FM Update

On the Satisfiability of Context-free String Constraints with Subword-Ordering

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What is a String Constraint?

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Set of variables V

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Alphabet A

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- **Membership Constraints**

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- **Membership Constraints**

$$\boxed{x \in L}$$

What is a String Constraint?

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REGULAR

- **Membership Constraints**

$$x \in L$$

What is a String Constraint?

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- **Membership Constraints**

$$x \in L$$

REGULAR

CONTEXT-FREE

⋮

What is a String Constraint?

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- **Membership Constraints**

$$\boxed{x \in L}$$

What is a String Constraint?

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- **Membership Constraints**

$$\boxed{x \in L}$$

$$L = a^n b^n$$

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$$\boxed{x \in L}$$

What is a String Constraint?

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- **Membership Constraints**

$$\boxed{x \in L}$$

- **Relational Constraints**

What is a String Constraint?

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- **Membership Constraints**

$$x \in L$$

- **Relational Constraints**

$$x = yz$$

What is a String Constraint?

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Alphabet A

- **Membership Constraints**

$$x \in L$$

- **Relational Constraints**

$$x = yz$$

$$\texttt{Len}(x) = 3 \times \texttt{Len}(y)$$

What is a String Constraint?

Set of variables V

Alphabet A

- **Membership Constraints**

$$x \in L$$

- **Relational Constraints**

$$x = yz$$

$$\texttt{Len}(x) = 3 \times \texttt{Len}(y)$$

$$T(x) = y$$

What is a String Constraint?

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Alphabet A

- **Membership Constraints**

$$x \in L$$

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$$x = yz$$

$$\texttt{Len}(x) = 3 \times \texttt{Len}(y)$$

$$T(x) = y$$

$$x \preceq yy$$

What is a String Constraint?

Set of variables V

Alphabet A

- **Membership Constraints**

$$x \in L$$

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$$x = yz$$

$$\texttt{Len}(x) = 3 \times \texttt{Len}(y)$$

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...

What is a String Constraint?

Set of variables V

Alphabet A

- **Membership Constraints**

$$x \in L$$

- **Relational Constraints**

$$x = yz$$

$$\texttt{Len}(x) = 3 \times \texttt{Len}(y)$$

$$T(x) = y$$

$$x \preceq yy$$

What is a String Constraint?

Set of variables V

Alphabet A

- **Membership Constraints**

$$x \in L$$

- **Relational Constraints**

$$x = yz \qquad \texttt{Len}(x) = 3 \times \texttt{Len}(y)$$

$$T(x) = y \qquad x \preceq yy$$

What is a String Constraint?

Set of variables V

Alphabet A

- **Membership Constraints** $x \in L$
- **Relational Constraints** $x = yz$, $\texttt{Len}(x) = 3 \times \texttt{Len}(y)$, $T(x) = y$, $x \preceq yy$

$\bigwedge$

What is a String Constraint?

Set of variables V

Alphabet A

"Satisfiability"

- **Membership Constraints** $x \in L$

- **Relational Constraints** $x = yz$, $\texttt{Len}(x) = 3 \times \texttt{Len}(y)$, $T(x) = y$, $x \preceq yy$

$$\bigwedge$$

Some known results

- **Membership Constraints** $x \in L$

- **Relational Constraints** $x = yz$ $\quad \texttt{Len}(x) = 3 \times \texttt{Len}(y)$

  $T(x) = y$ $\quad x \preceq yy$ ...

Some known results

- Membership Constraints

$x \in L$

- Relational Constraints

$x = yz$

$\mathtt{Len}(x) = 3 \times \mathtt{Len}(y)$

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Some known results

- Membership Constraints

$x \in L$

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$x = yz$

$\mathrm{Len}(x) = 3 \times \mathrm{Len}(y)$

$T(x) = y$

$x \preceq yy$

...

Decidable! [Makanin 1977]

G S Makanin. The problem of solvability of equations in a free semigroup. Mathematics of the USSR-Sbornik, 32(2), 1977. doi: 10.1070/SM1977v032n02ABEH002376.

Some known results

- Membership Constraints

$x \in L$ ← **Regular**

- Relational Constraints

$x = yz$ $\quad$ $\texttt{Len}(x) = 3 \times \texttt{Len}(y)$

$T(x) = y$ $\quad$ $x \preceq yy$ $\quad$ ...

Some known results

- Membership Constraints

$x \in L$ ← **Regular**

- Relational Constraints

$x = yz$

$\mathtt{Len}(x) = 3 \times \mathtt{Len}(y)$

$T(x) = y$

$x \preceq yy$

...

Decidable [Schulz 1992]

Klaus U. Schulz. Makanin's algorithm for word equations-two improvements and a gener- alization. In K. U. Schulz, editor, Word Equations and Related Topics, pages 85–150, Berlin, Heidelberg, 1992. Springer Berlin Heidelberg. ISBN 978-3-540-46737-3.

Some known results

- Membership Constraints

$x \in L$

- Relational Constraints

$x = yz$

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Some known results

- Membership Constraints

$x \in L$

- Relational Constraints

$x = yz$

$\texttt{Len}(x) = 3 \times \texttt{Len}(y)$

$T(x) = y$

$x \preceq yy$

...

Open!

Some known results

- Membership Constraints $x \in L$
- Relational Constraints $x = yz$ $\texttt{Len}(x) = 3 \times \texttt{Len}(y)$

$T(x) = y$ $x \preceq yy$ ...

Some known results

- Membership Constraints $x \in L$
- Relational Constraints $x = yz$ $\mathrm{Len}(x) = 3 \times \mathrm{Len}(y)$

$T(x) = y$ $x \preceq yy$ ...

Undecidable [Bjorner et. al. 2009]

Bjørner, N., Tillmann, N., Voronkov, A. (2009). Path Feasibility Analysis for String-Manipulating Programs. In: Kowalewski, S., Philippou, A. (eds) Tools and Algorithms for the Construction and Analysis of Systems. TACAS 2009. Lecture Notes in Computer Science, vol 5505. Springer, Berlin, Heidelberg. https://doi.org/10.1007/978-3-642-00768-2_27

Some known results

- Membership Constraints $x \in L$
- Relational Constraints $x = yz$ $\mathtt{Len}(x) = 3 \times \mathtt{Len}(y)$

$T(x) = y$ $x \preceq yy$ ...

Our Setting

- **Membership Constraints**

$$x \in L$$

- **Relational Constraints**

$$x = yz$$

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$$T(x) = y$$

$$x \preceq yy$$

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Our Setting

- **Membership Constraints** $x \in L$
- **Relational Constraints** $x = yz$ $\text{Len}(x) = 3 \times \text{Len}(y)$ $T(x) = y$ $x \preceq yy$ ...

Our Setting

- **Membership Constraints** $x \in L$ ← **Context-free**

- **Relational Constraints**

$x = yz$ $\mathrm{Len}(x) = 3 \times \mathrm{Len}(y)$

$T(x) = y$ $x \preceq yy$ ...

Our Setting

- **Membership Constraints** $x \in L$ ← **Context-free**

- **Relational Constraints**

$x = yz$

$\mathtt{Len}(x) = 3 \times \mathtt{Len}(y)$

$T(x) = y$

$x \preceq \mathtt{Shuffle}(yy)$

...

Our Setting

- **Membership Constraints** $x \in L$ ← **Context-free**

- **Relational Constraints** $x = yz$ $\texttt{Len}(x) = 3 \times \texttt{Len}(y)$

$T(x) = y$ $x \preceq \texttt{Shuffle}(yy)$ **...**

`Shuffle`

Our Setting

- **Membership Constraints** $x \in L$ ⟵ **Context-free**

- **Relational Constraints** $x = yz$ $\texttt{Len}(x) = 3 \times \texttt{Len}(y)$

  $T(x) = y$ $x \preceq \texttt{Shuffle}(yy)$ **...**

`Shuffle`

u = abab

v = baabaa

Our Setting

- **Membership Constraints** $x \in L$ ← **Context-free**

- **Relational Constraints**

$x = yz$ | $\texttt{Len}(x) = 3 \times \texttt{Len}(y)$

$T(x) = y$ | $x \preceq \texttt{Shuffle}(yy)$ ...

`Shuffle`

u = abab

{ }

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Our Setting

- **Membership Constraints** $x \in L$ ← **Context-free**

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Shuffle

u = abab

{ abbaababaa ,

}

v = baabaa

Our Setting

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Shuffle

u = abab

{ abbaababaa ,

baaabbabaa ,

}

v = baabaa

Our Setting

- **Membership Constraints** $x \in L$ ← **Context-free**

- **Relational Constraints** $x = yz$ $\quad$ $\texttt{Len}(x) = 3 \times \texttt{Len}(y)$

  $T(x) = y$ $\quad$ $x \preceq \texttt{Shuffle}(yy)$ ...

`Shuffle`

u = abab

{ abbaababaa ,

baaabbabaa ,

... }

v = baabaa

Our Setting

- **Membership Constraints** $x \in L$ ← **Context-free**

- **Relational Constraints**

$x = yz$

$\texttt{Len}(x) = 3 \times \texttt{Len}(y)$

$T(x) = y$

$x \preceq \texttt{Shuffle}(yy)$

...

Our Setting

- **Membership Constraints** $x \in L$ ← **Context-free**
- **Relational Constraints** $x = yz$ $\quad \texttt{Len}(x) = 3 \times \texttt{Len}(y)$ $\quad T(x) = y$ $\quad x \preceq \texttt{Shuffle}(yy)$ ...

Subword

Our Setting

- **Membership Constraints** $x \in L$ ← **Context-free**

- **Relational Constraints** $x = yz$ $\texttt{Len}(x) = 3 \times \texttt{Len}(y)$

  $T(x) = y$ $x \preceq \texttt{Shuffle}(yy)$ ...

