On Linear Time Decidability of Differential Privacy for Programs with Unbounded Inputs

Joint work with Rohit Chadha and Prasad Sistla

Mahesh Viswanathan, IARCS Verification Seminar Series, October 2022

Accessing Sensitive Information

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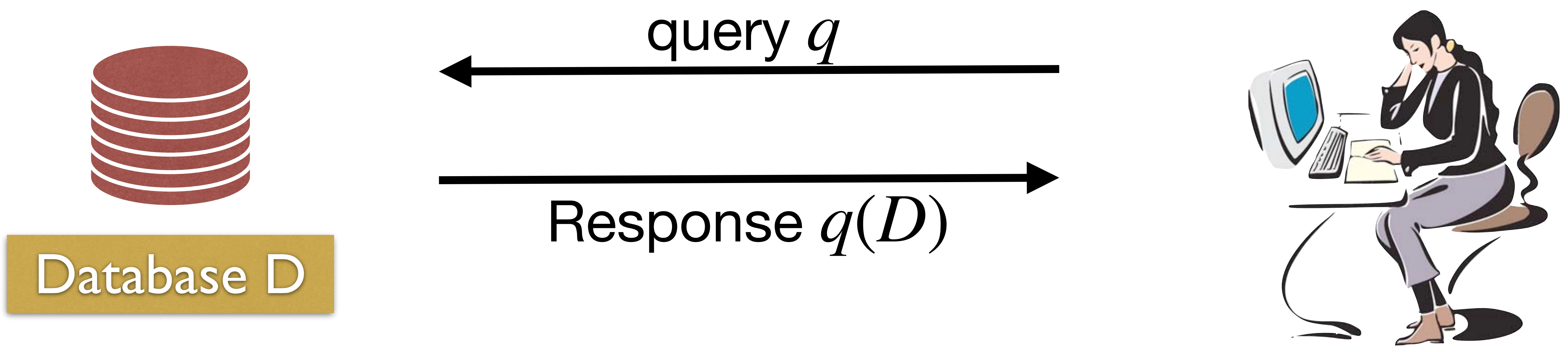

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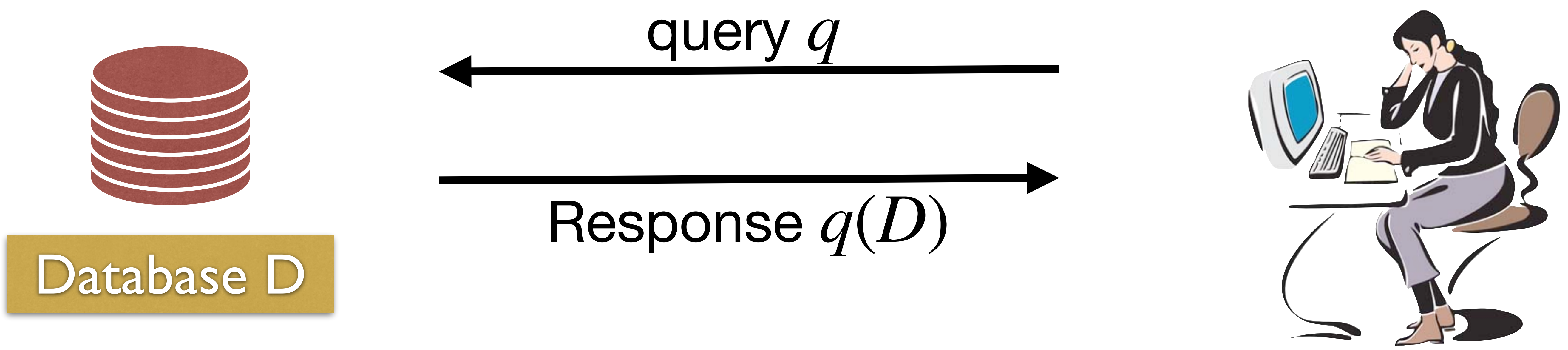

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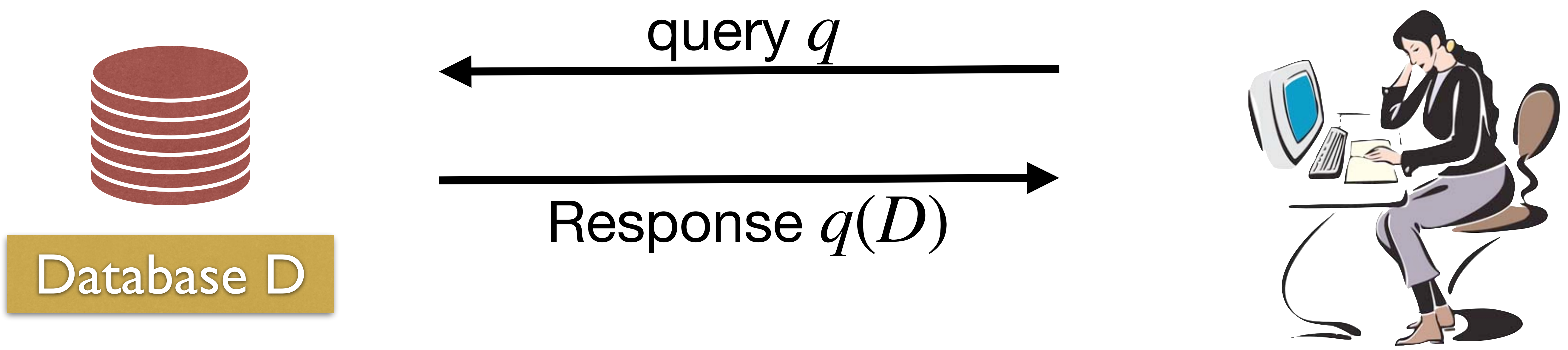

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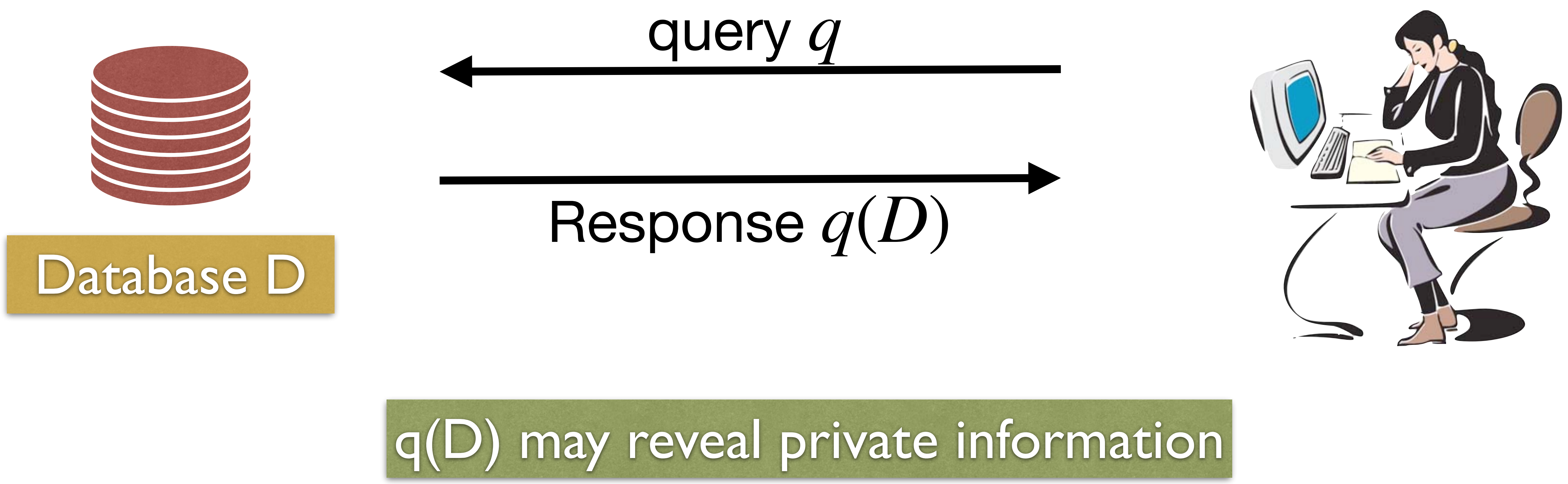

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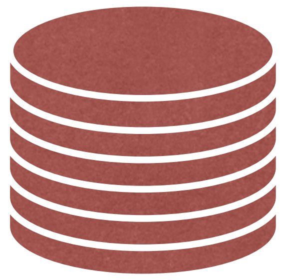
- [Dinur-Nissim 2003] Entire database can be reconstructed using answers to a few random aggregate queries

Differential Privacy Framework

Dwork, McSherry, Nissim, Smith 2006



Database D

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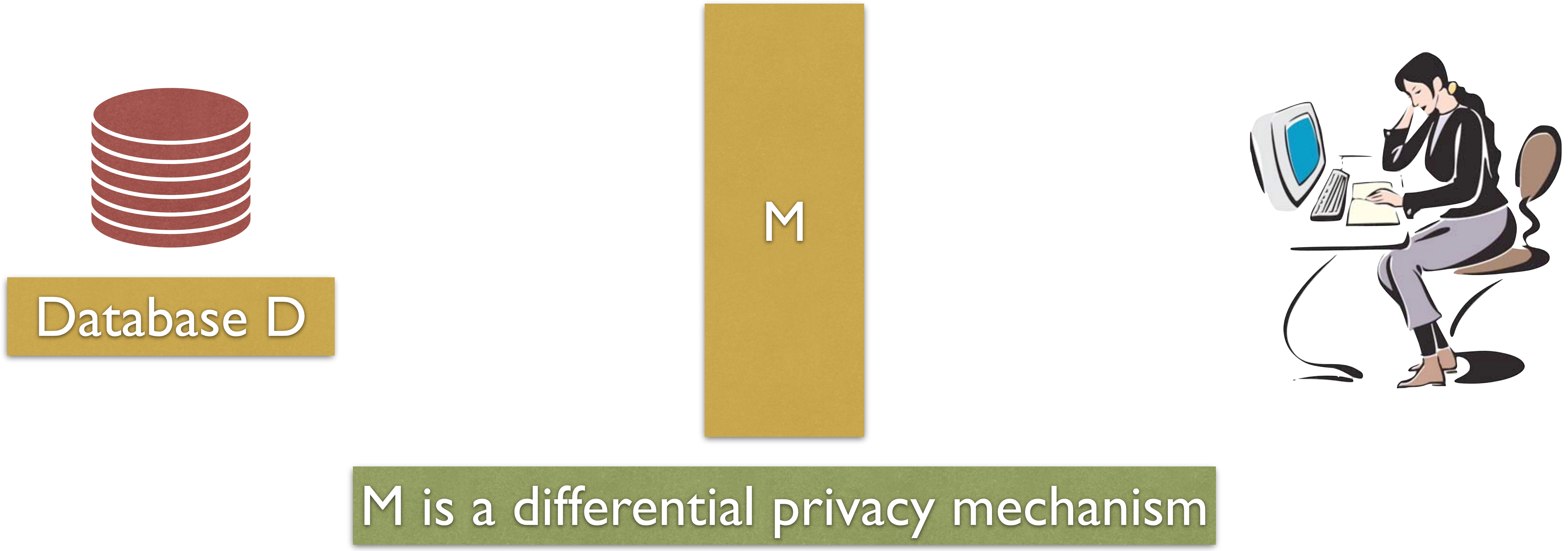

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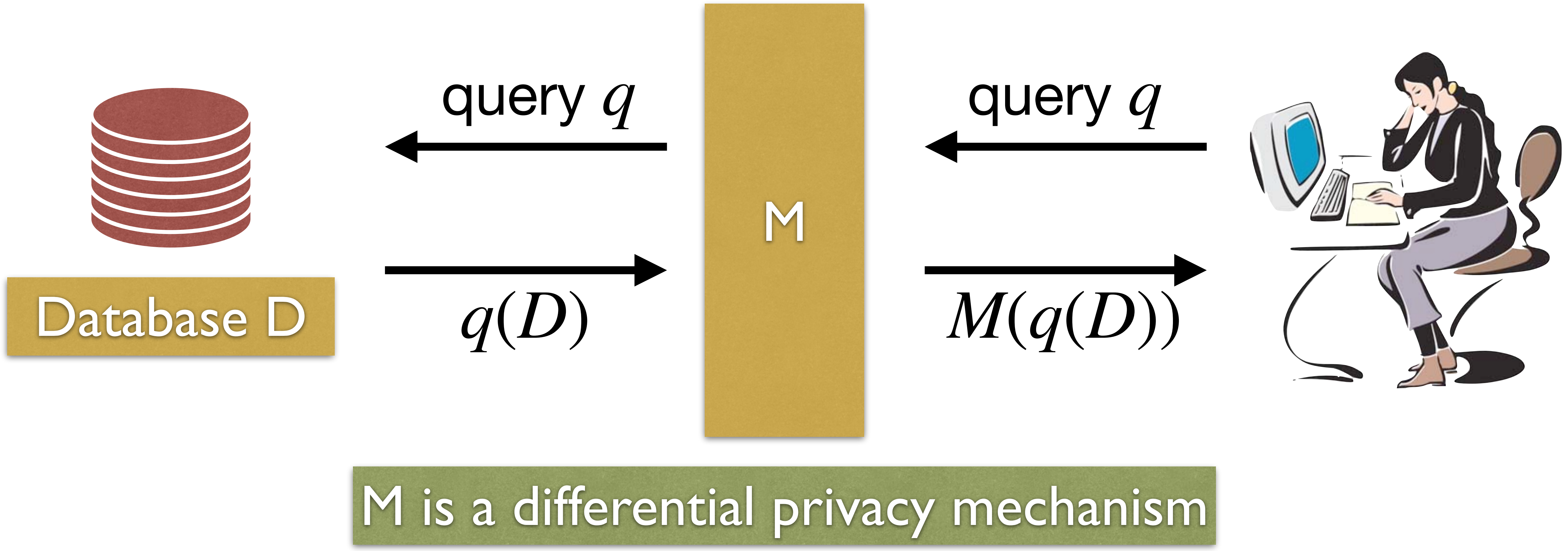

