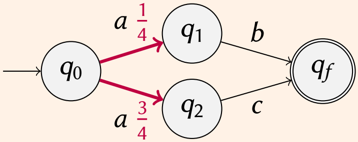
Focus on automata over finite words

Probabilistic automata

Probabilistic automata



Probabilistic automata



$Pr(ab \text{ is accepted}) = \frac{1}{4}$

$Pr(ac \text{ is accepted}) = \frac{3}{4}$

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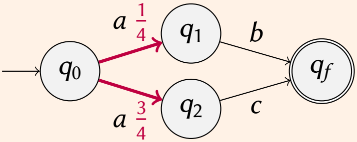

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$$\mathcal{L}(\mathcal{P}_\lambda) \quad = \quad \{w \mid Pr(w \text{ is accepted}) \quad \geq \quad \lambda\}$$

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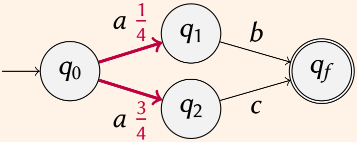

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$\mathcal{L}(\mathcal{P}_{\frac{1}{4}}) = \{ab, ac\}$
$\mathcal{L}(\mathcal{P}_{\frac{3}{4}}) = \{ac\}$

Stochastic Resolver

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Resolve using randomised (memoryless) strategy

Resolver $\mathcal{R} : Q \times \Sigma \mapsto \Delta(Q)$

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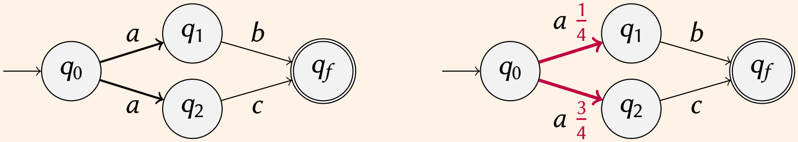
Resolver $\mathcal{R} : Q \times \Sigma \mapsto \Delta(Q)$

$\mathcal{R}$ produces probabilistic finite automaton (PFA) from NFA $\mathcal{A}$



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Resolver accepts all words in $\mathcal{L}(\mathcal{A})$ with probability above a threshold

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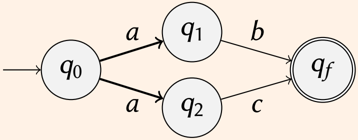

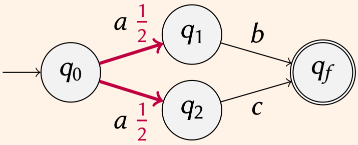

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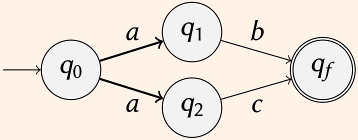

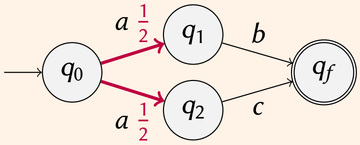

$\mathcal{R}(q_0, a, q_1) = p \mid \mathcal{R}(q_0, a, q_2) = 1 - p$

> λ-resolvable with $\lambda = \min(p, 1 - p)$

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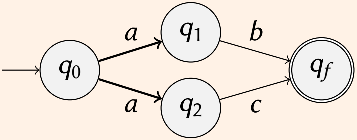

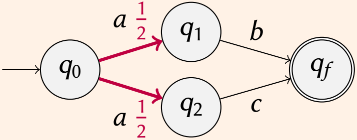

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Next : Importance of threshold

Resolvability with specific threshold

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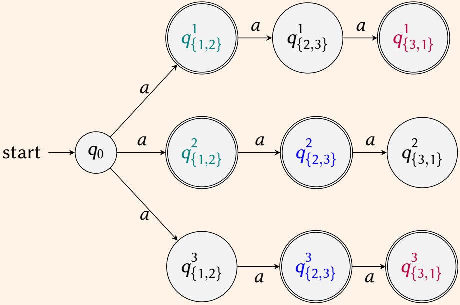

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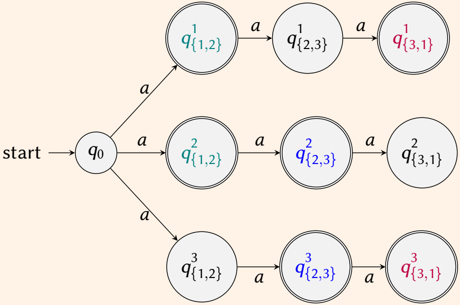