Subword $\preceq$

Our Setting

- **Membership Constraints** $x \in L$ ← **Context-free**

- **Relational Constraints** $x = yz$ $\mathtt{Len}(x) = 3 \times \mathtt{Len}(y)$

$T(x) = y$ $x \preceq \mathtt{Shuffle}(yy)$ **...**

abbaaababab

Subword $\preceq$

Our Setting

- **Membership Constraints** $x \in L$ ← **Context-free**
- **Relational Constraints**

$x = yz$ $\quad$ $\mathtt{Len}(x) = 3 \times \mathtt{Len}(y)$

$T(x) = y$ $\quad$ $x \preceq \mathtt{Shuffle}(yy)$ ...

abbaaababab

Subword $\preceq$

bbabb

Our Setting

- **Membership Constraints** $x \in L$ ← **Context-free**

- **Relational Constraints**

$x = yz$ $\quad$ $\texttt{Len}(x) = 3 \times \texttt{Len}(y)$

$T(x) = y$ $\quad$ $x \preceq \texttt{Shuffle}(yy)$ $\quad$ **...**

Subword $\preceq$

abbaaababab

bbabb

Satisfiability is Undecidable!

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Intersection non-emptiness of CFL

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Intersection non-emptiness of CFL

$V = \{x, y\}$ Alphabet A

Satisfiability is Undecidable!

Intersection non-emptiness of CFL

$V = \{x, y\}$ Alphabet A

$x \in L_1$ $y \in L_2$

Satisfiability is Undecidable!

Intersection non-emptiness of CFL

$V = \{x, y\}$	Alphabet A
$x \in L_1$	$y \in L_2$
$x \preceq y$	$y \preceq x$

Satisfiability is Undecidable!

Intersection non-emptiness of CFL

$V = \{x, y\}$ Alphabet A

$x \in L_1$ $y \in L_2$

$x \preceq y$ $y \preceq x$ $\implies x = y$

Satisfiability is Undecidable!

Intersection non-emptiness of CFL

$V = \{x, y\}$ Alphabet A

$x \in L_1$ $y \in L_2$

$x \preceq y$ $y \preceq x$ $\implies x = y$

Undecidable!

Satisfiability is Undecidable!

Even for regular membership

Satisfiability is Undecidable!

Even for regular membership

Reduction from PCP

Satisfiability is Undecidable!

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Reduction from PCP

PCP instance over alphabet Σ:

Satisfiability is Undecidable!

PCP instance over alphabet Σ:

Satisfiability is Undecidable!

PCP instance over alphabet Σ:

Given two vector of strings U and V, each having n elements:

Satisfiability is Undecidable!

PCP instance over alphabet Σ:

Given two vector of strings U and V, each having n elements:

	1	2	...	n
U	u_1	u_2	...	u_n
V	v_1	v_2	...	v_n

Satisfiability is Undecidable!

PCP instance over alphabet Σ:

Given two vector of strings U and V, each having n elements:

	1	2	...	n
U	u_1	u_2	...	u_n
V	v_1	v_2	...	v_n

$\exists \; i_1, i_2, \ldots, i_k$

sequence of indices

$u_{i_1} \cdot u_{i_2} \cdots u_{i_k}$

=

$v_{i_1} \cdot v_{i_2} \cdots v_{i_k}$

?

Satisfiability is Undecidable!

Reduction from PCP

PCP instance over alphabet Σ:

Given two vector of strings U and V, each having n elements:

	1	2	...	n
U	u_1	u_2	...	u_n
V	v_1	v_2	...	v_n

$\exists \quad i_1, i_2, \ldots, i_k$

$$u_{i_1} \cdot u_{i_2} \cdots u_{i_k}$$

$$=$$

$$v_{i_1} \cdot v_{i_2} \cdots v_{i_k}$$

sequence of indices

?

Satisfiability is Undecidable!

Reduction from PCP

PCP instance over alphabet Σ:

Given two vector of strings U and V, each having n elements:

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sequence of indices

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Satisfiability is Undecidable!

Reduction from PCP

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Given two vector of strings U and V, each having n elements:

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$\exists \; i_1, i_2, \ldots, i_k$

sequence of indices

$u_{i_1} \cdot u_{i_2} \cdots u_{i_k}$

=

$v_{i_1} \cdot v_{i_2} \cdots v_{i_k}$

?

Regular String Constraint:

Satisfiability is Undecidable!

Reduction from PCP

PCP instance over alphabet Σ:

Given two vector of strings U and V, each having n elements:

	1	2	...	n
U	u_1	u_2	...	u_n
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$\exists$ $i_1, i_2, \ldots, i_k$

sequence of indices

$u_{i_1} \cdot u_{i_2} \cdots u_{i_k}$

=

$v_{i_1} \cdot v_{i_2} \cdots v_{i_k}$

?

Regular String Constraint:

Variable set: $\{x_i, x_s, x_u, x_v\}$

Satisfiability is Undecidable!

Reduction from PCP

PCP instance over alphabet Σ:

Given two vector of strings U and V, each having n elements:

	1	2	3	4
U	baab	ab	bab	ba
V	b	aa	babb	aba

Regular String Constraint:

Variable set: $\{x_i, x_s, x_u, x_v\}$

Satisfiability is Undecidable!

Reduction from PCP

PCP instance over alphabet Σ:

Given two vector of strings U and V, each having n elements:

	1	2	3	4
U	baab	ab	bab	ba
V	b	aa	babb	aba

Regular String Constraint:

Variable set: $\{x_i, x_s, x_u, x_v\}$

$$x_i \in (1 + 2 + 3 + 4)^*$$

Satisfiability is Undecidable!

Reduction from PCP

PCP instance over alphabet Σ:

Given two vector of strings U and V, each having n elements:

	1	2	3	4
U	baab	ab	bab	ba
V	b	aa	babb	aba

Regular String Constraint:

Variable set: $\{x_i, x_s, x_u, x_v\}$

$x_i \in (1 + 2 + 3 + 4)^*$

$x_s \in \Sigma^*$

Satisfiability is Undecidable!

Reduction from PCP

PCP instance over alphabet Σ:

Given two vector of strings U and V, each having n elements:

	1	2	3	4
U	baab	ab	bab	ba
V	b	aa	babb	aba

Regular String Constraint:

Variable set: $\{x_i, x_s, x_u, x_v\}$

$$x_i \in (1 + 2 + 3 + 4)^*$$
$$x_s \in \Sigma^*$$
$$x_u \in (1.baab + 2.ab + 3.bab + 4.ba)^*$$

Satisfiability is Undecidable!

Reduction from PCP

PCP instance over alphabet Σ:

Given two vector of strings U and V, each having n elements:

	1	2	3	4
U	baab	ab	bab	ba
V	b	aa	babb	aba

Regular String Constraint:

Variable set: $\{x_i, x_s, x_u, x_v\}$

$$x_i \in (1 + 2 + 3 + 4)^*$$
$$x_s \in \Sigma^*$$
$$x_u \in (1.baab + 2.ab + 3.bab + 4.ba)^*$$
$$x_v \in (1.b + 2.aa + 3.babb + 4.aba)^*$$

Satisfiability is Undecidable!

Reduction from PCP

PCP instance over alphabet Σ:

Given two vector of strings U and V, each having n elements:

	1	2	...	n
U	u_1	u_2	...	u_n
V	v_1	v_2	...	v_n

$\exists$ $i_1, i_2, \ldots, i_k$

sequence of indices

$u_{i_1} \cdot u_{i_2} \cdots u_{i_k}$

=

$v_{i_1} \cdot v_{i_2} \cdots v_{i_k}$

?

Regular String Constraint:

Variable set: $\{x_i, x_s, x_u, x_v\}$

Satisfiability is Undecidable!

Reduction from PCP

PCP instance over alphabet Σ:

Given two vector of strings U and V, each having n elements:

	1	2	...	n
U	u_1	u_2	...	u_n
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$\exists \; i_1, i_2, \ldots, i_k$

sequence of indices

$$u_{i_1} \cdot u_{i_2} \cdots u_{i_k} = v_{i_1} \cdot v_{i_2} \cdots v_{i_k} \; ?$$

Regular String Constraint:

Variable set: $\{x_i, x_s, x_u, x_v\}$

$$x_i = \left(\bigcup_{j \in [n]} j \right)^* \qquad x_s = \Sigma^*$$

$$x_u = \left(\bigcup_{j \in [n]} j.u_j \right)^* \qquad x_v = \left(\bigcup_{j \in [n]} j.v_j \right)^*$$

Satisfiability is Undecidable!

Reduction from PCP

PCP instance over alphabet Σ:

Given two vector of strings U and V, each having n elements:

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Regular String Constraint:

Variable set: $\{x_i, x_s, x_u, x_v\}$

$$x_i = \left(\bigcup_{j \in [n]} j\right)^*$$

$$x_s = \Sigma^*$$

$$x_u = \left(\bigcup_{j \in [n]} j.u_j\right)^*$$

$$x_v = \left(\bigcup_{j \in [n]} j.v_j\right)^*$$

$$x_i \preceq x_u$$

Satisfiability is Undecidable!

Reduction from PCP

PCP instance over alphabet Σ:

Given two vector of strings U and V, each having n elements:

	1	2	...	n
U	u_1	u_2	...	u_n
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$\exists \quad i_1, i_2, \ldots, i_k$

sequence of indices

$$u_{i_1} \cdot u_{i_2} \cdots u_{i_k} = v_{i_1} \cdot v_{i_2} \cdots v_{i_k}$$

?

Regular String Constraint:

Variable set: $\{x_i, x_s, x_u, x_v\}$

$$x_i = \left(\bigcup_{j \in [n]} j \right)^* \qquad x_s = \Sigma^*$$

$$x_u = \left(\bigcup_{j \in [n]} j.u_j \right)^* \qquad x_v = \left(\bigcup_{j \in [n]} j.v_j \right)^*$$

$$x_i \preceq x_u \qquad x_s \preceq x_u$$

Satisfiability is Undecidable!

Reduction from PCP

PCP instance over alphabet Σ:

Given two vector of strings U and V, each having n elements:

	1	2	...	n
U	u_1	u_2	...	u_n
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$\exists$ $i_1, i_2, \ldots, i_k$

sequence of indices

$u_{i_1} \cdot u_{i_2} \cdots u_{i_k}$

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Regular String Constraint:

Variable set: $\{x_i, x_s, x_u, x_v\}$

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$$x_u = \left(\bigcup_{j \in [n]} j.u_j\right)^* \qquad x_v = \left(\bigcup_{j \in [n]} j.v_j\right)^*$$

$$x_i \preceq x_u \qquad x_s \preceq x_u \qquad x_u \preceq \texttt{Shuffle}(x_i, x_s)$$

Satisfiability is Undecidable!