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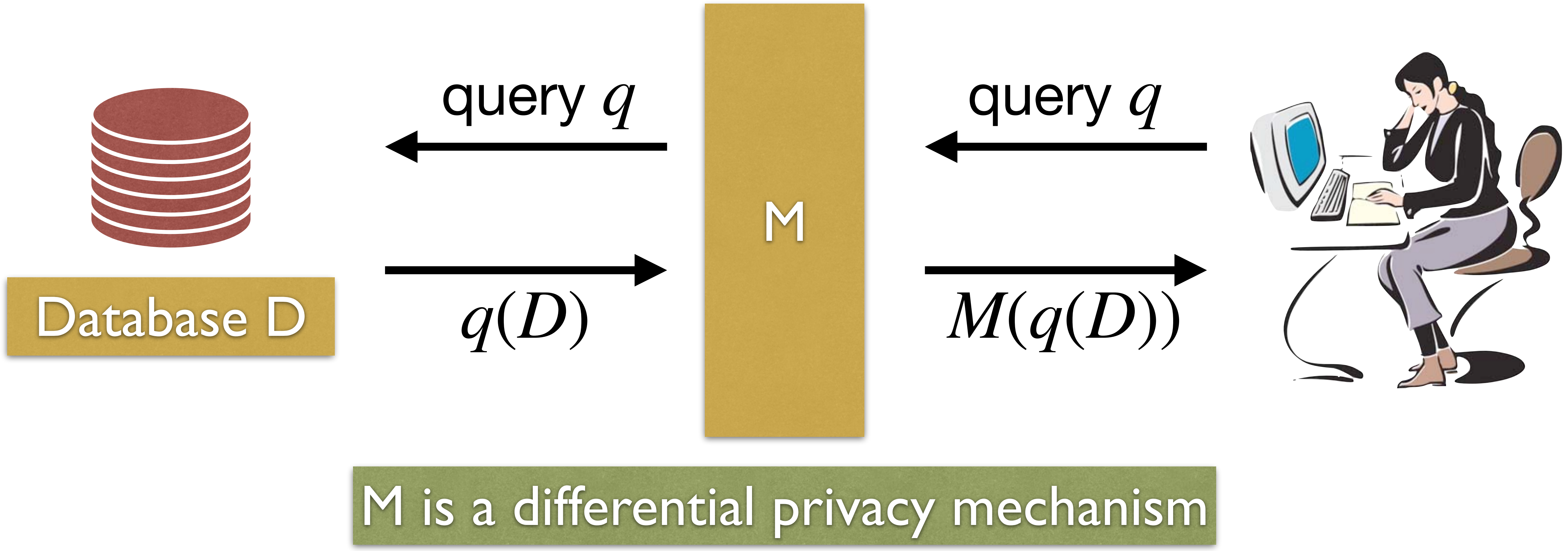

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- Add noise to $q(D)$ and share the noisy response $M(q(D))$
- Trade "accuracy" for "privacy"

Randomized Response

An Example

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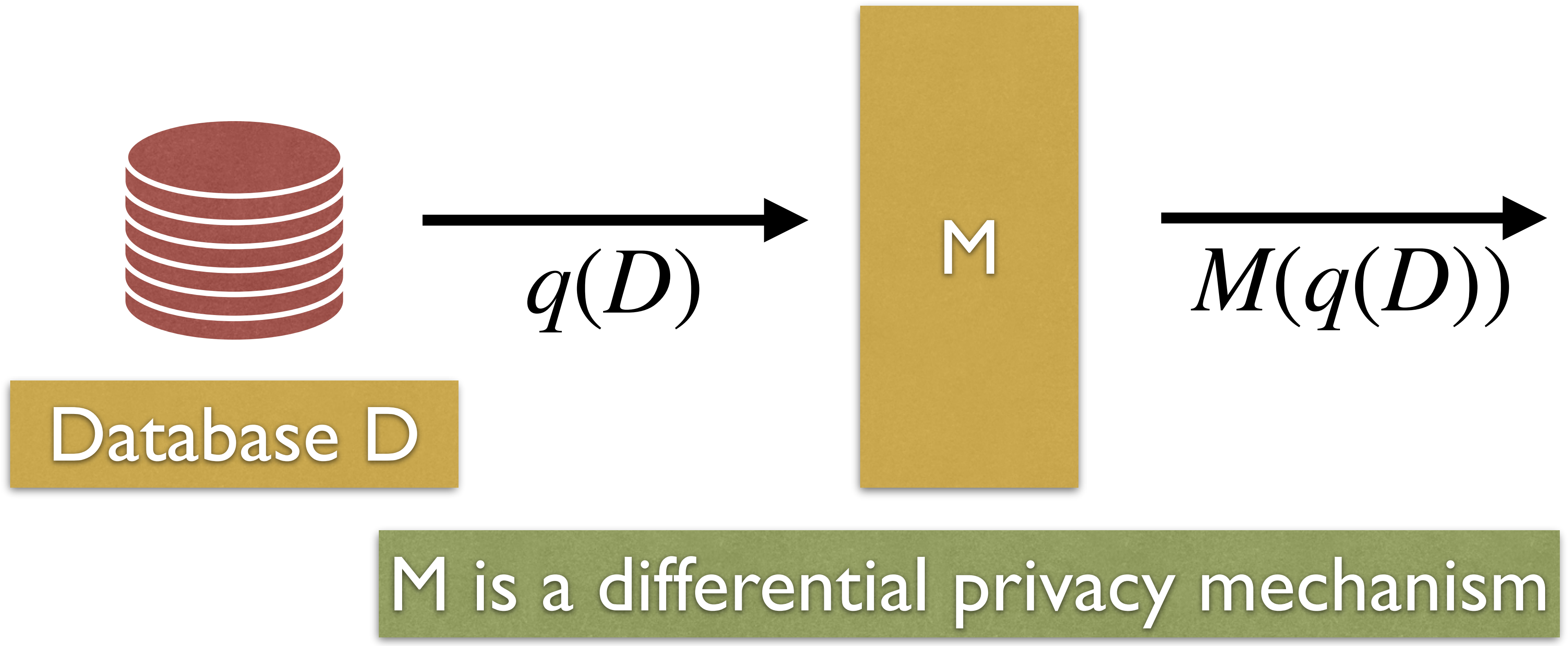
- “Privacy” arises from the plausible deniability of any outcome
- If p is the fraction of smokers in a population, then the expected number of “Yes” responses is $(1/4)(1-p) + (3/4)p = (1/4) + (p/2)$

Differential Privacy

Definition [Dwork, McSherry, Nissim, Smith 2006]



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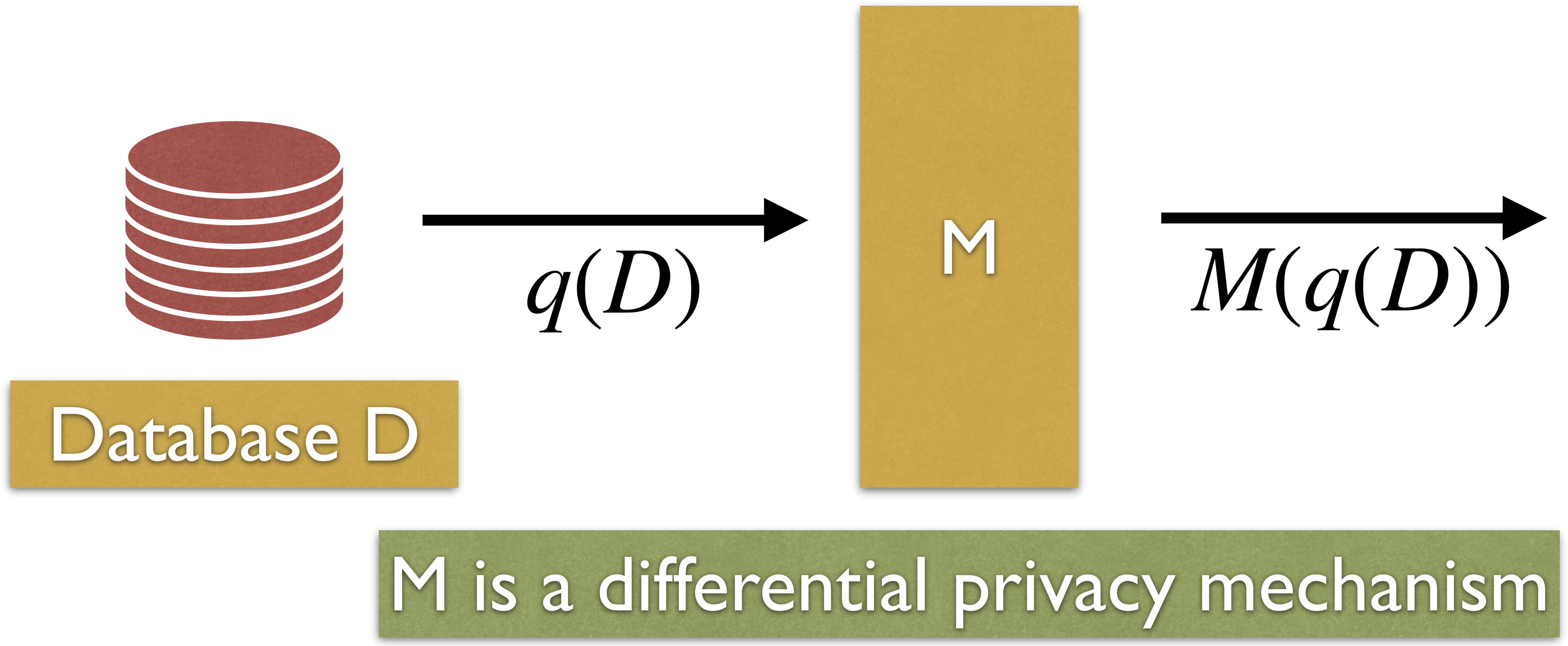

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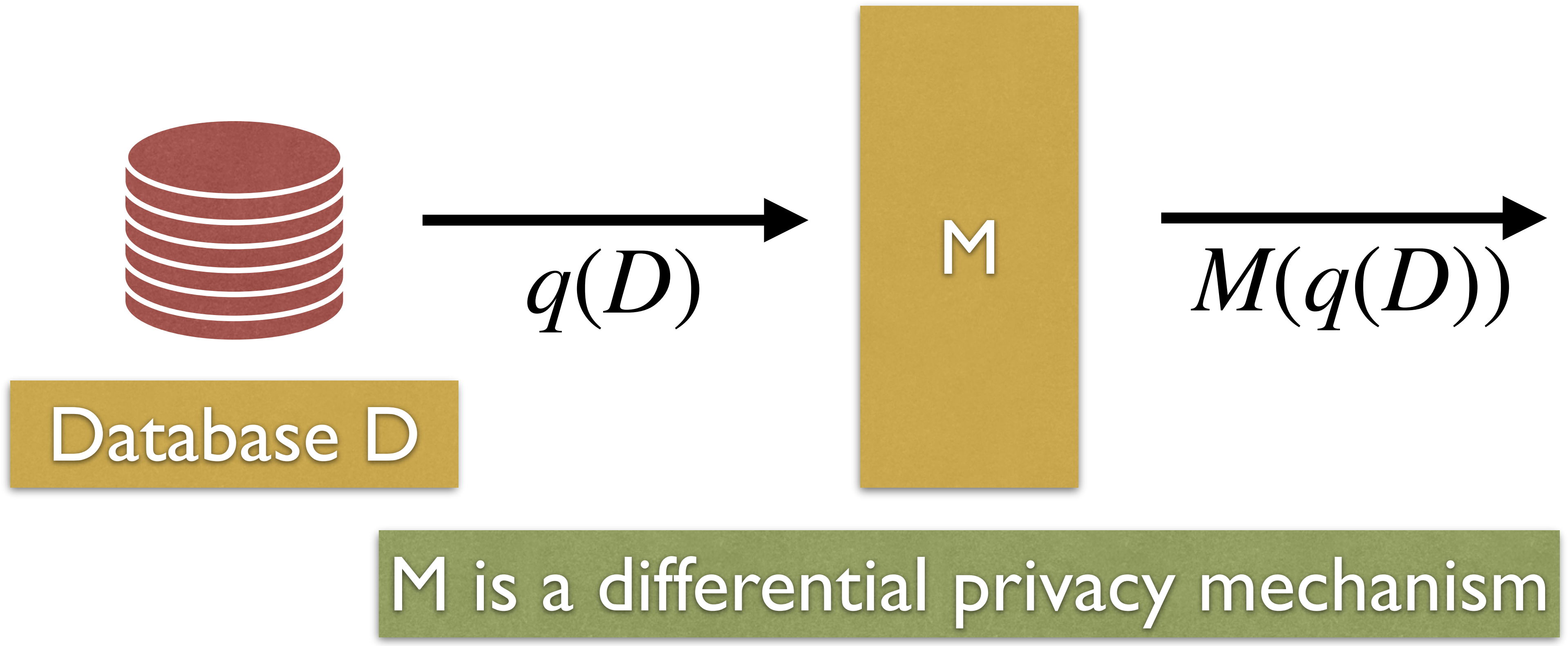