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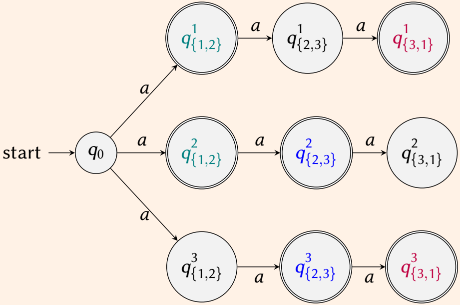

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Cannot do better that $\frac{2}{3}$

Hierarchy of λ-resolvability

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Theorem

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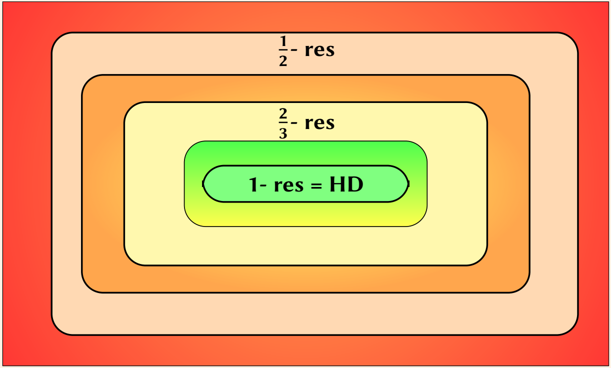

Set of all NFA λ-resolvable for some $\lambda > 0$

Resolvability with any threshold

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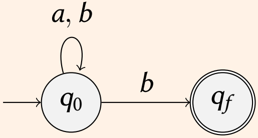

$Pr(b^i \text{ is accepted}) \to 0$

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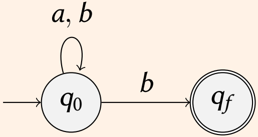

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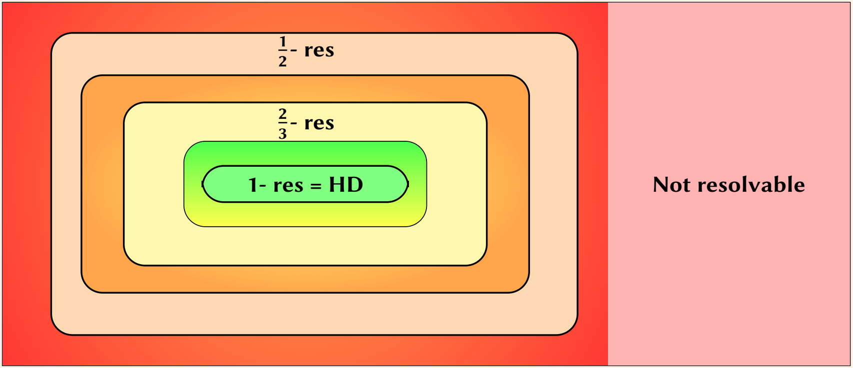

Set of all NFA

Possible questions..

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For automata over finite words

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Positive Resolvability
Given $\mathcal{A}$ $\exists \lambda \in (0, 1]$ s.t. $\mathcal{A}$ is λ-resolvable?

Complexity

Complexity

		unambiguous	**finitely-ambiguous**	**general**
Positive-resolvability	unary	NL	coNP-hard Σ_2^P	
	non-unary	NL-complete	PSPACE-complete	open
λ-**resolvability**	unary	P	coNP-hard **decidable**	open
	non-unary	NL-hard P	PSPACE-hard **decidable**	undecidable

Undecidability of λ-resolvability

Reduction from universality of PFA

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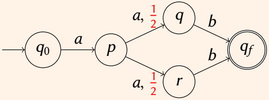

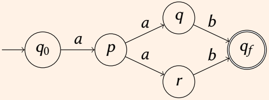

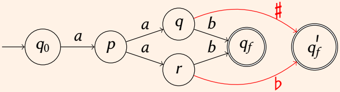

Optimal resolver $\mathcal{R}$ should have same support as PFA

Undecidability of λ-resolvability

Reduction from universality of PFA

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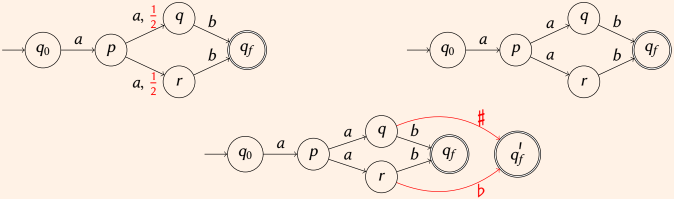