Reduction from PCP

PCP instance over alphabet Σ:

Given two vector of strings U and V, each having n elements:

	1	2	...	n
U	u_1	u_2	...	u_n
V	v_1	v_2	...	v_n

$\exists$ $i_1, i_2, \ldots, i_k$

sequence of indices

$u_{i_1} \cdot u_{i_2} \cdots u_{i_k}$

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Regular String Constraint:

Variable set: $\{x_i, x_s, x_u, x_v\}$

$x_i = \left(\bigcup_{j \in [n]} j \right)^*$ $\qquad$ $x_s = \Sigma^*$

$x_u = \left(\bigcup_{j \in [n]} j.u_j \right)^*$ $\qquad$ $x_v = \left(\bigcup_{j \in [n]} j.v_j \right)^*$

$x_i \preceq x_u$ $\qquad$ $x_s \preceq x_u$ $\qquad$ $x_u \preceq \texttt{Shuffle}(x_i, x_s)$

$x_i \preceq x_v$

Satisfiability is Undecidable!

Reduction from PCP

PCP instance over alphabet Σ:

Given two vector of strings U and V, each having n elements:

	1	2	...	n
U	u_1	u_2	...	u_n
V	v_1	v_2	...	v_n

$\exists$ $i_1, i_2, \ldots, i_k$

sequence of indices

$u_{i_1} \cdot u_{i_2} \cdots u_{i_k}$ = $v_{i_1} \cdot v_{i_2} \cdots v_{i_k}$?

Regular String Constraint:

Variable set: $\{x_i, x_s, x_u, x_v\}$

$$x_i = \left(\bigcup_{j \in [n]} j\right)^* \qquad x_s = \Sigma^*$$

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$$x_i \preceq x_u \qquad x_s \preceq x_u \qquad x_u \preceq \texttt{Shuffle}(x_i, x_s)$$

$$x_i \preceq x_v \qquad x_s \preceq x_v$$

Satisfiability is Undecidable!

Reduction from PCP

PCP instance over alphabet Σ:

Given two vector of strings U and V, each having n elements:

	1	2	...	n
U	u_1	u_2	...	u_n
V	v_1	v_2	...	v_n

$\exists$ $i_1, i_2, \ldots, i_k$

sequence of indices

$u_{i_1} \cdot u_{i_2} \cdots u_{i_k}$

=

$v_{i_1} \cdot v_{i_2} \cdots v_{i_k}$

?

Regular String Constraint:

Variable set: $\{x_i, x_s, x_u, x_v\}$

$$x_i = \left(\bigcup_{j \in [n]} j \right)^*$$

$$x_s = \Sigma^*$$

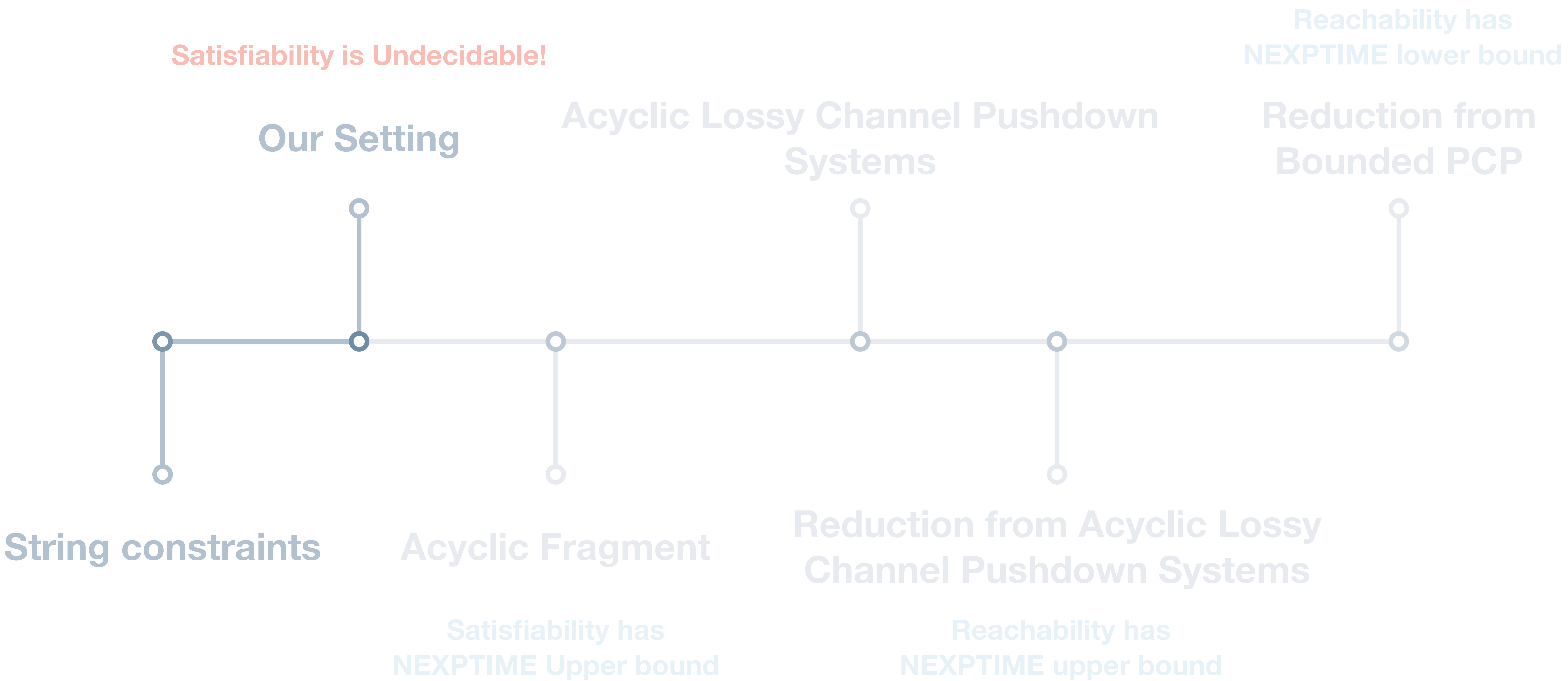
$$x_u = \left(\bigcup_{j \in [n]} j.u_j \right)^*$$

$$x_v = \left(\bigcup_{j \in [n]} j.v_j \right)^*$$

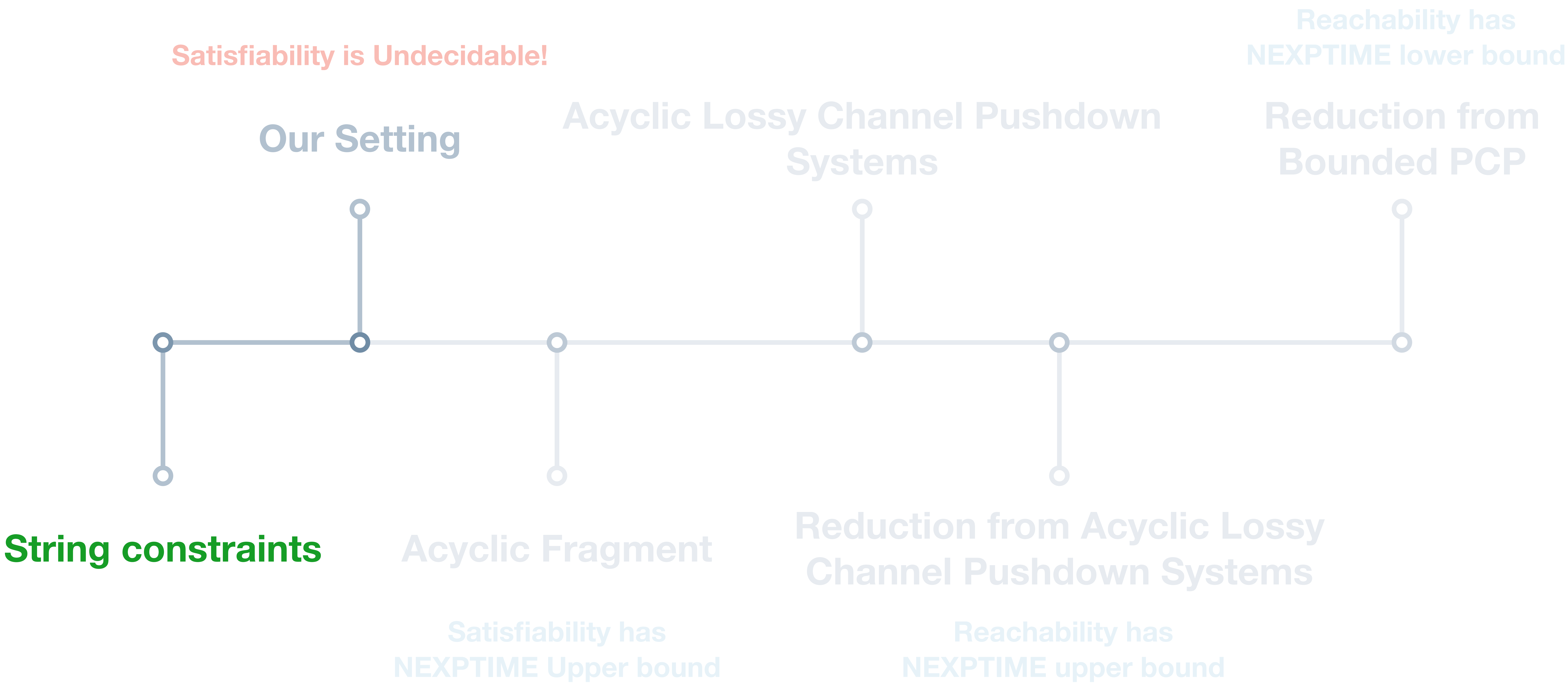
$x_i \preceq x_u$ $\quad$ $x_s \preceq x_u$ $\quad$ $x_u \preceq \texttt{Shuffle}(x_i, x_s)$

$x_i \preceq x_v$ $\quad$ $x_s \preceq x_v$ $\quad$ $x_v \preceq \texttt{Shuffle}(x_i, x_s)$

Outline


Satisfiability is Undecidable!
Our Setting
Acyclic Lossy Channel Pushdown Systems
Reachability has NEXPTIME lower bound
Reduction from Bounded PCP
String constraints
Acyclic Fragment
Satisfiability has NEXPTIME Upper bound
Reduction from Acyclic Lossy Channel Pushdown Systems
Reachability has NEXPTIME upper bound