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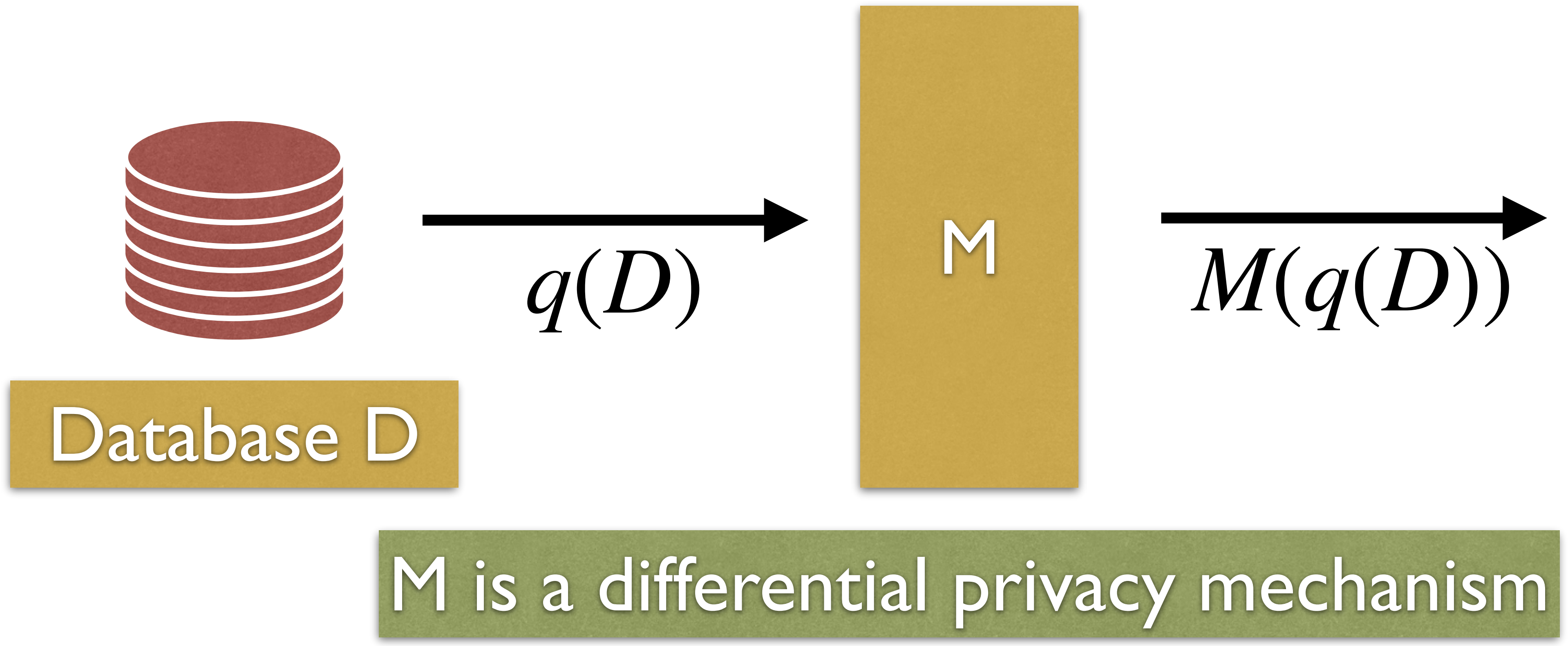

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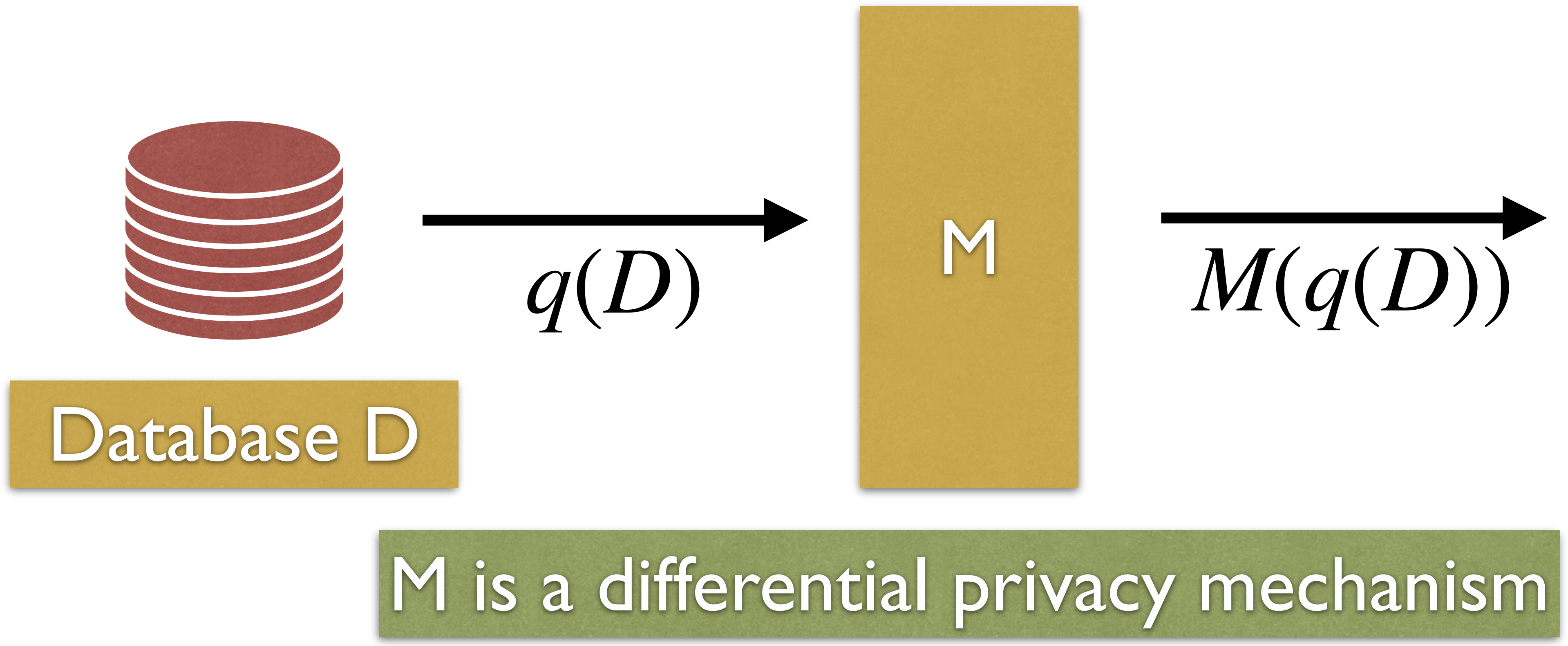

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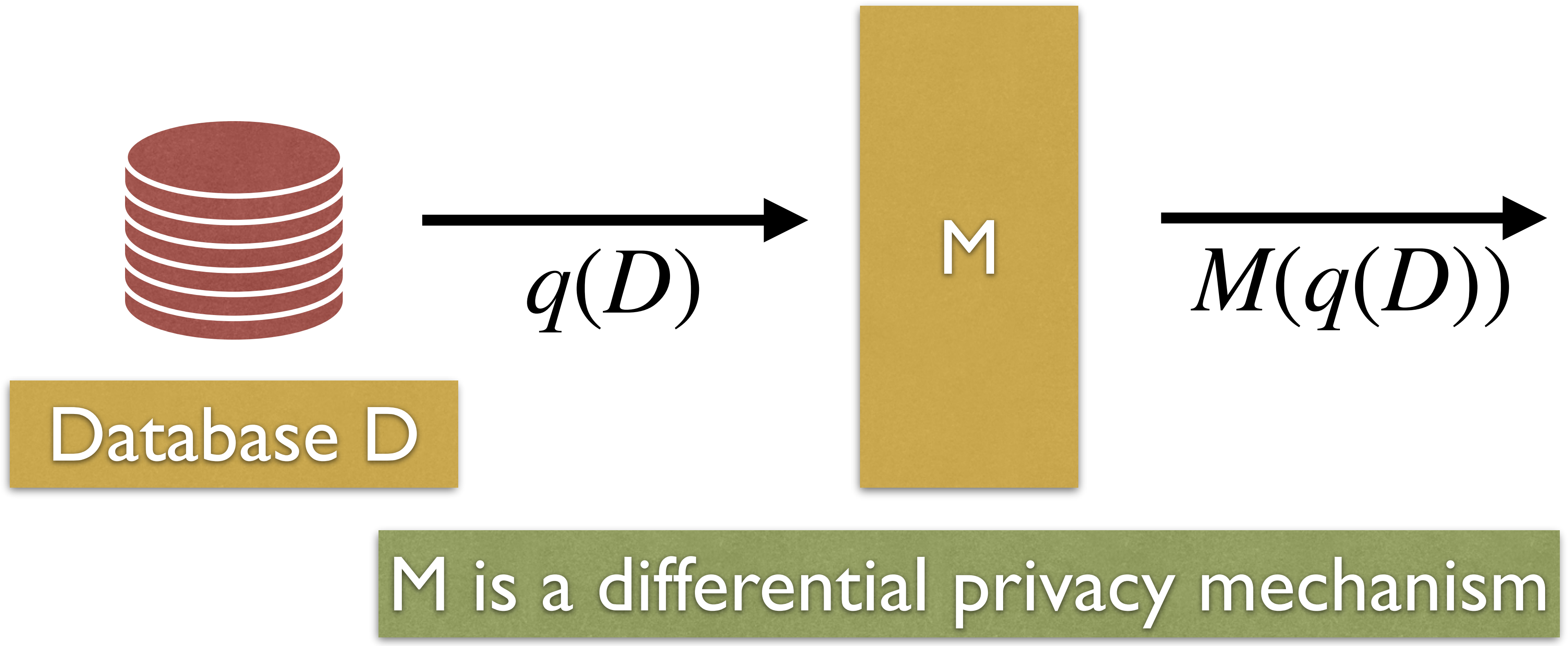

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Adjacency: Inputs q_1 and q_2 are adjacent if $|q_1[i] - q_2[i]| \leq 1$.