Optimal resolver $\mathcal{R}$ should have same support as PFA

Corollary : λ-RES is undecidable even for fixed λ. ($\lambda = \frac{1}{4}$)

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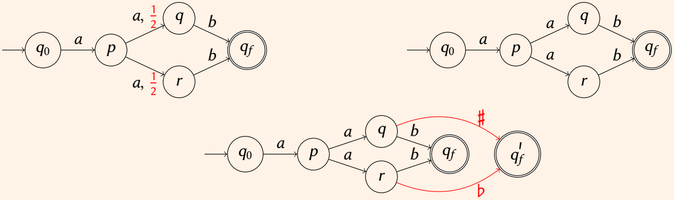

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Decidable for finitely ambiguous NFA

Positive resolvability

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Decidability still open for general case

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Decidability still open for general case

Theorem

Positive-Resolvability is decidable for

- unary NFA
- finitely ambiguous NFA

Finitely Ambiguous Automata

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k-ambiguous automata : every word w has at most k accepting runs

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Finitely ambiguous : k-ambiguous for some k
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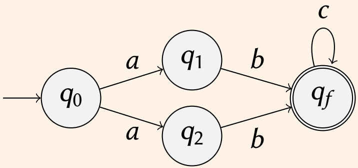
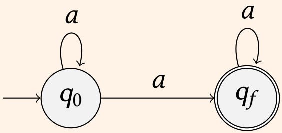


2-ambiguous



a^k has k runs

Complexity of positive resolvability

Theorem

The positive resolvability problem is

- PSPACE-complete for finitely ambiguous automata
- NL-complete for unambiguous automata

Complexity of positive resolvability

Theorem

The positive resolvability problem is

- PSPACE-complete for finitely ambiguous automata
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Next : Positive resolvability for finitely ambiguous automata

Positive resolvability

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When is a resolver $\mathcal{R}$ good for positive resolvability of $\mathcal{A}$?

Positive resolvability

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No diminishing sequence of words in resulting PFA

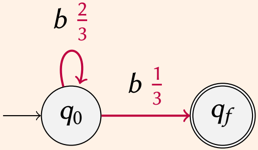
> There is no sequence of words $w_1, \ldots, w_i, \ldots$ in $\mathcal{L}(\mathcal{A})$ s.t. $\lim_{i\to\infty} Pr_{\mathcal{R}}(w) = 0$

Positive resolvability

When is a resolver $\mathcal{R}$ good for positive resolvability of $\mathcal{A}$?

No diminishing sequence of words in resulting PFA

There is no sequence of words $w_1, \ldots, w_i, \ldots$ in $\mathcal{L}(\mathcal{A})$ s.t. $\lim_{i\to\infty} Pr_{\mathcal{R}}(w) = 0$



Diminishig sequence : $b, bb, \cdots, b^i, \cdots$

$Pr_{\mathcal{R}}(b^i \text{ is accepted }) = (\frac{2}{3})^{i-1}\frac{1}{3}$

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Support of $\mathcal{R}$: $\{(q, a, q') \mid \mathcal{R}(q, a, q') > 0\}$

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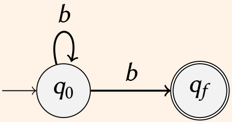
Bad support : A support over which no probability assignments works

Bad support



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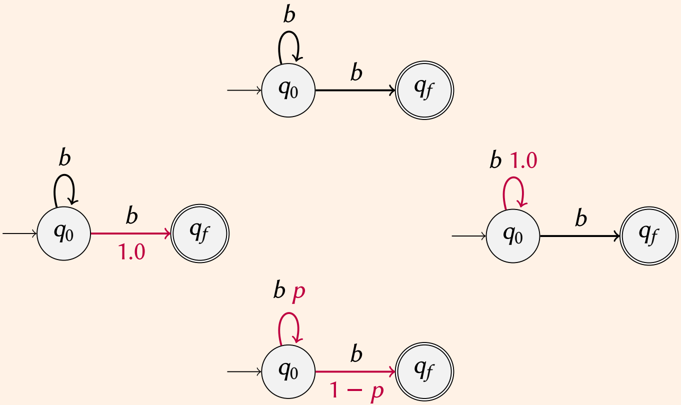