Outline



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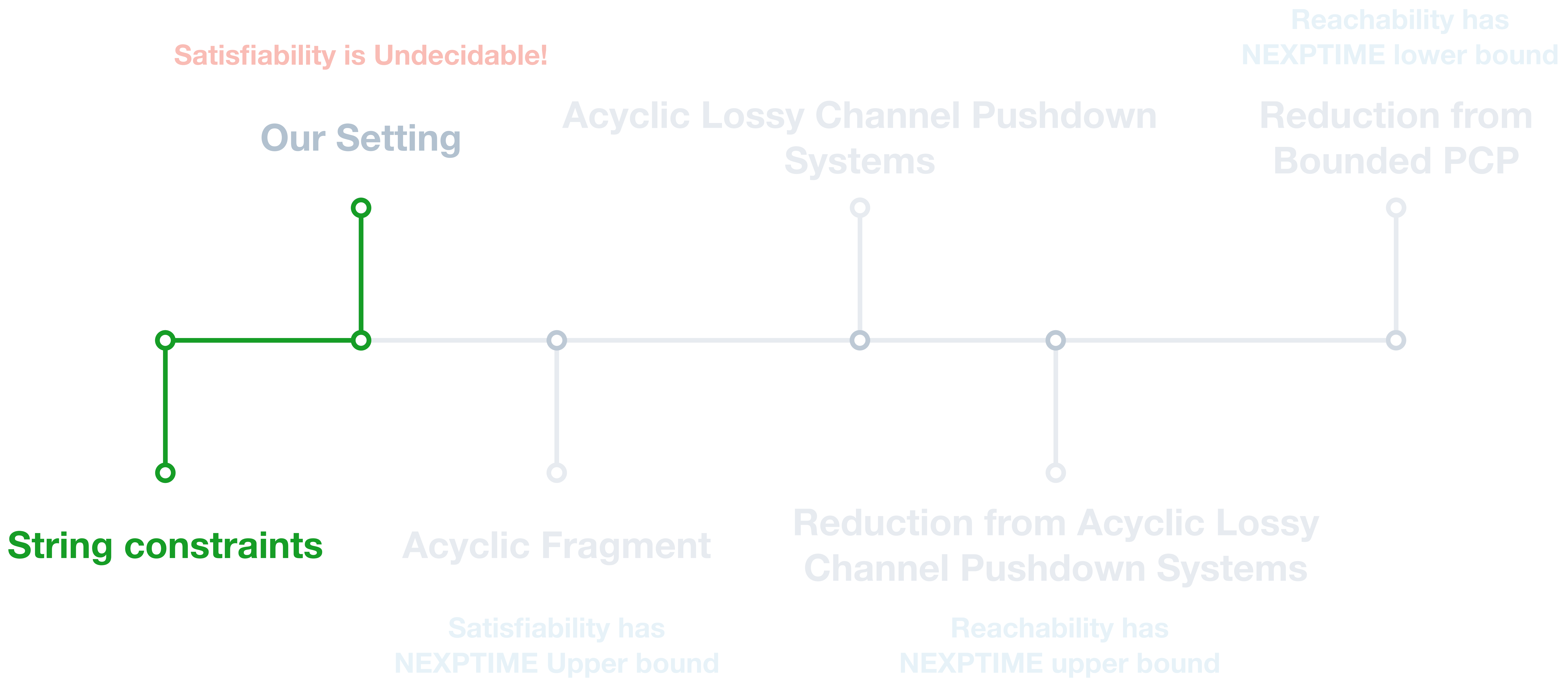
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Outline

Satisfiability is Undecidable!
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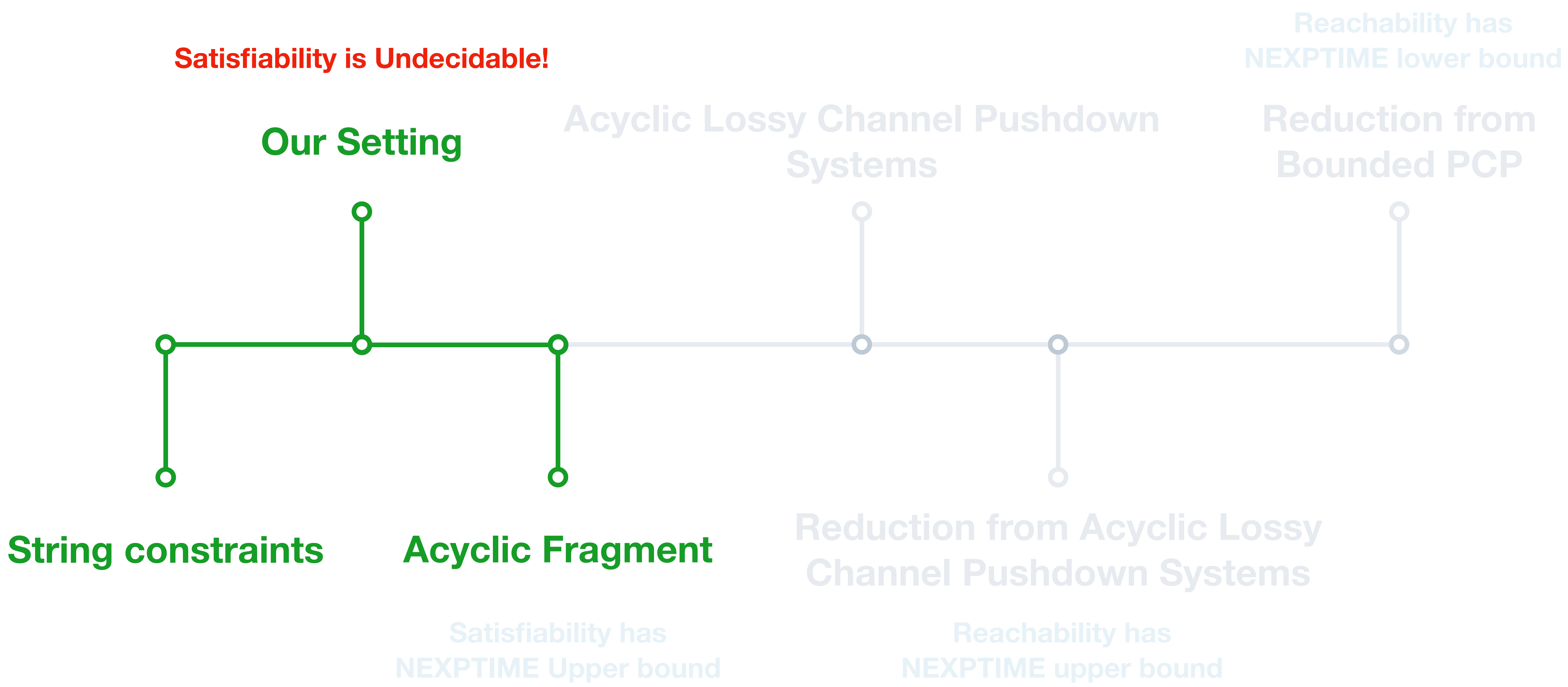
Outline


Reachability has
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Reachability has
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Acyclic Fragment

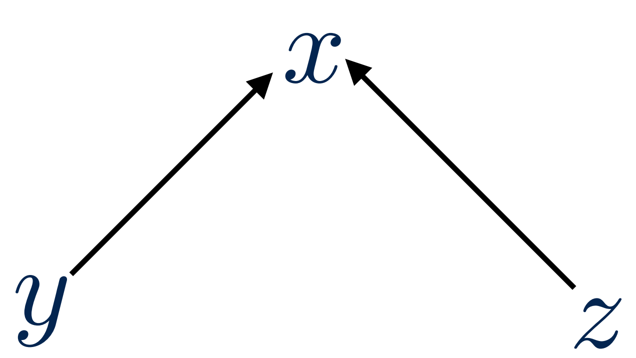
Acyclic Fragment

Acyclic Fragment

$x \preceq \texttt{Shuffle}(yz)$

Acyclic Fragment

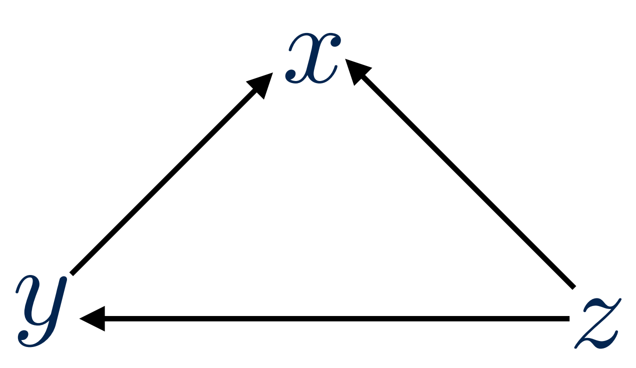
$x \preceq \texttt{Shuffle}(yz)$



Acyclic Fragment

$x \preceq \texttt{Shuffle}(yz)$

$y \preceq z$



Acyclic Fragment

$x \preceq \texttt{Shuffle}(yz)$

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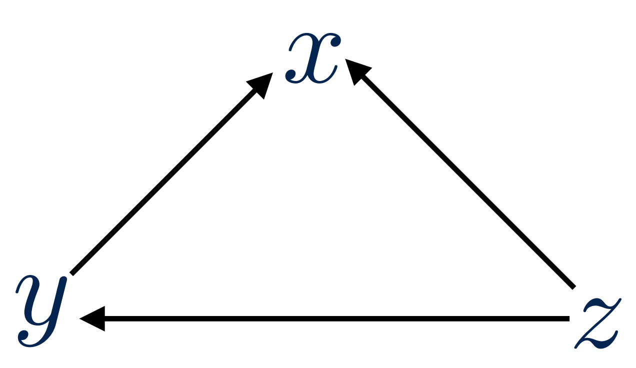

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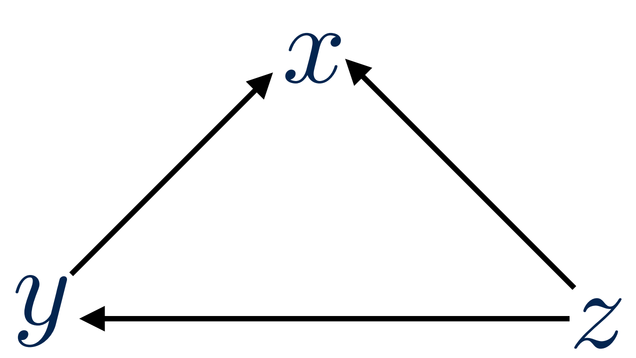


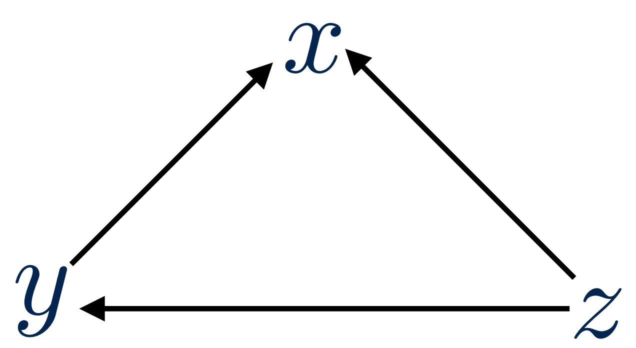
✔

Acyclic!