Definition: M is $d\epsilon$-differentially private if for any pair of adjacent inputs q_1, q_2 and each subset S of outputs

$$\Pr(M(q_1) \in S) \leq e^{d\epsilon} \Pr(M(q_2) \in S)$$

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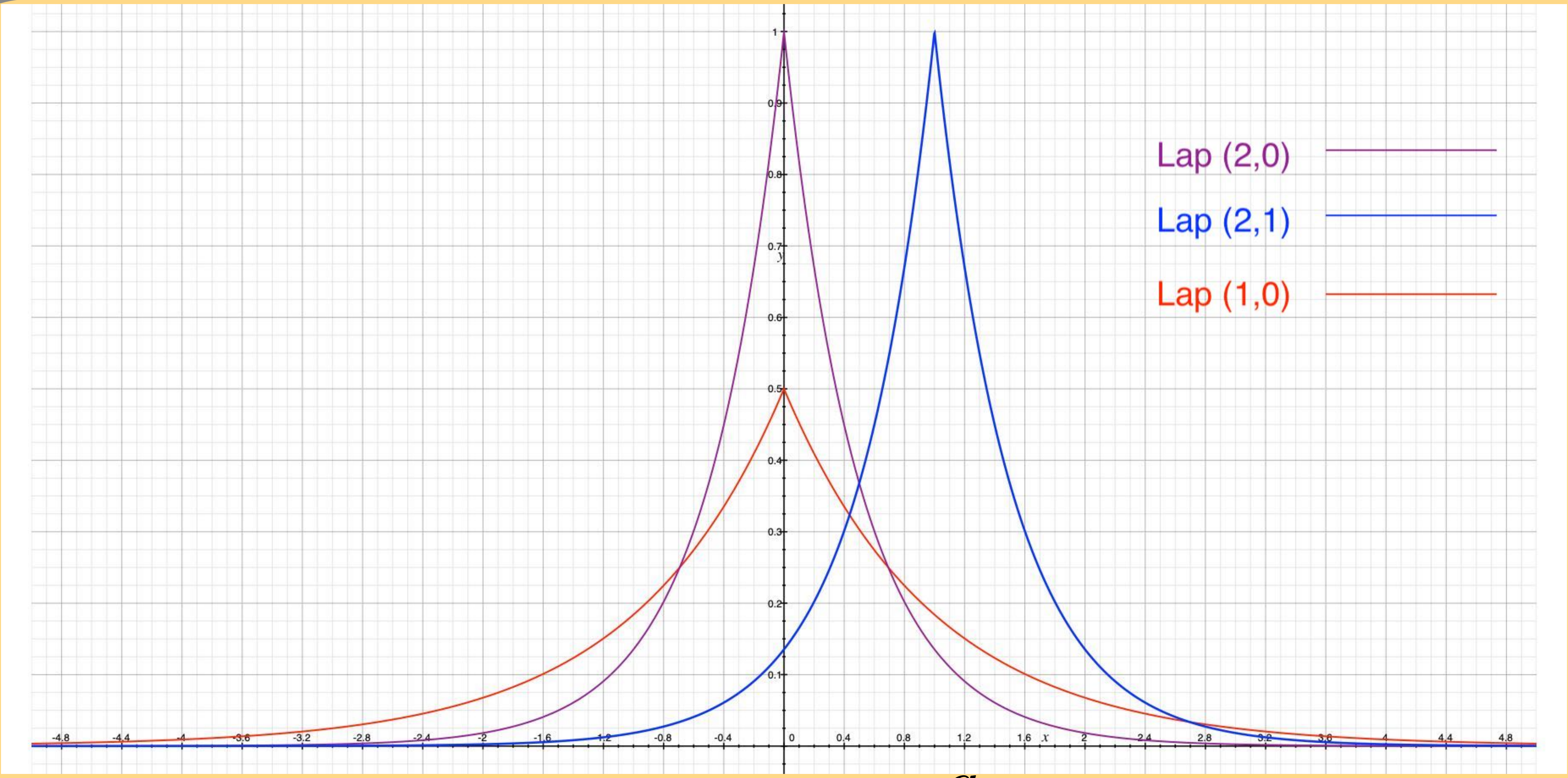

pdf of Lap(a, μ) is $\frac{a}{2}e^{-a|x-\mu|}$

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$\forall \epsilon$, for all adjacent inputs q_1, q_2, for all $o \in \bot^* \top$

$\Pr[\mathrm{SVT}(q_1) = o] \leq e^{\epsilon} \Pr[\mathrm{SVT}(q_2) = o]$

Ensuring Privacy is subtle

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For all d, not $d\epsilon$-differentially private.

[BCJSV20] Given a program $M(\epsilon)$, the problem of determining if it is $d\epsilon$-differentially private is undecidable.

This Talk

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An automaton model to describe some differential privacy algorithms

- The model processes an (unbounded) stream of real-valued query answers (input) and produces a stream of outputs (finite or real-valued)
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Identify necessary and sufficient conditions when an automaton is differentially private

- Necessary and sufficient conditions can be checked in linear time
- Algorithm certifies differential privacy or produces counter-examples

Prior Work

- Constructing Privacy proofs: [Reed, Pierce 2010], [Gaboardi, Haeberlen, Hsu, Narayan, Pierce 2013], [Barthe, Kopf, Olmedo, Zanella-Beguelin 2013], [Barthe, Gaboardi, Gregoire, Hsu, Strub 2016], [Zhang, Kifer 2017], [Albarghouthi, Hsu 2018], [Wang, Ding, Wang, Kifer, Zhang 2019], [de Amorim, Gaboardi, Hsu, Katsumata 2019]

- Discovering privacy bugs: [Ding, Wang, Wang, Zhang, Kifer 2018], [Bichsel, Gehr, Drechsler-Cohen, Tsankov, Vechev 2018], [Wang, Ding, Kifer, Zhang 2020]

- Decision Procedures: [Barthe, Chadha, Jagannath, Sistla, V. 2019]

- **This talk:** Decision procedure for unbounded inputs

Differentially Private Automata (DiPA)

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- If an input is read, then control state is changed based on a comparison between insample and stored value x. The transition has an output.
- Based on the transition, the stored value x maybe be updated to insample.

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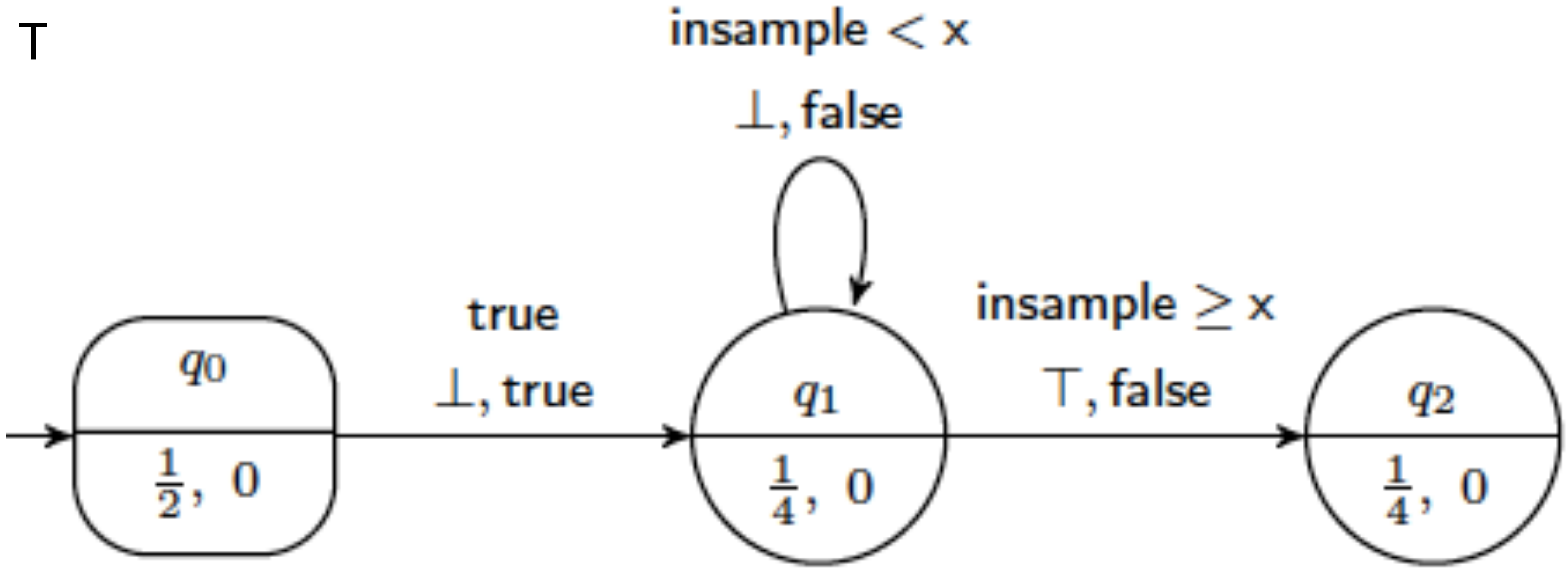

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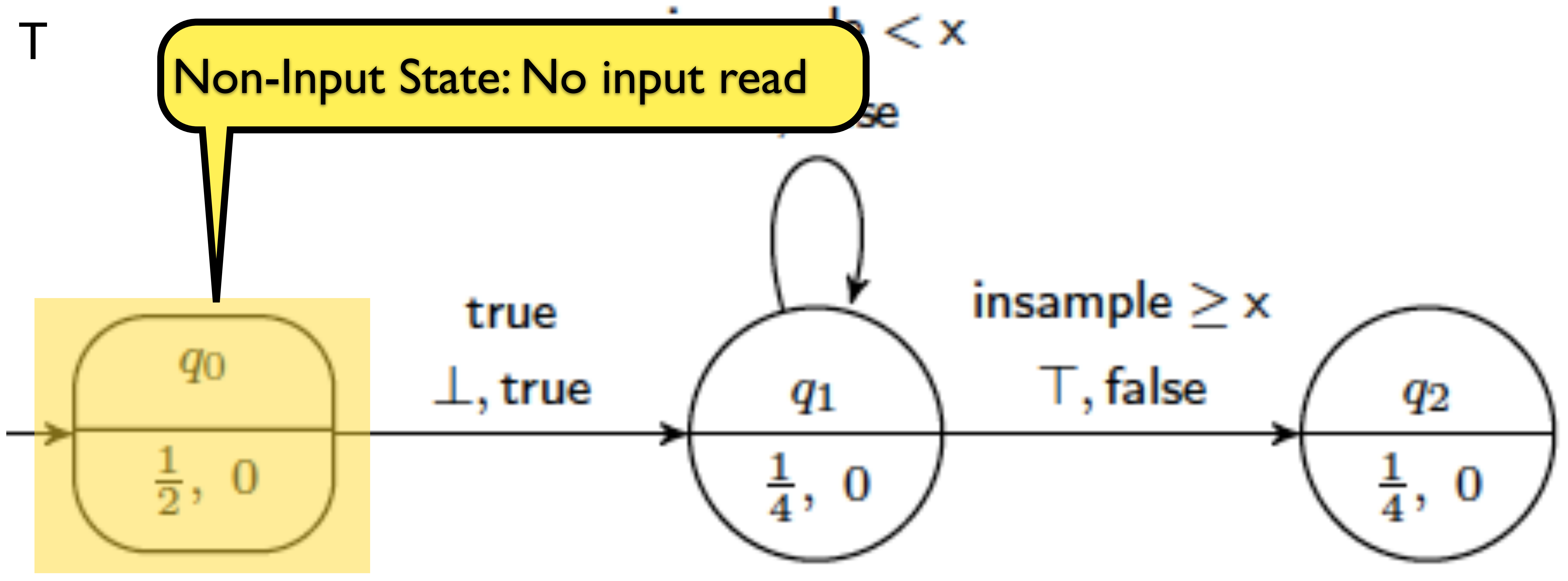

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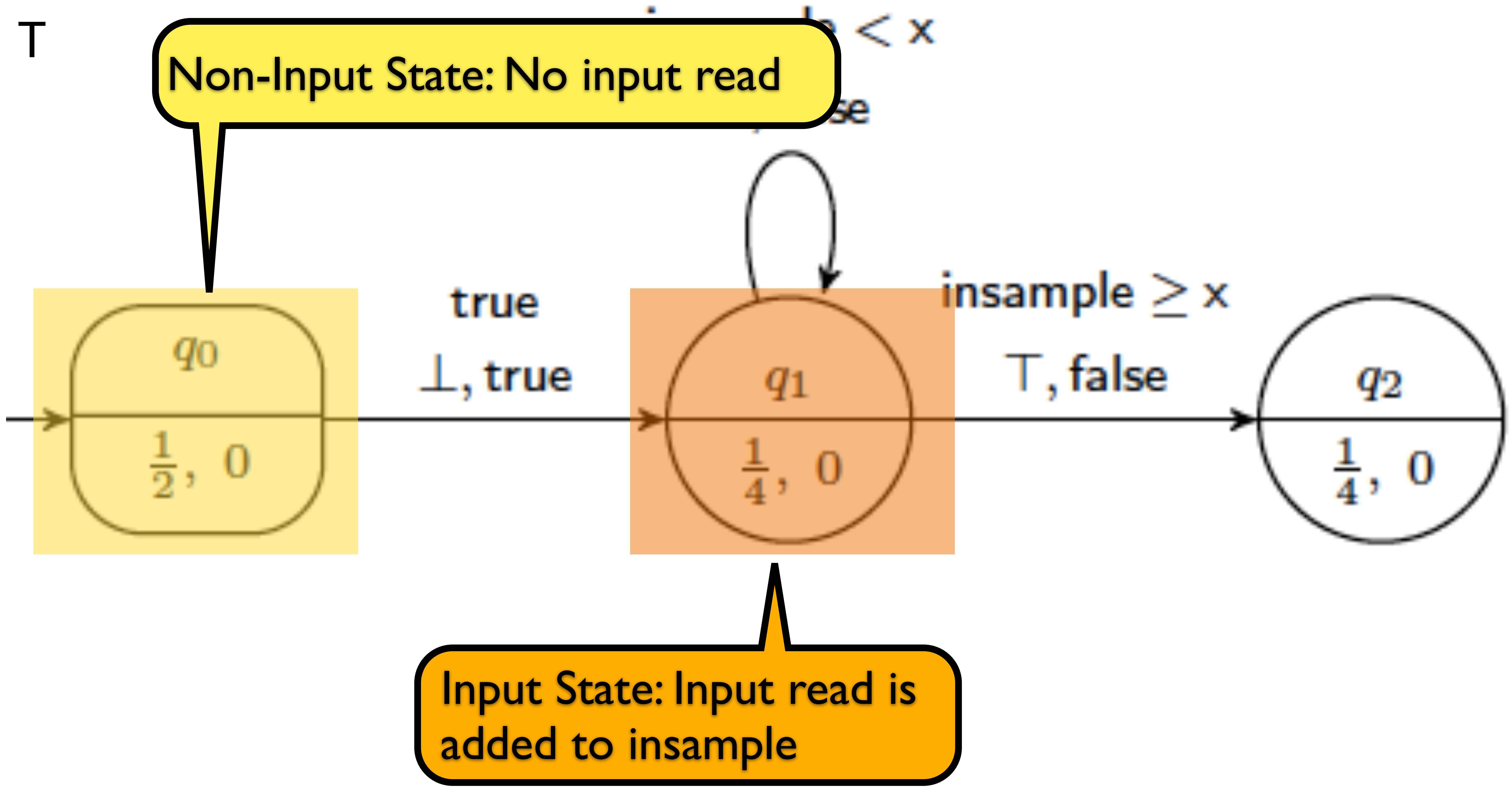