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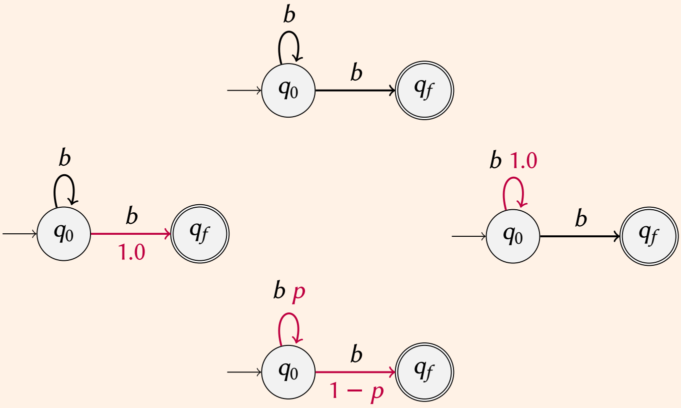

- $\mathcal{L}(\mathcal{A}) \neq \mathcal{L}(\mathcal{A}_{\mathcal{S}})$
- Some condition equivalent to existence of diminishing sequence

Idea behind algorithm

Check if support is bad using
the two conditions

Unambiguous case : Positive resolvability

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First step : trim the automata

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Now there is unique support S, with $\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathcal{A}_{\mathcal{S}})$

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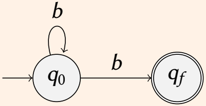

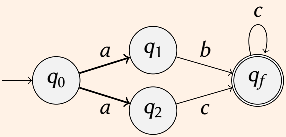

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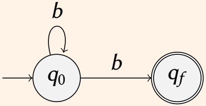

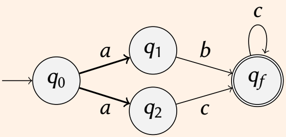

Positive resolvability for unambiguous automata is in NL

Finitely ambiguous case : Positive resolvability

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Generalise bad support from unambiguous to k-ambiguous

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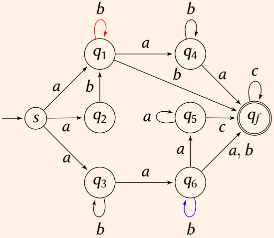

a b b a b b a c

$s \to q_1 \to q_1 \to q_1 \to q_4 \to q_4 \to q_4 \to q_f \to q_f$

$s \to q_2 \to q_1 \to q_1 \to q_4 \to q_4 \to q_4 \to q_f \to q_f$

$s \to q_3 \to q_3 \to q_3 \to q_6 \to q_6 \to q_6 \to q_5 \to q_f$

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4-ambiguous automata

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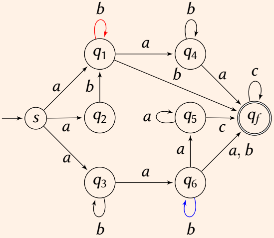

	a	b	b	a	b	b	a	c
s	$\to q_1$	$\to q_1$	$\to q_1$	$\to q_4$	$\to q_4$	$\to q_4$	$\to q_f$	$\to q_f$
s	$\to q_2$	$\to q_1$	$\to q_1$	$\to q_4$	$\to q_4$	$\to q_4$	$\to q_f$	$\to q_f$
s	$\to q_3$	$\to q_3$	$\to q_3$	$\to q_6$	$\to q_6$	$\to q_6$	$\to q_5$	$\to q_f$
s	$\to q_3$	$\to q_3$	$\to q_3$	$\to q_6$	$\to q_6$	$\to q_6$	$\to q_f$	$\to q_f$

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Diminishing sequence : $abb^i abb^i ac$

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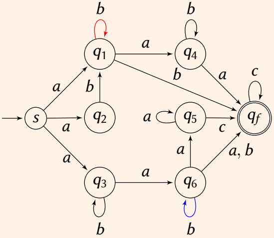

	a	b	b	a	b	b	a	c
s	$\to q_1$	$\to q_1$	$\to q_1$	$\to q_4$	$\to q_4$	$\to q_4$	$\to q_f$	$\to q_f$
s	$\to q_2$	$\to q_1$	$\to q_1$	$\to q_4$	$\to q_4$	$\to q_4$	$\to q_f$	$\to q_f$
s	$\to q_3$	$\to q_3$	$\to q_3$	$\to q_6$	$\to q_6$	$\to q_6$	$\to q_5$	$\to q_f$
s	$\to q_3$	$\to q_3$	$\to q_3$	$\to q_6$	$\to q_6$	$\to q_6$	$\to q_f$	$\to q_f$