Acyclic Fragment

$x \preceq \texttt{Shuffle}(yz)$

$y \preceq z$



Acyclic!

Acyclic Fragment

$x \preceq \texttt{Shuffle}(yz)$

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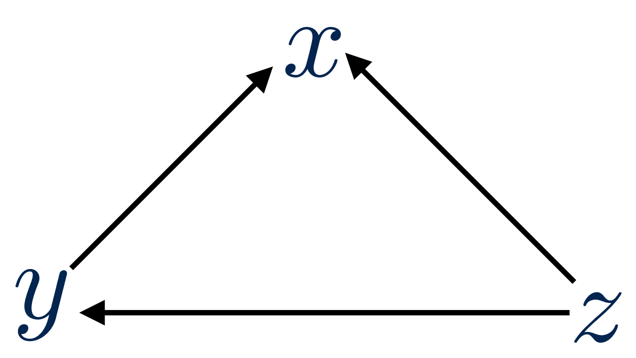

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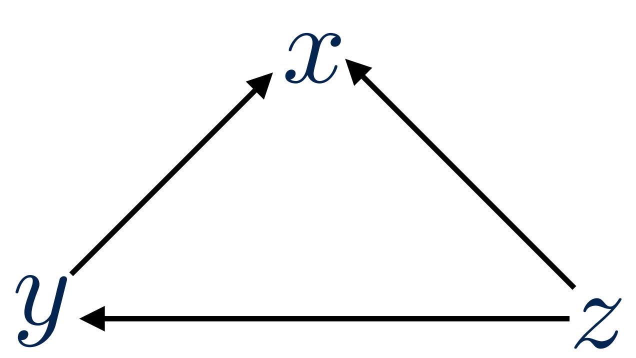
$x \preceq \texttt{Shuffle}(yz)$

Acyclic Fragment

$x \preceq \texttt{Shuffle}(yz)$

$y \preceq z$



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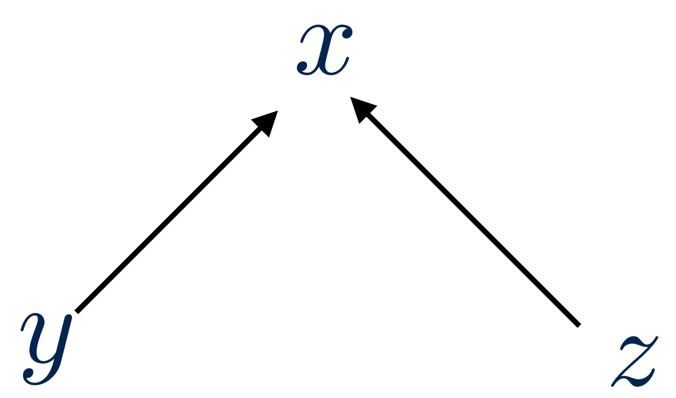

Acyclic Fragment

$x \preceq \texttt{Shuffle}(yz)$

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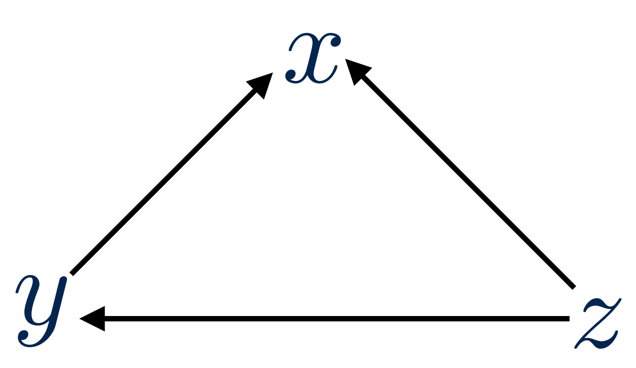
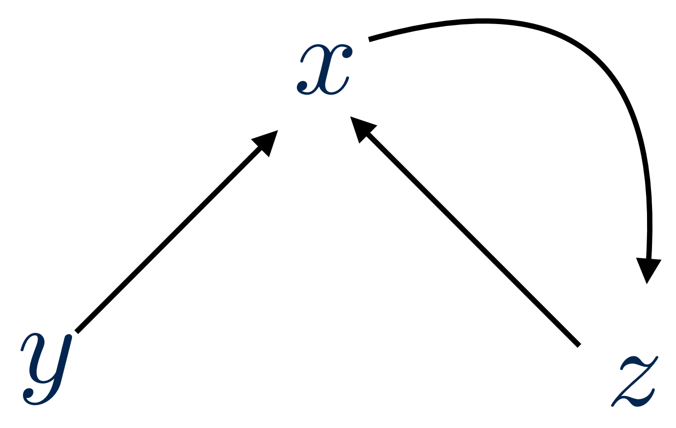


Acyclic!

$x \preceq \texttt{Shuffle}(yz)$

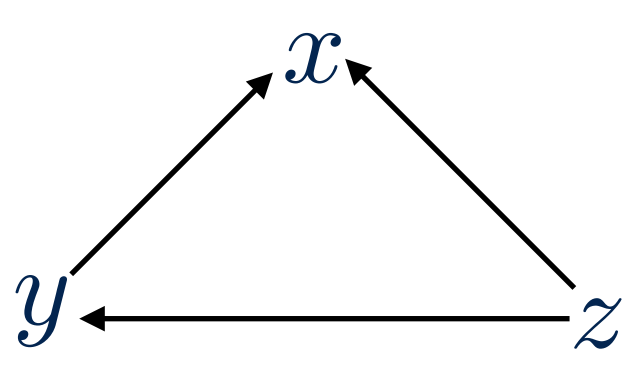
$z \preceq x$



Acyclic Fragment

$x \preceq \texttt{Shuffle}(yz)$

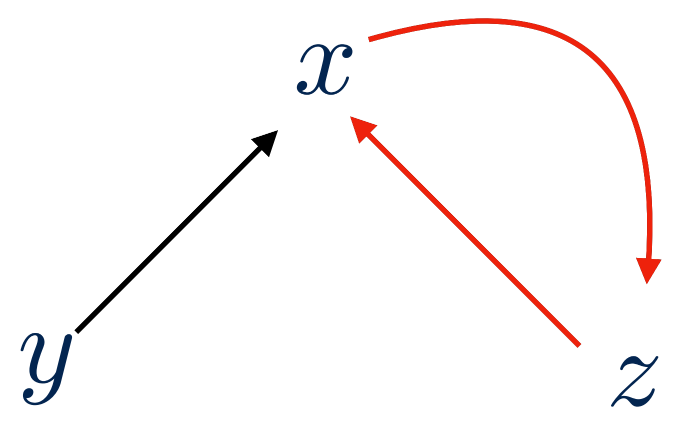
$y \preceq z$



Acyclic!

$x \preceq \texttt{Shuffle}(yz)$

$z \preceq x$

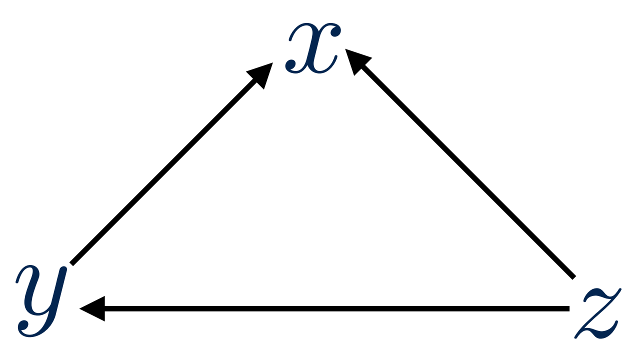


Cycle!

Acyclic Fragment

$x \preceq \texttt{Shuffle}(yz)$

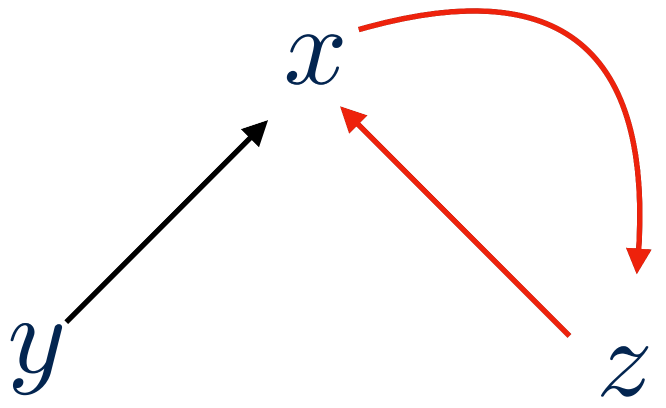
$y \preceq z$



Acyclic!

$x \preceq \texttt{Shuffle}(yz)$

$z \preceq x$



Cycle!

Main Result

Main Result

Satisfiability of Acyclic Fragment is NEXPTIME Complete!

NEXPTIME Upper bound

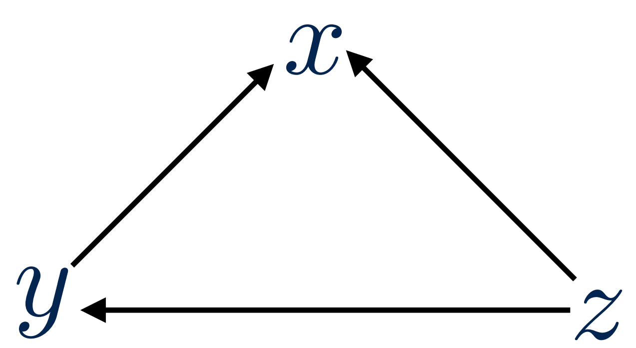
NEXPTIME Upper bound

small model property.