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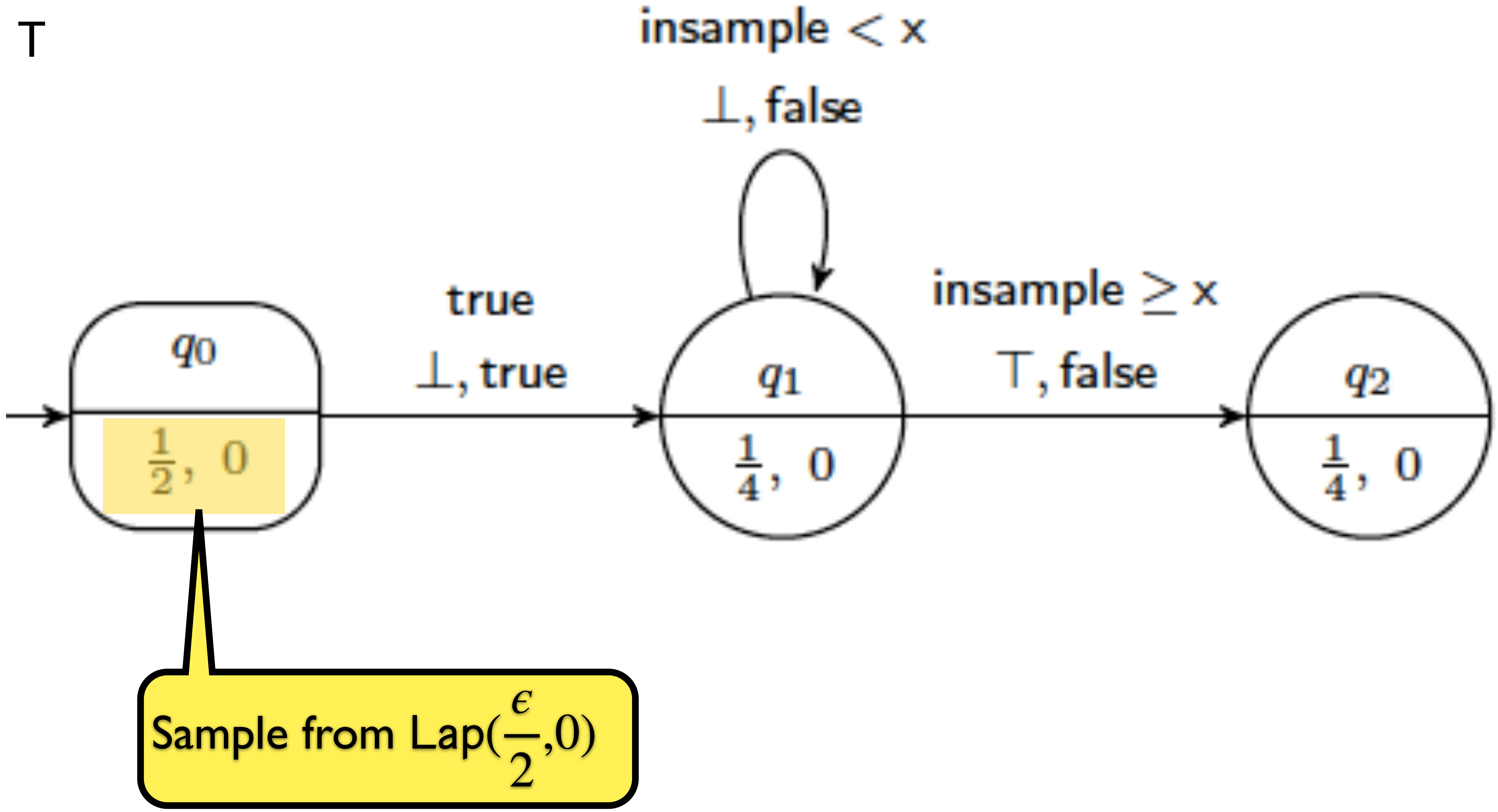

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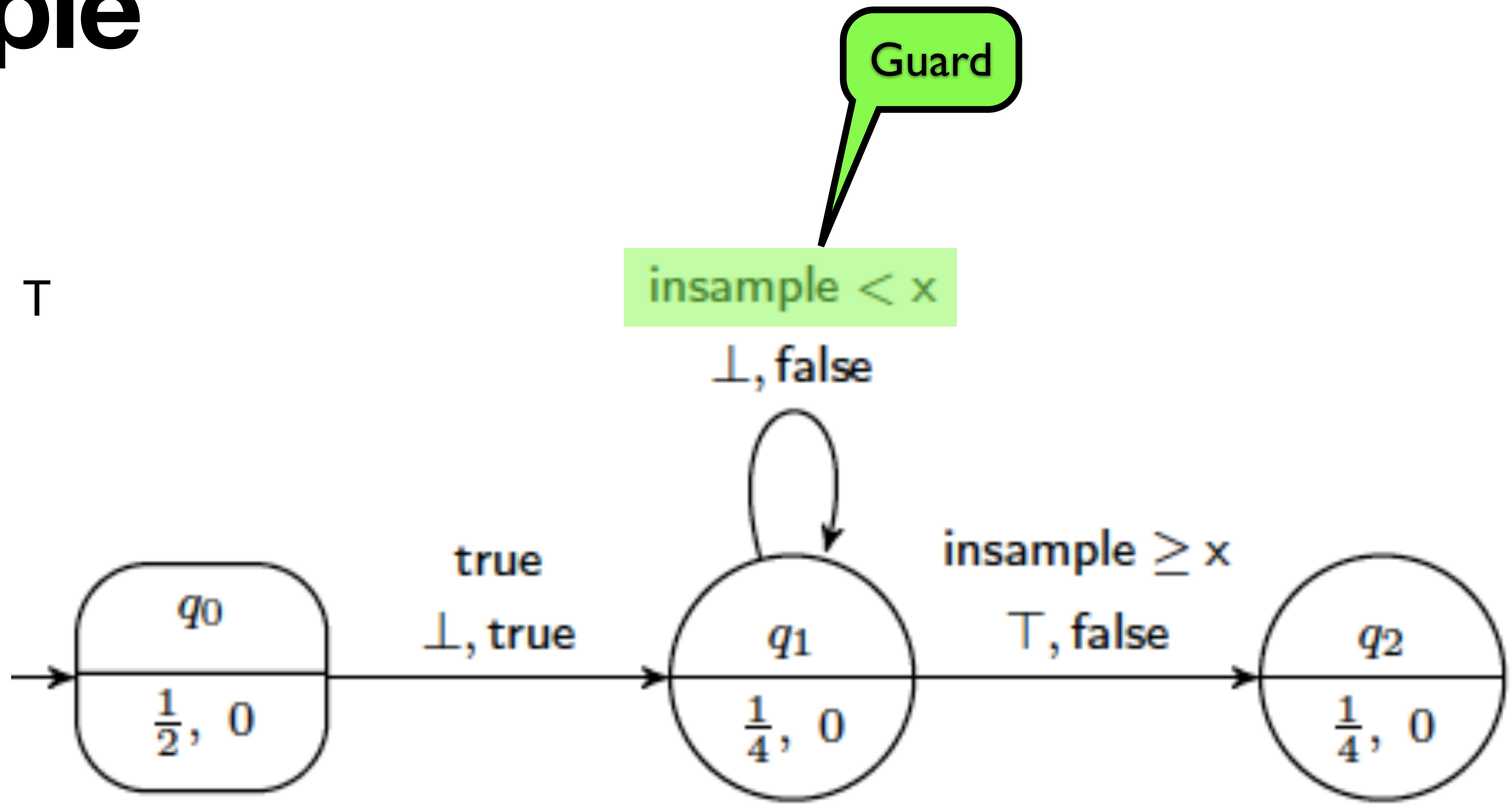

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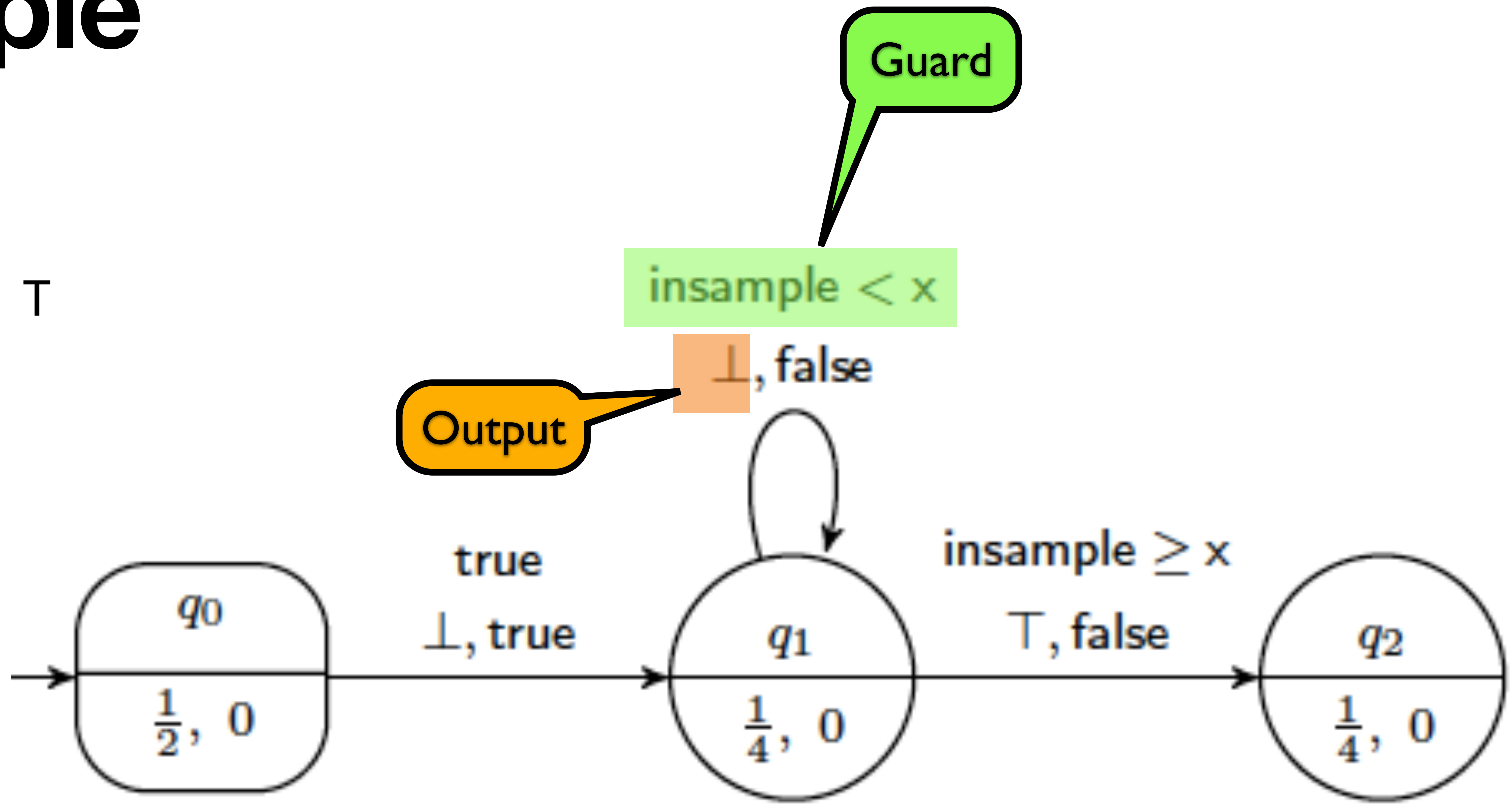