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Diminishing sequence : abb^iabb^iac

Need to conserve number of runs in the pumped words

Need to store more

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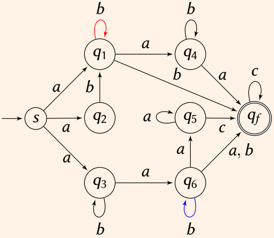

		a	b	b	a	b	b	a	c
	s	q_1	q_1	q_1	q_4	q_4	q_4	q_f	q_f
	s	q_2	q_1	q_1	q_4	q_4	q_4	q_f	q_f
	s	q_3	q_3	q_3	q_6	q_6	q_6	q_5	q_f
	s	q_3	q_3	q_3	q_6	q_6	q_6	q_f	q_f
R	$\varnothing$	$\varnothing$	$\{q_f\}$	$\{q_f\}$	$\varnothing$	$\{q_f\}$	$\{q_f\}$	$\varnothing$	$\varnothing$

R stores states from which there is no accepting run of suffix after reading prefix from start state

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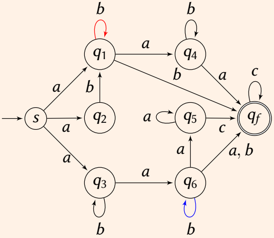

		a		b		b		a		b		b		a		c	
	s	→	q_1	→	q_1	→	q_1	→	q_4	→	q_4	→	q_4	→	q_f	→	q_f
	s	→	q_2	→	q_1	→	q_1	→	q_4	→	q_4	→	q_4	→	q_f	→	q_f
	s	→	q_3	→	q_3	→	q_3	→	q_6	→	q_6	→	q_6	→	q_5	→	q_f
	s	→	q_3	→	q_3	→	q_3	→	q_6	→	q_6	→	q_6	→	q_f	→	q_f
R	$\varnothing$		$\varnothing$		$\{q_f\}$		$\{q_f\}$		$\varnothing$		$\{q_f\}$		$\{q_f\}$		$\varnothing$		$\varnothing$

R stores states from which there is no accepting run of suffix after reading prefix from start state

> Bad support S : every run has non-det transition in some SCC of product $\mathcal{A}_S^k \times Q$ under this transition system

PSPACE algorithm

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- Guess support
- Guess a short word in the transition system witnessing bad support

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Ambiguity can be exponential

Store useful abstractions of the system and guess word on the fly

Automata on Infinite words ?

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Theorem

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Independent work for 1-resolvability (almost sure acceptance)
[Henzinger, Prakash, Thejaswini'25]

Application : 1-resolvable Büchi automaton used in faster Markov Chain verification for UBA specifications
[Li, P, Schewe, Tang'25]

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Given unary $\mathcal{A}$, is $\mathcal{A}$ λ-resolvable?

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Decidable for unary : analysis of periodic behaviour of support matrix

Requires analysis of matrices obtained as product of support matrices of each letter

What's next?

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- Closing complexity gaps
- Other models : Pushdown, Timed Automata, VASS,...
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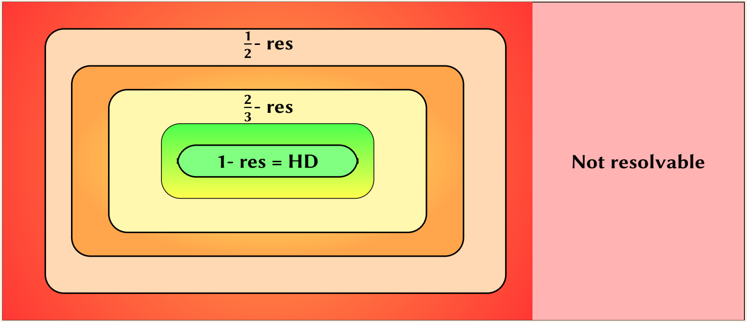
Thank You

Summary : Automata on finite words



		unambiguous	finitely-ambiguous	general
Positive-resolvability	unary	NL	coNP-hard Σ_2^P	
	non-unary	NL-complete	PSPACE-complete	open
λ-resolvability	unary	P	coNP-hard **decidable**	open
	non-unary	NL-hard P	PSPACE-hard **decidable**	undecidable