NEXPTIME Upper bound

$x \preceq \texttt{Shuffle}(yz)$

$y \preceq z$



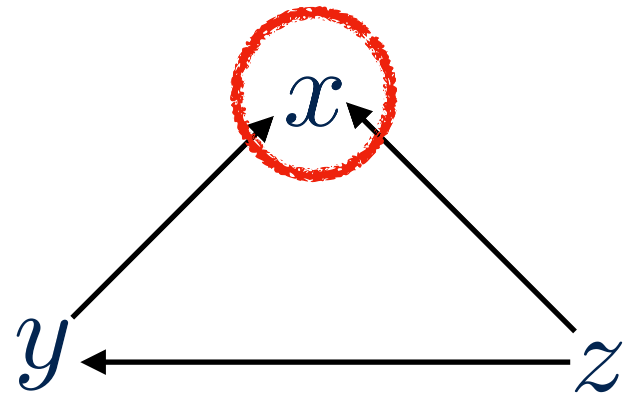
Acyclic!

small model property.

NEXPTIME Upper bound

$x \preceq \texttt{Shuffle}(yz)$

$y \preceq z$



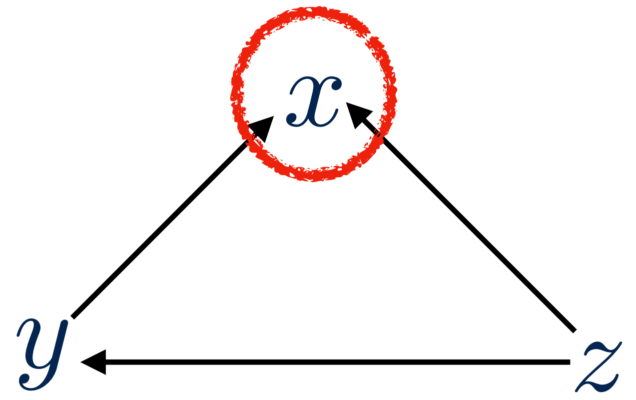
Acyclic!

small model property.

NEXPTIME Upper bound

$x \preceq \texttt{Shuffle}(yz)$

$y \preceq z$



Acyclic!

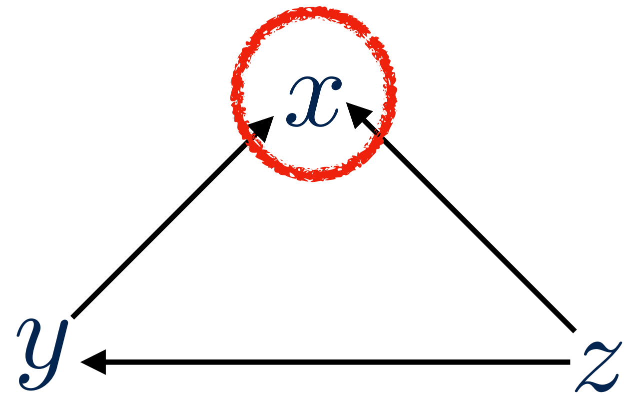
small model property.

Grammar in CNF form

NEXPTIME Upper bound

$x \preceq \texttt{Shuffle}(yz)$

$y \preceq z$



Acyclic!

small model property.

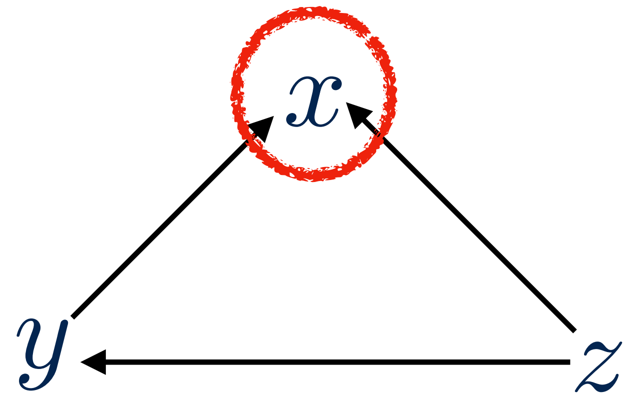
Grammar in CNF form

Parse tree

NEXPTIME Upper bound

$x \preceq \texttt{Shuffle}(yz)$

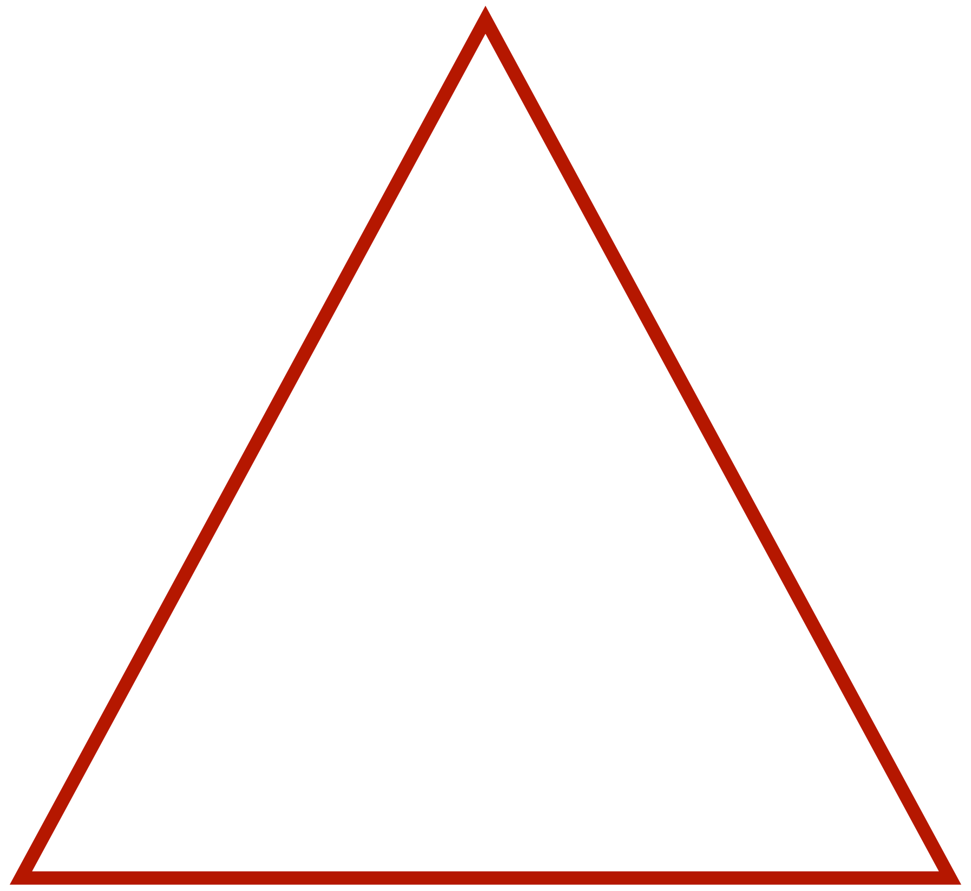
$y \preceq z$



Acyclic!

small model property.