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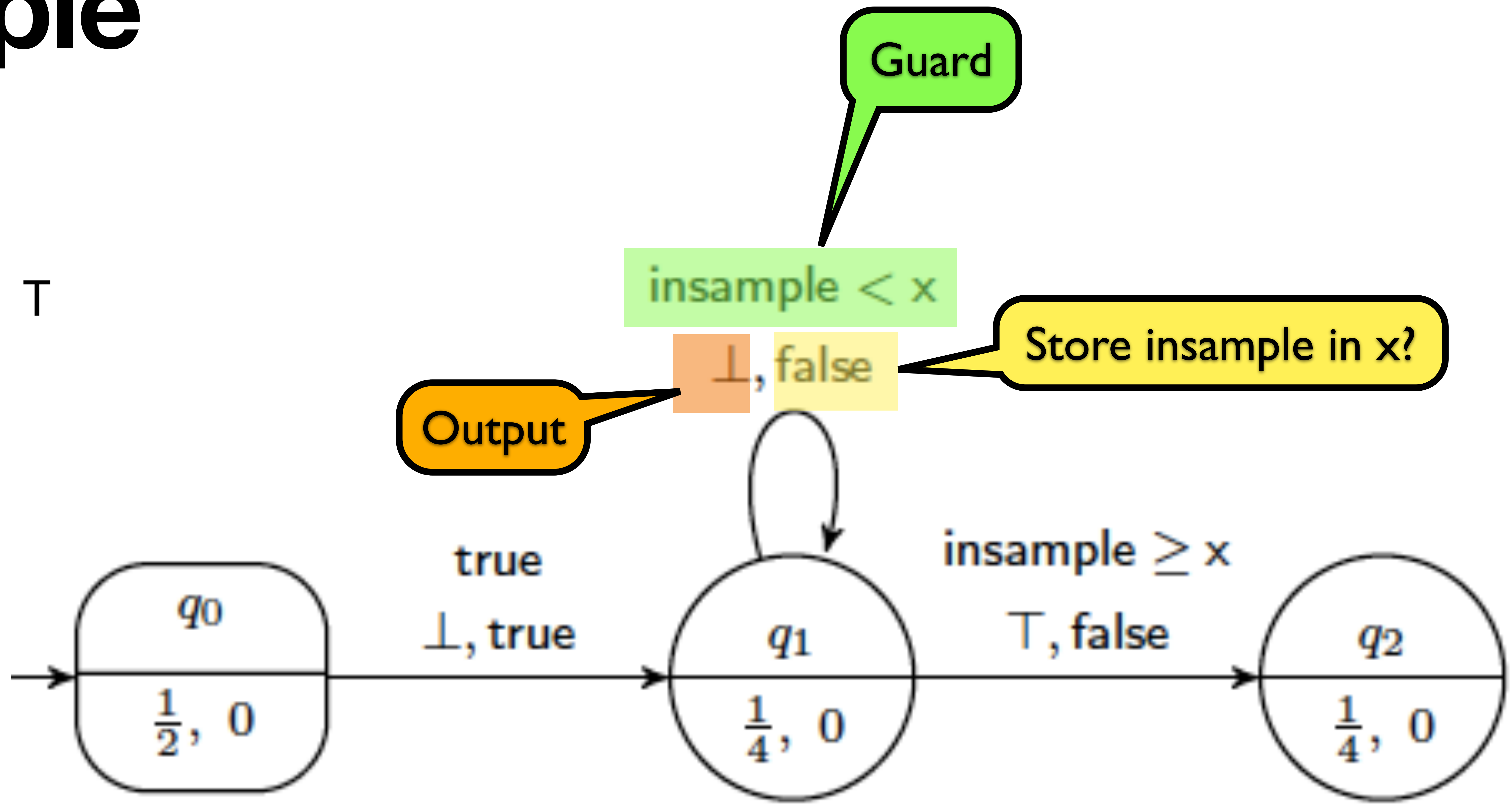

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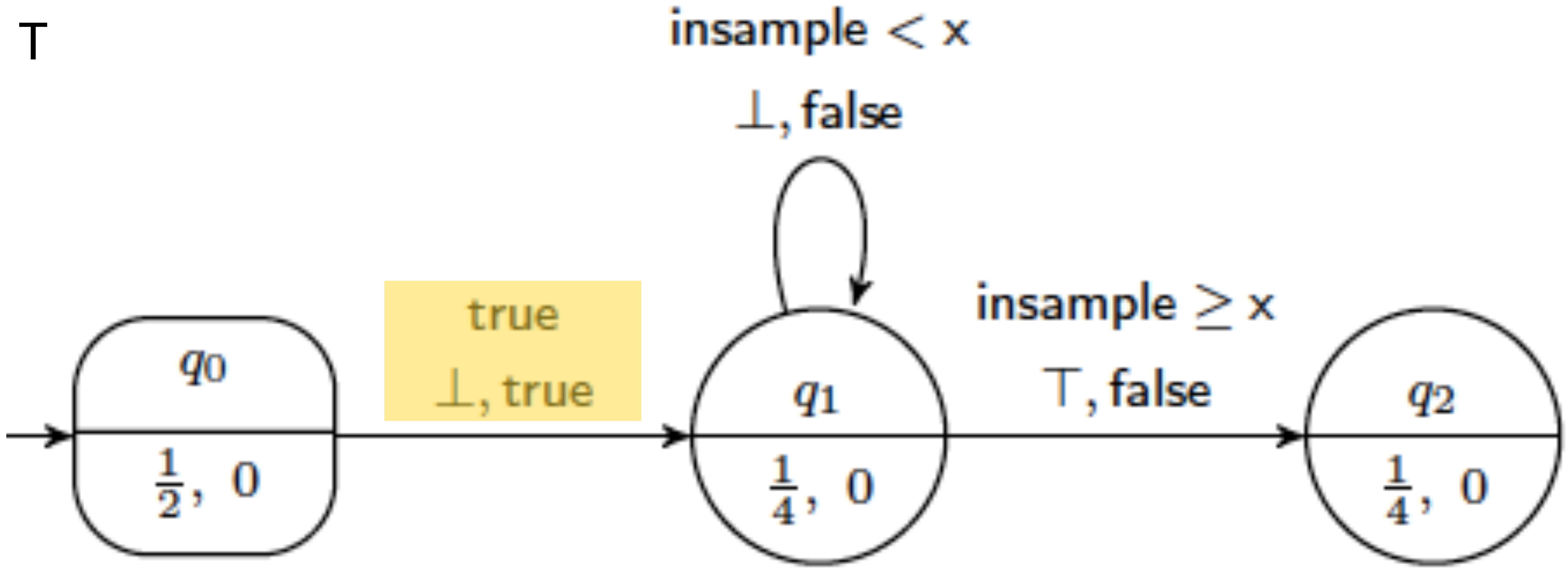

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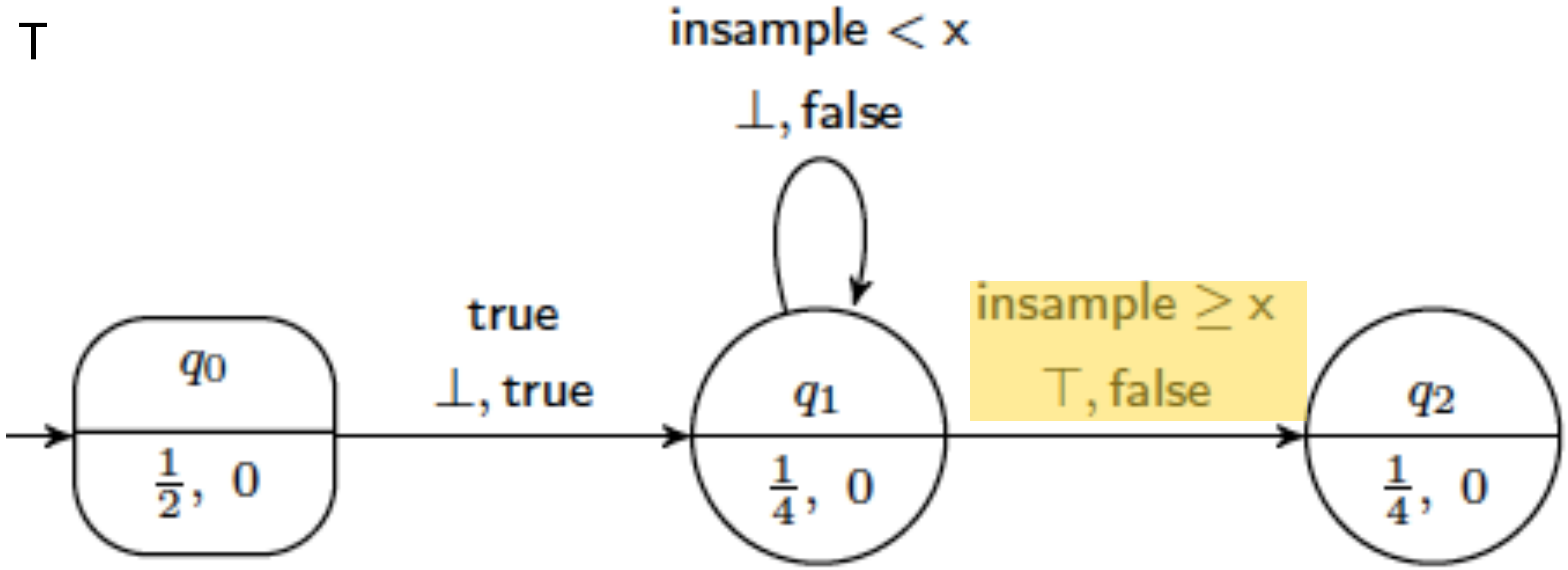

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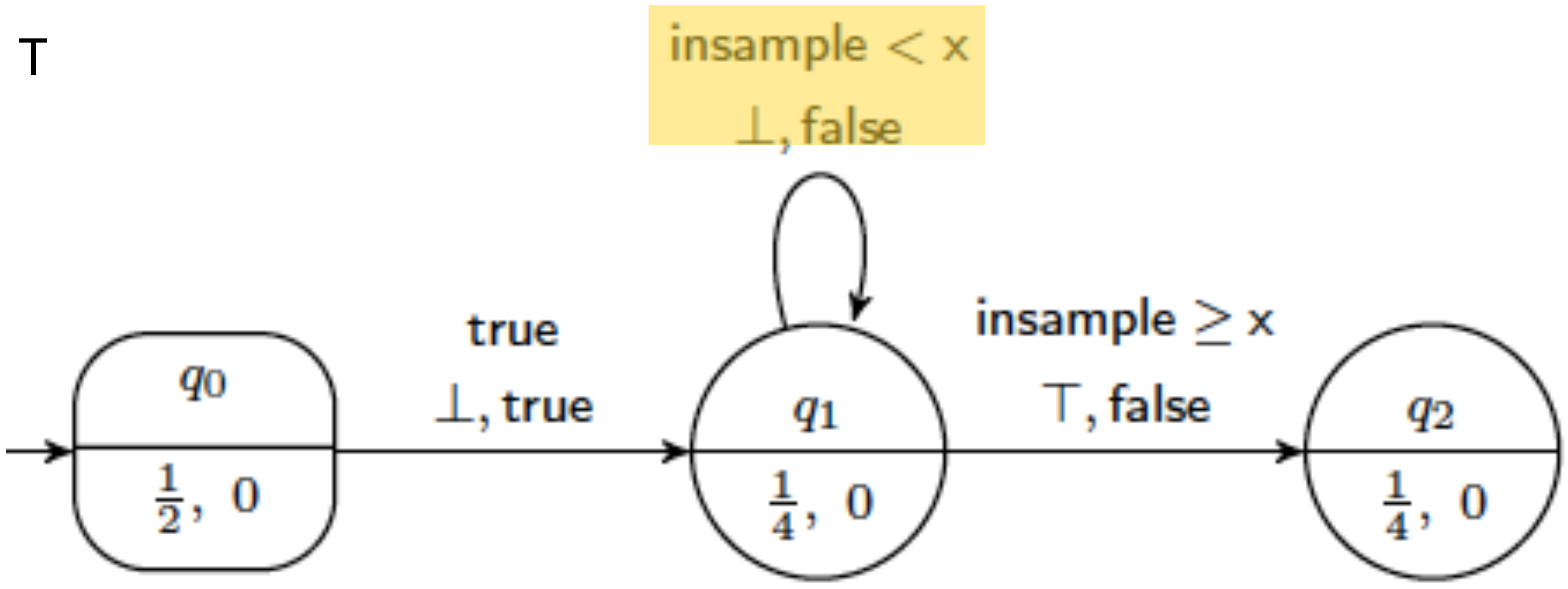

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Probability: For path ρ from initial state, Pr$[\epsilon_0, \rho]$ is the probability when security parameter is ϵ_0

Computational Problem and Results

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$d\epsilon$-Differential Privacy: DiPA $\mathscr{A}$ is $d\epsilon$-differentially private (for $d > 0$) if for all $\epsilon > 0$, and any two paths ρ_1, ρ_2 starting from the initial state such that outseq$(\rho_1) =$ outseq(ρ_1) and adjacent inseq(ρ_1) and inseq(ρ_2), Pr$[\epsilon, \rho_1] \leq e^{d\epsilon}$ Pr$[\epsilon, \rho_2]$.

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Differential Privacy: DiPA $\mathscr{A}$ is differentially private if $\exists d\,.\,\mathscr{A}$ is $d\epsilon$-differentially private.

Theorem: Given $\mathscr{A}$, there is a linear-time algorithm that can determine if $\mathscr{A}$ is differentially private.

Rest of the talk: Assume output alphabet is a finite set

Leaking Cycle

Cycle: Path that starts and ends in the same control state

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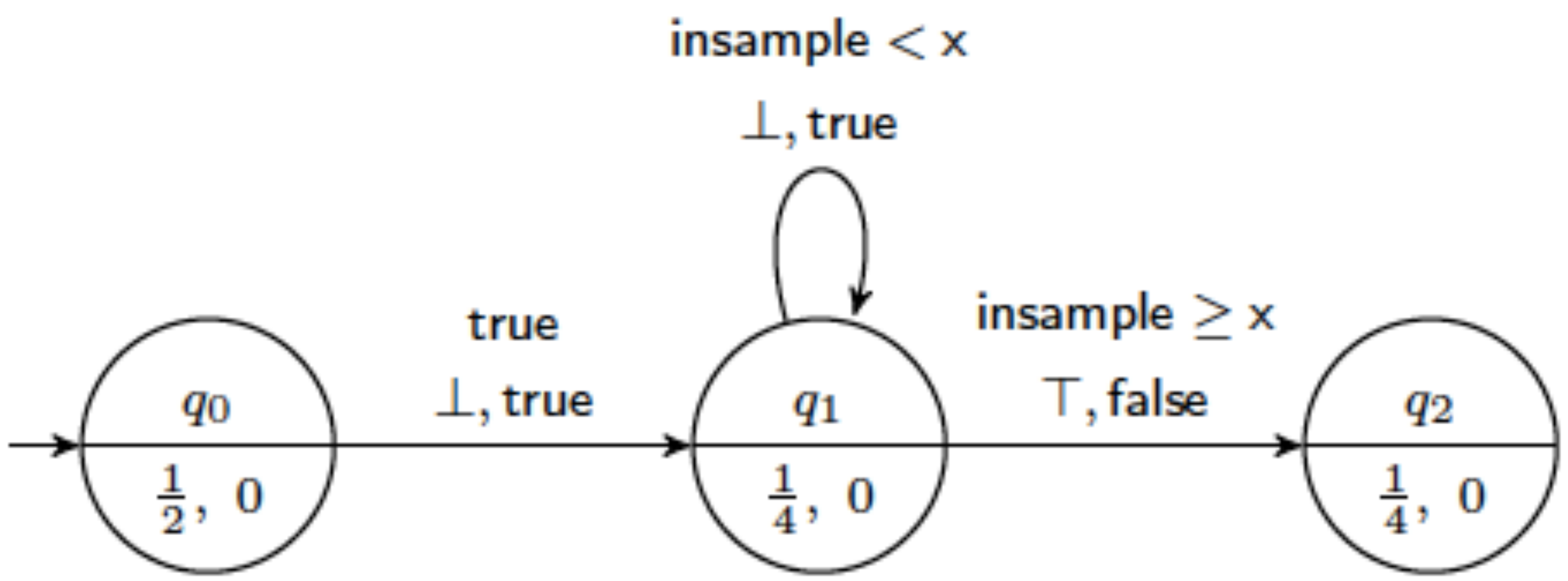

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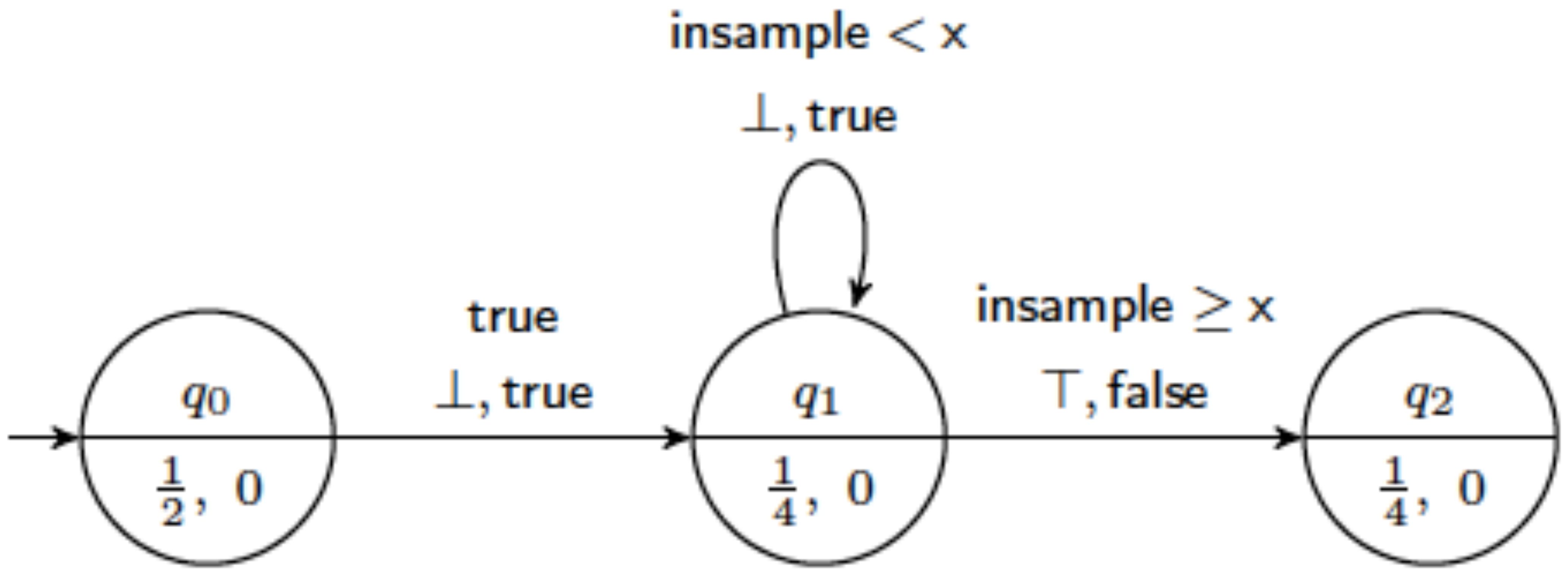

Self loop on q_1 is a leaking cycle.

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Leaking Pair: A pair of cycles (C_1, C_2) connected by a path ρ such that

- C_1 and C_2 have no assignment transitions,
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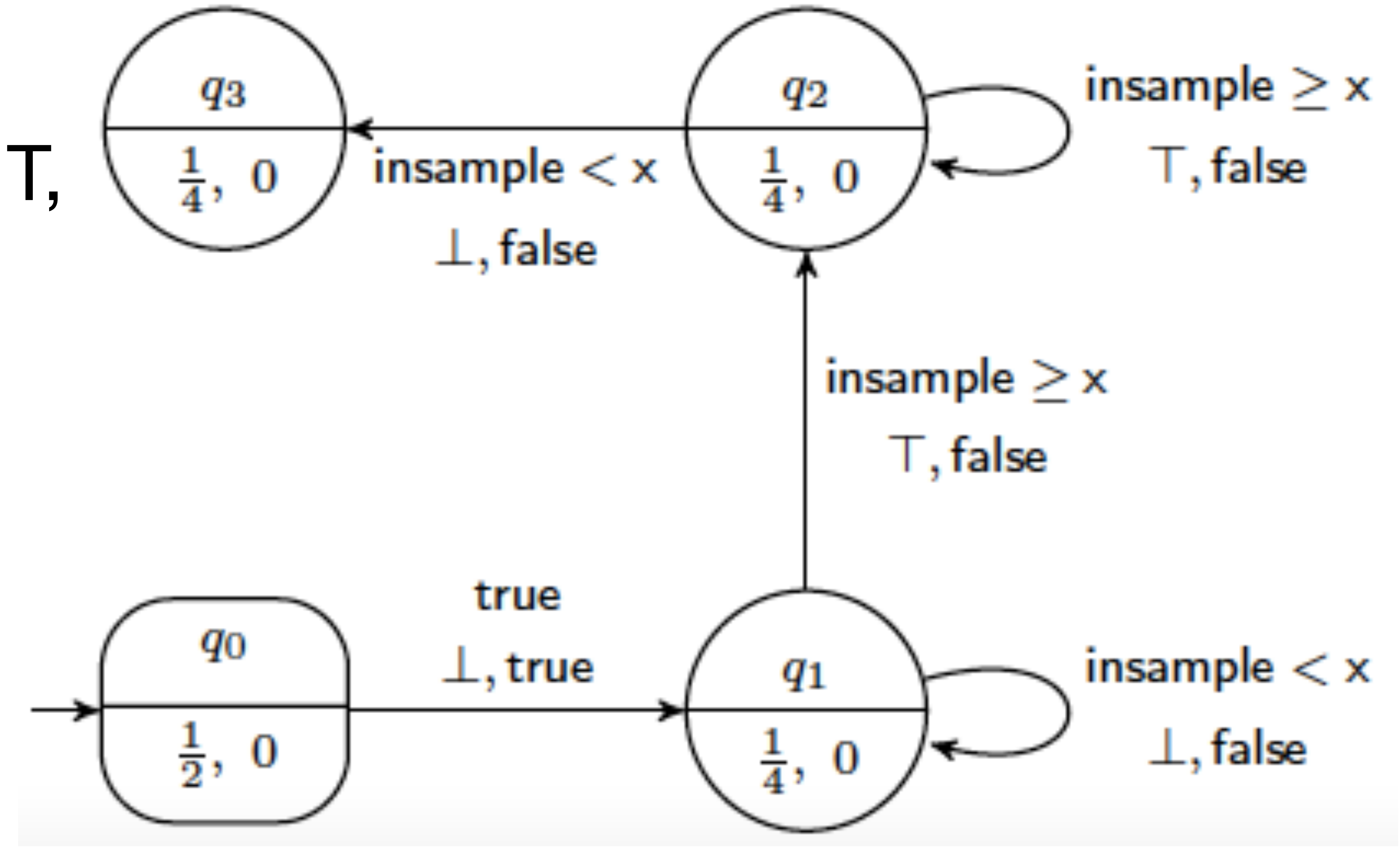

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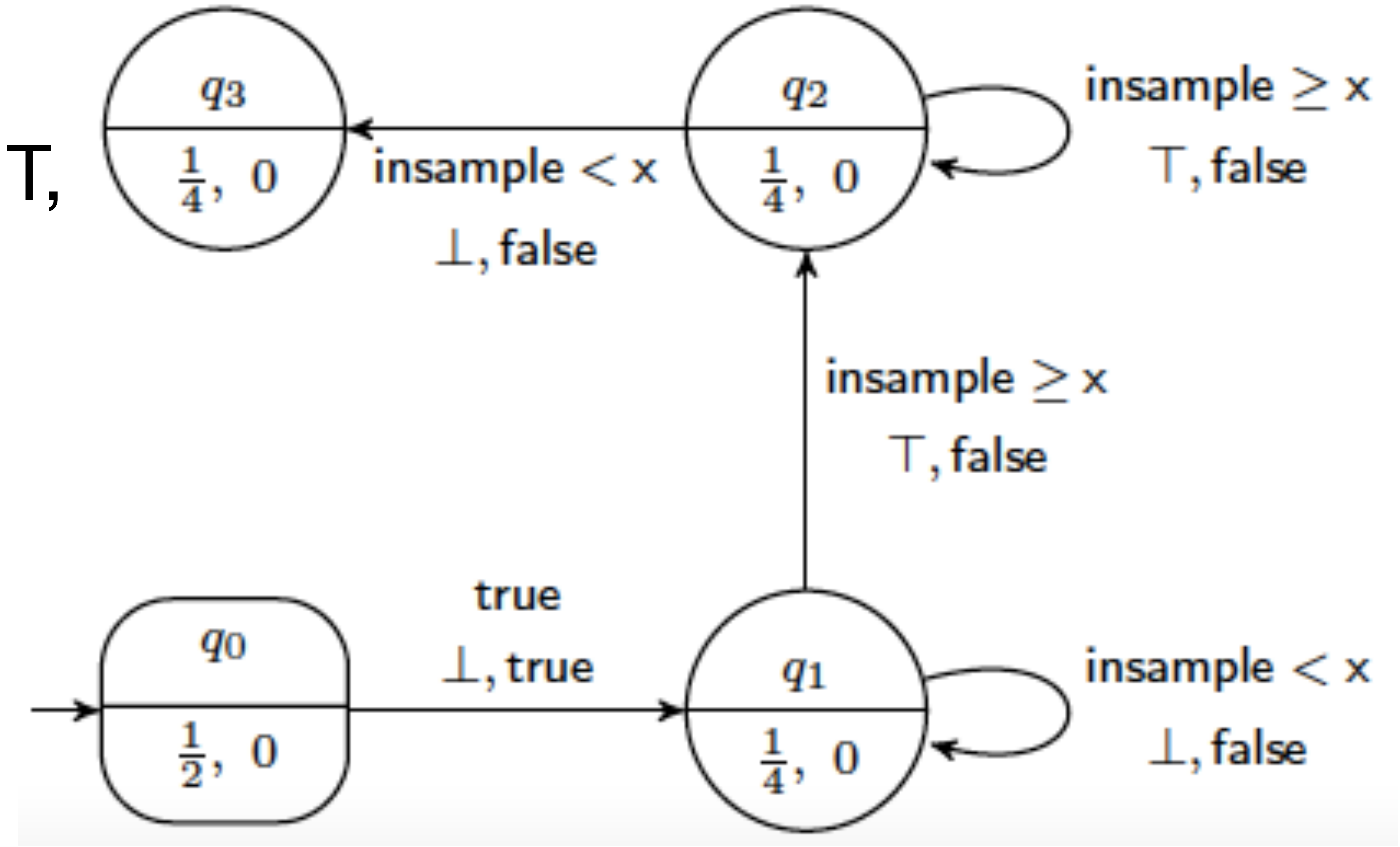

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Theorem: $\mathscr{A}$ is differentially private if and only if it is well formed.