Grammar in CNF form

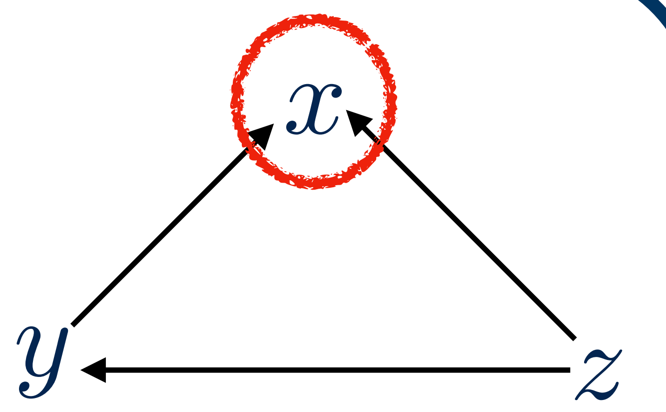


Parse tree

NEXPTIME Upper bound

$x \preceq \texttt{Shuffle}(yz)$

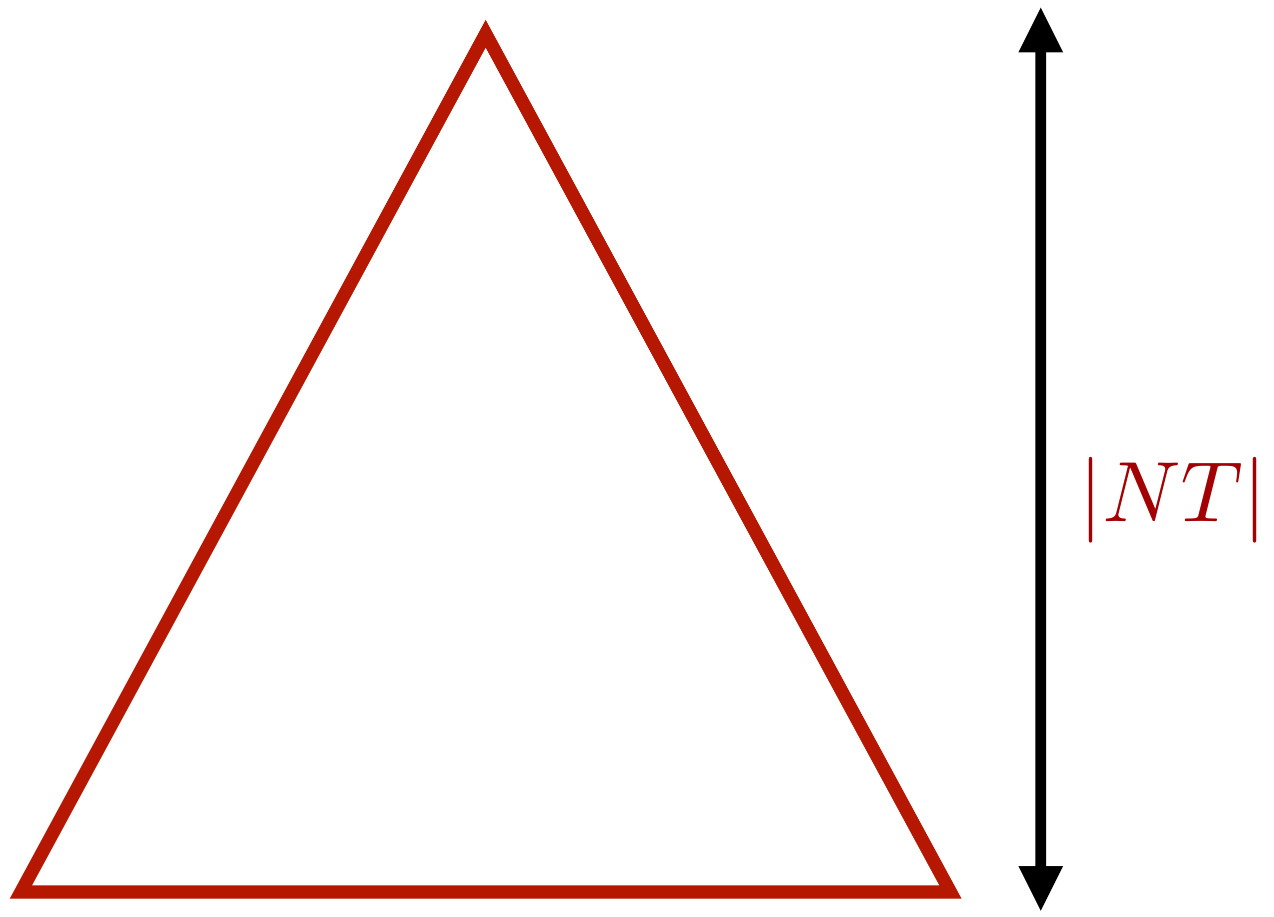
$y \preceq z$



Acyclic!

small model property.

Grammar in CNF form

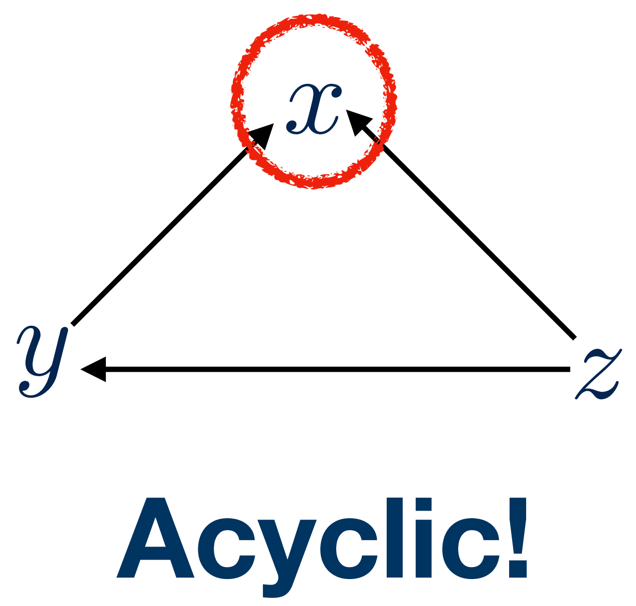


Parse tree

NEXPTIME Upper bound

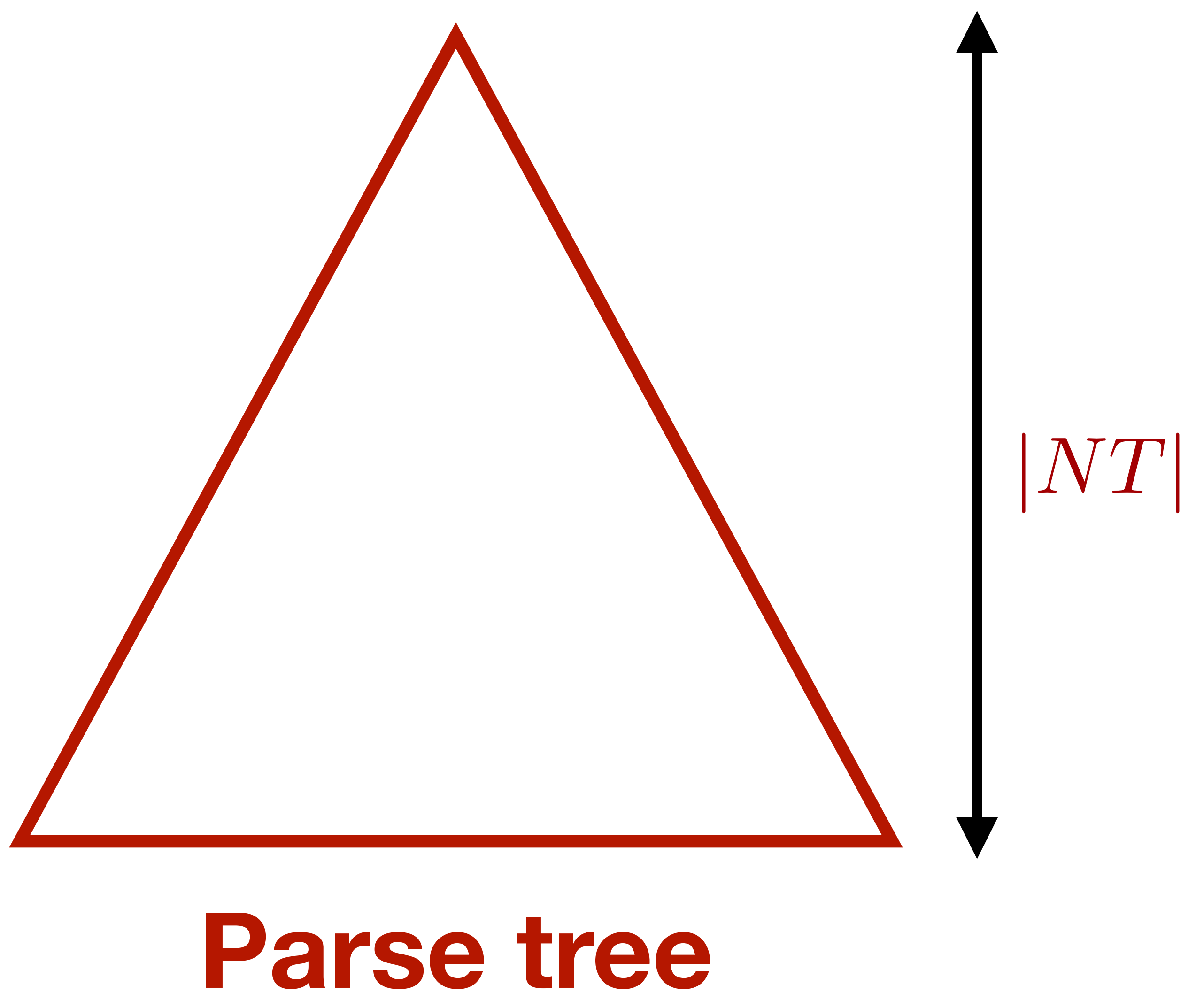
$x \preceq \mathtt{Shuffle}(yz)$

$y \preceq z$



small model property.

Grammar in CNF form

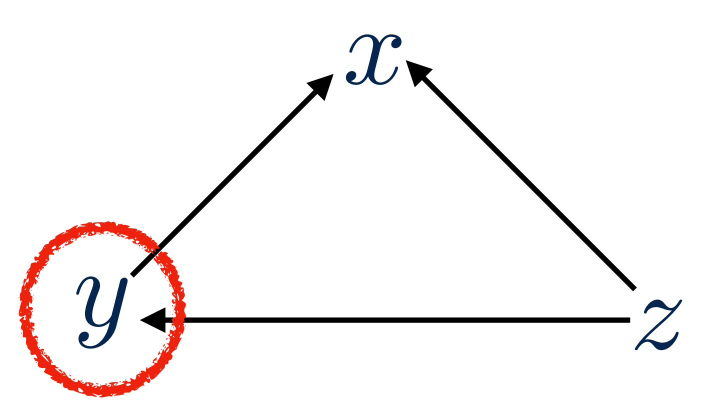


size of x $= \mathcal{O}(2^{|NT|})$

NEXPTIME Upper bound

$x \preceq \texttt{Shuffle}(yz)$

$y \preceq z$



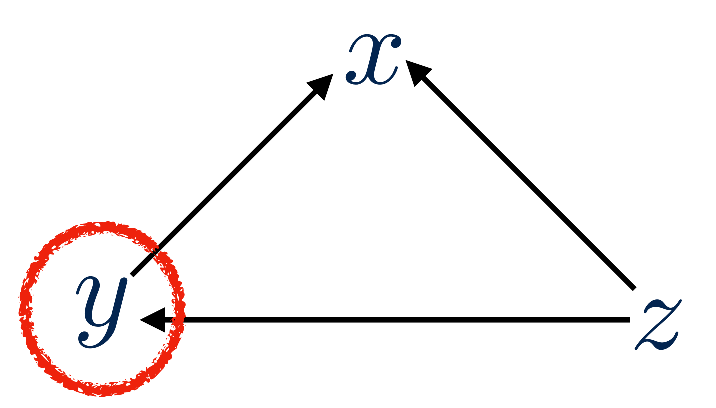
Acyclic!

small model property.

NEXPTIME Upper bound

$x \preceq \texttt{Shuffle}(yz)$

$y \preceq z$



Acyclic!

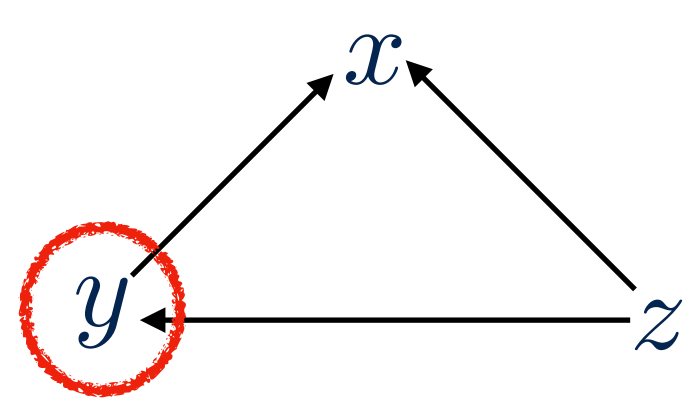
small model property.