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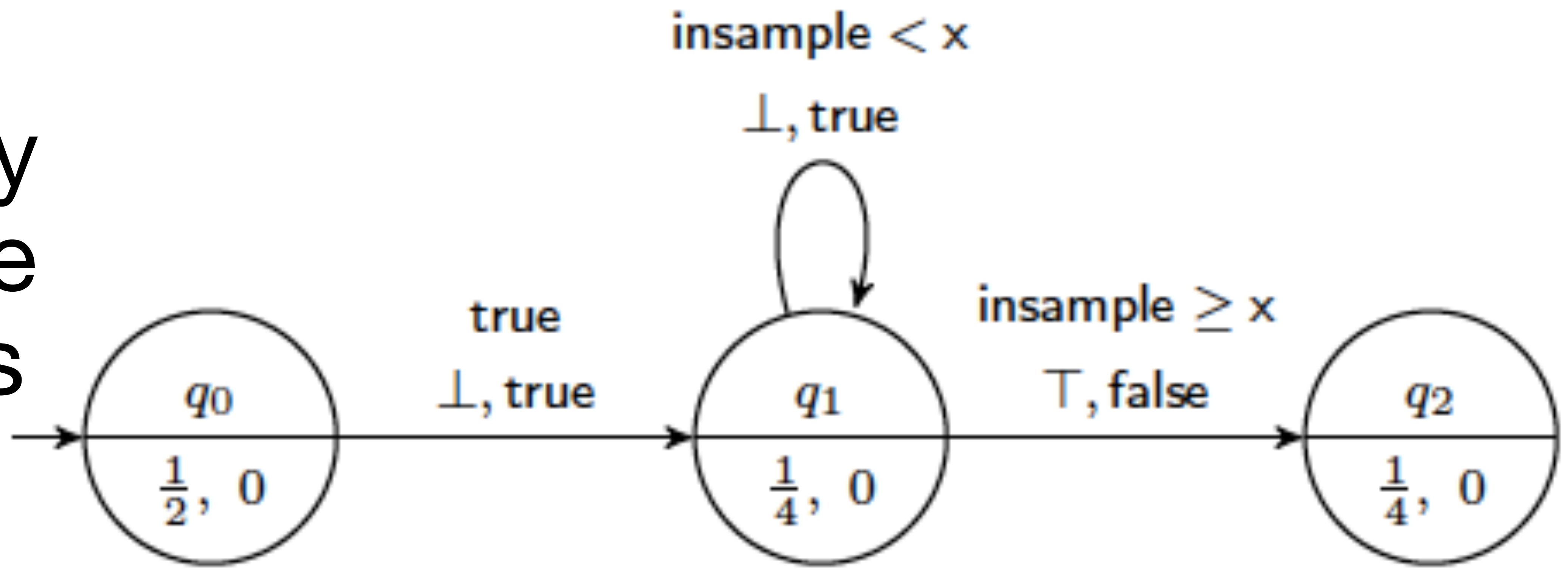

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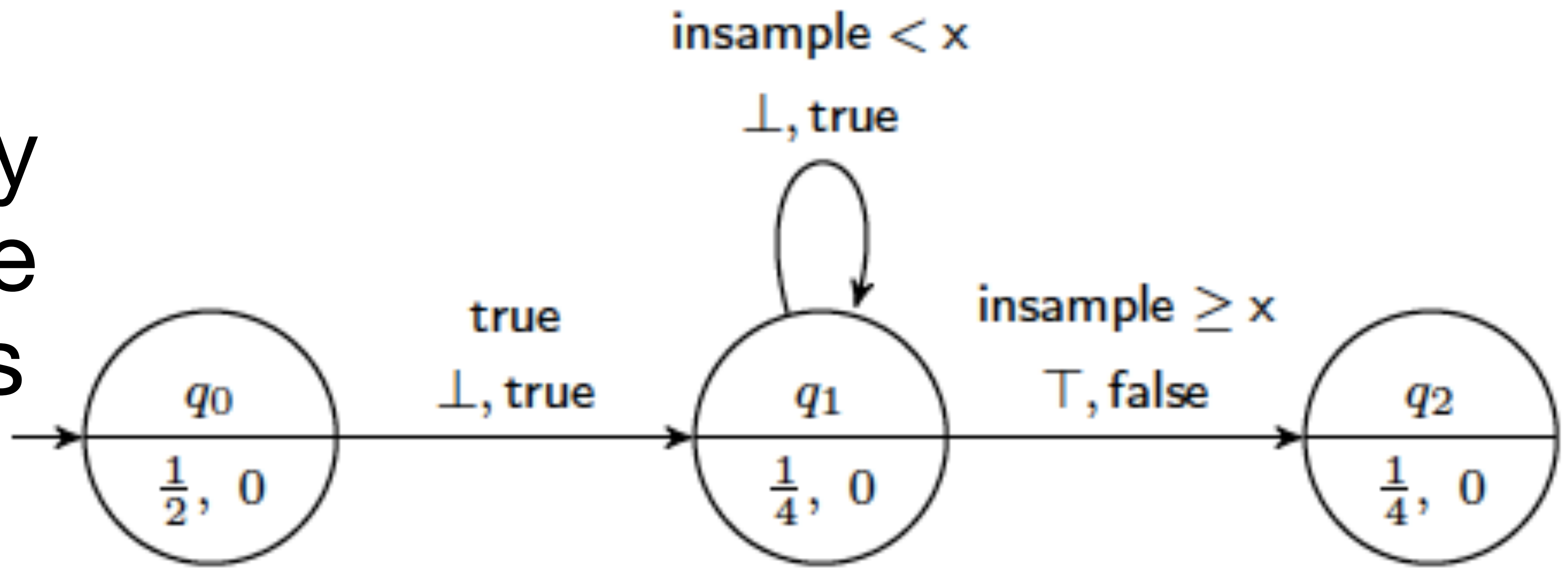

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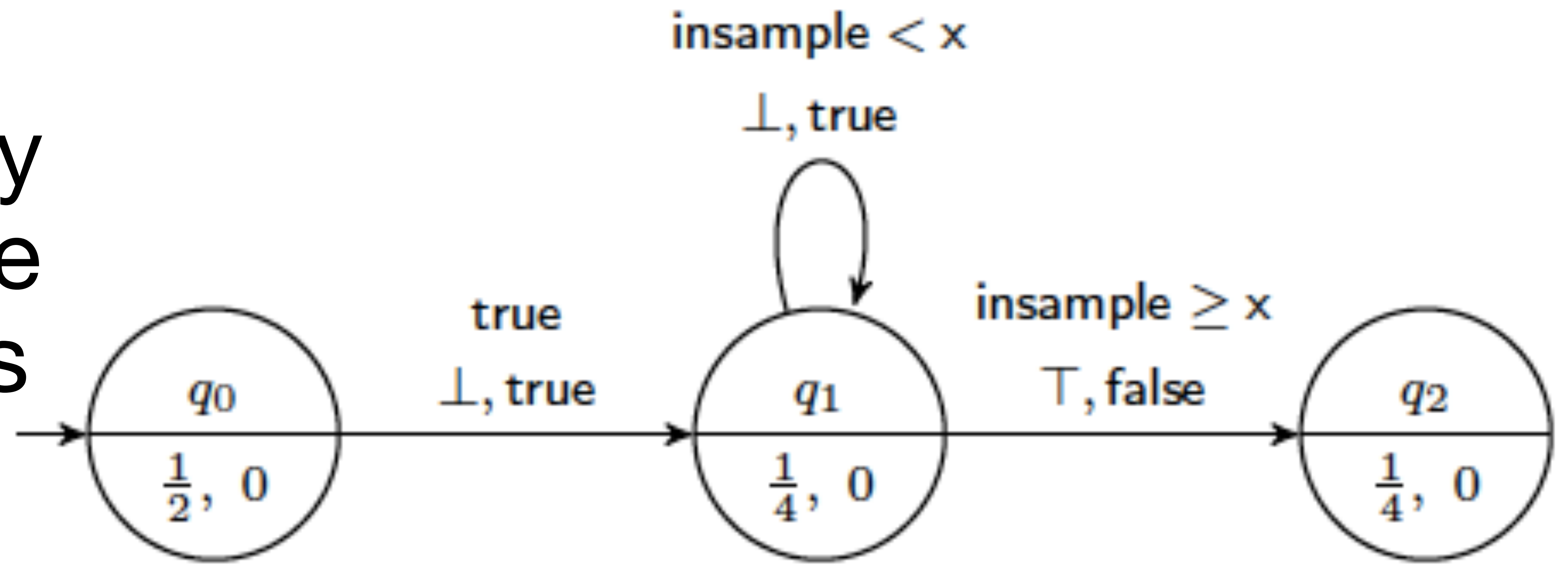

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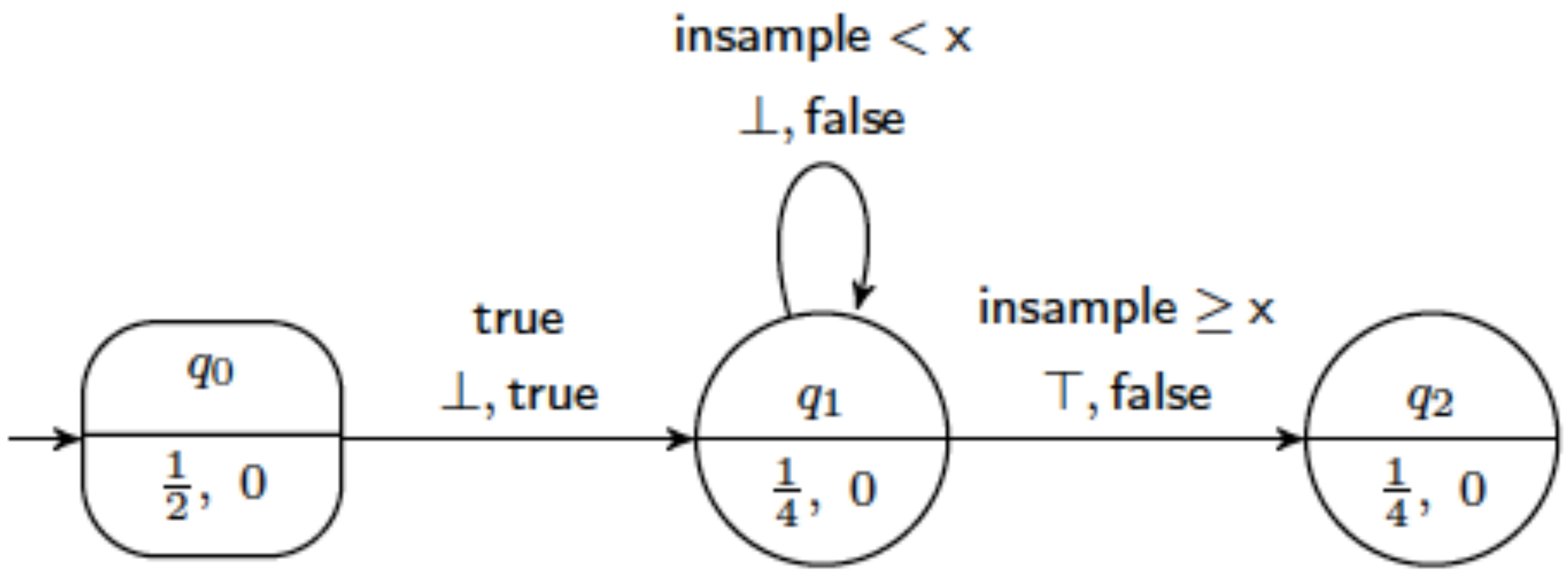
Example: cost(t_{11}) = 0, cost(t_{01}) = 1/2, cost(t_{12}) = 2*(1/4)=1/2.

wt$(\mathscr{A})$ = 1/2 + 1/2 = 1.

Thus $\mathscr{A}$ is ϵ-differentially private.



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- For example, to check if there is a reachable leaking cycle, check if there is a reachable SCC that has an assignment transition, and a transition whose guard compares the value stored in x.

Real valued outputs

The results on checking differential privacy can be extended to DiPA with real valued outputs.


CONCLUSIONS

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- Algorithm relies on checking conditions on the underlying graph of the automaton
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    - Computed values of “d” for algorithms match the theoretical bounds known

Possible next steps

- Results can be extended to automata with multiple storage variables and richer guard conditions
- How tight is the value of “d” computed by the algorithm?
- Checking other properties of automata-like models
  - $d\epsilon$-differential privacy
  - Accuracy