Will x still be embedded in y?

NEXTPTIME Upper bound

$x \preceq \mathtt{Shuffle}(yz)$

$y \preceq z$



Acyclic!

small model property.

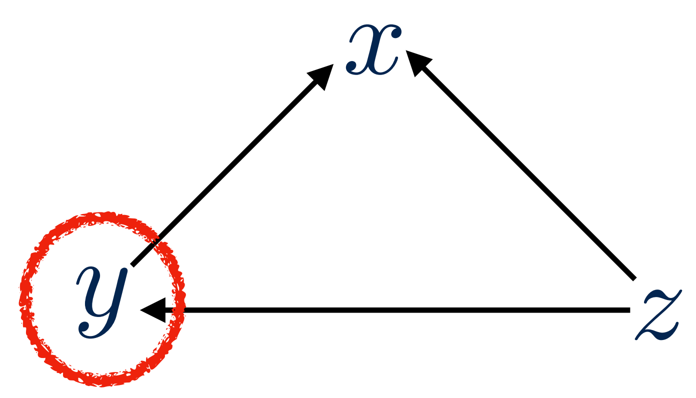
Will x still be embedded in y?

Lemma:

NEXPTIME Upper bound

$x \preceq \mathtt{Shuffle}(yz)$

$y \preceq z$



Acyclic!

small model property.

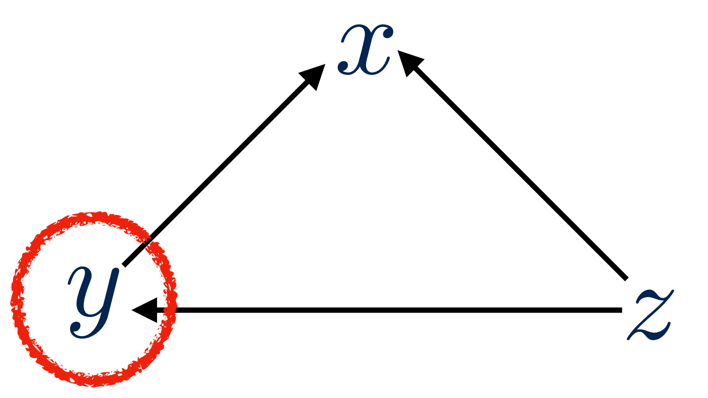
Will x still be embedded in y?

Lemma: if $w_1 \preceq w_2$ and $w_2 \in L$

NEXPTIME Upper bound

$x \preceq \mathtt{Shuffle}(yz)$

$y \preceq z$



Acyclic!

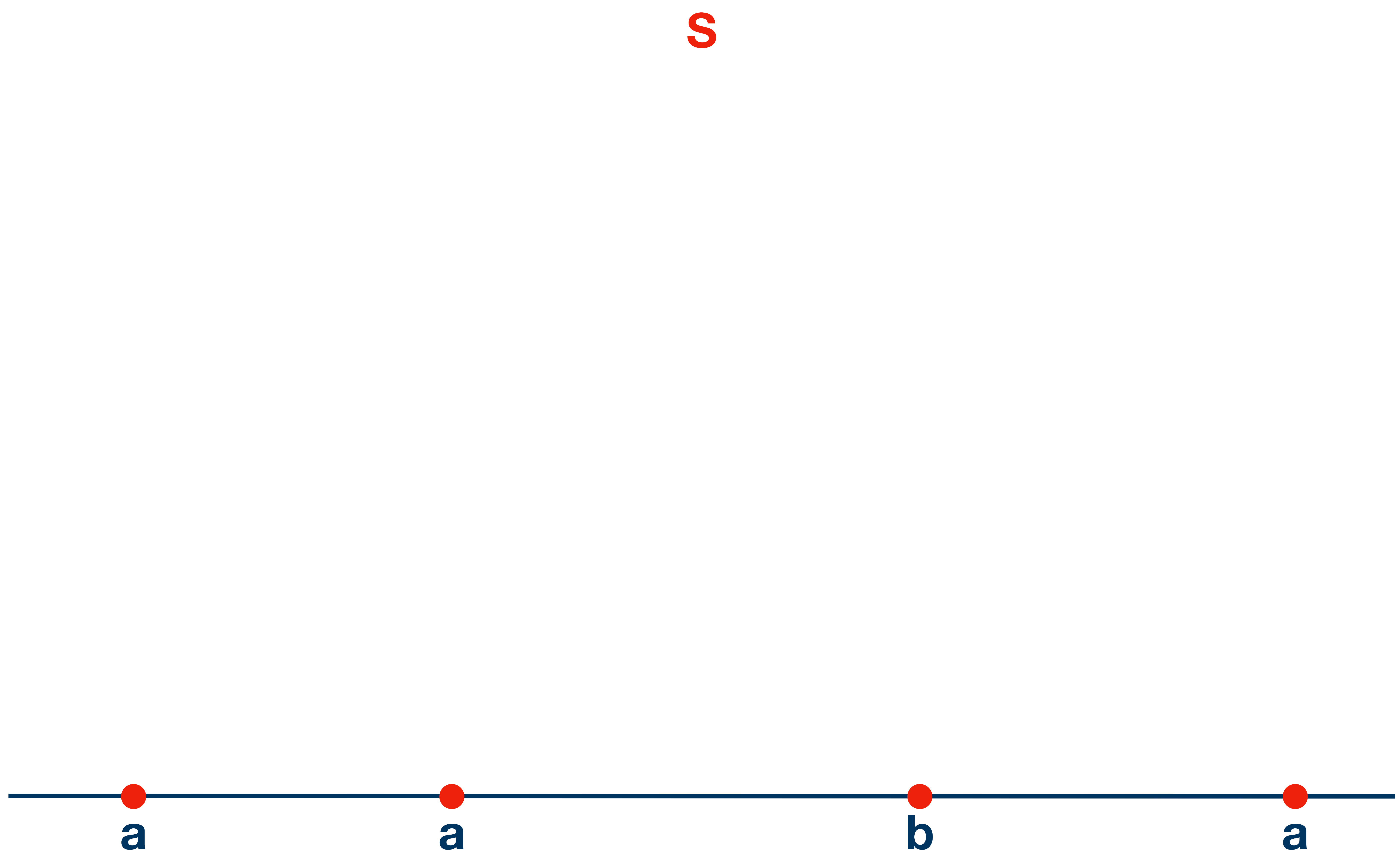
small model property.

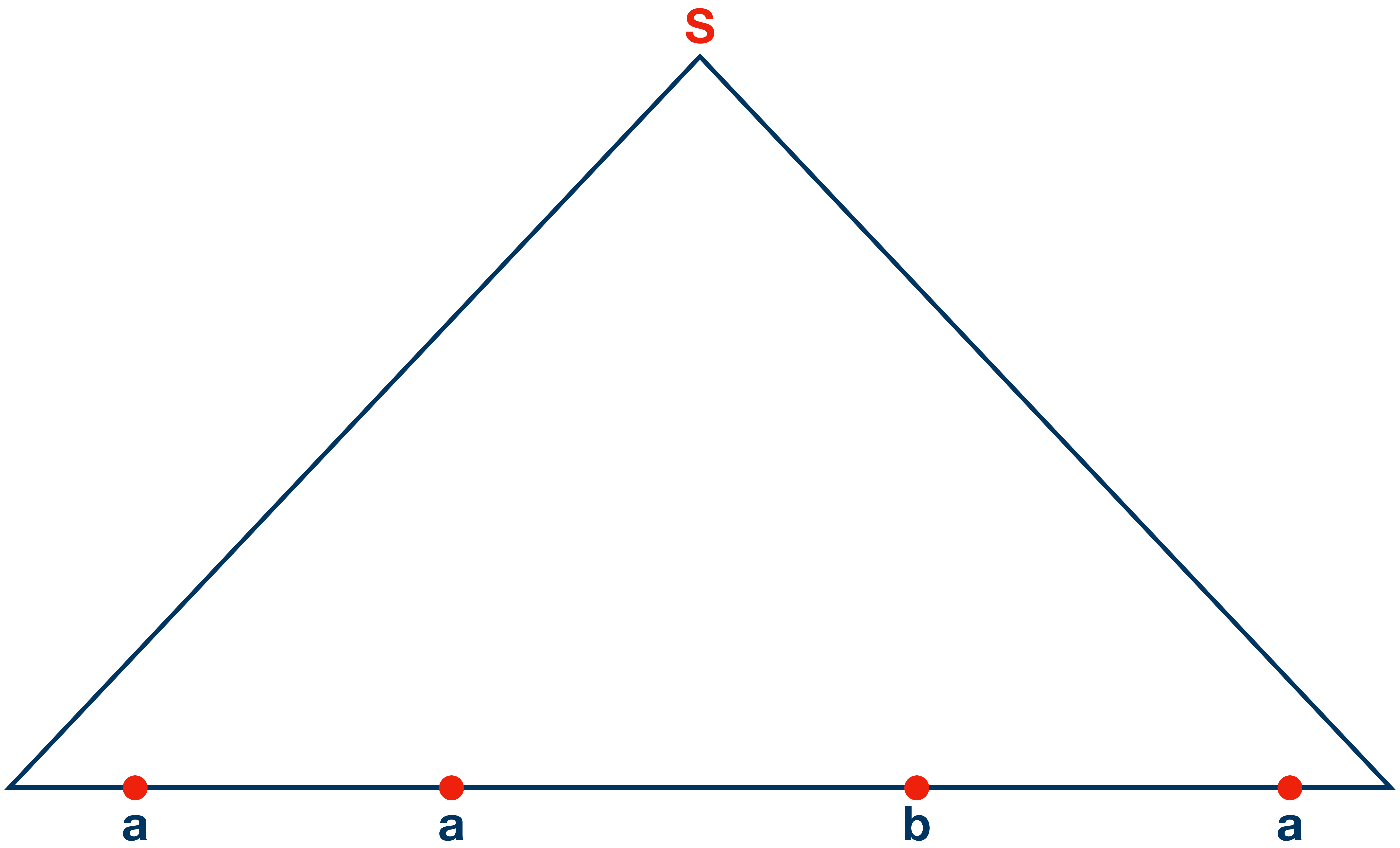
Will x still be embedded in y?

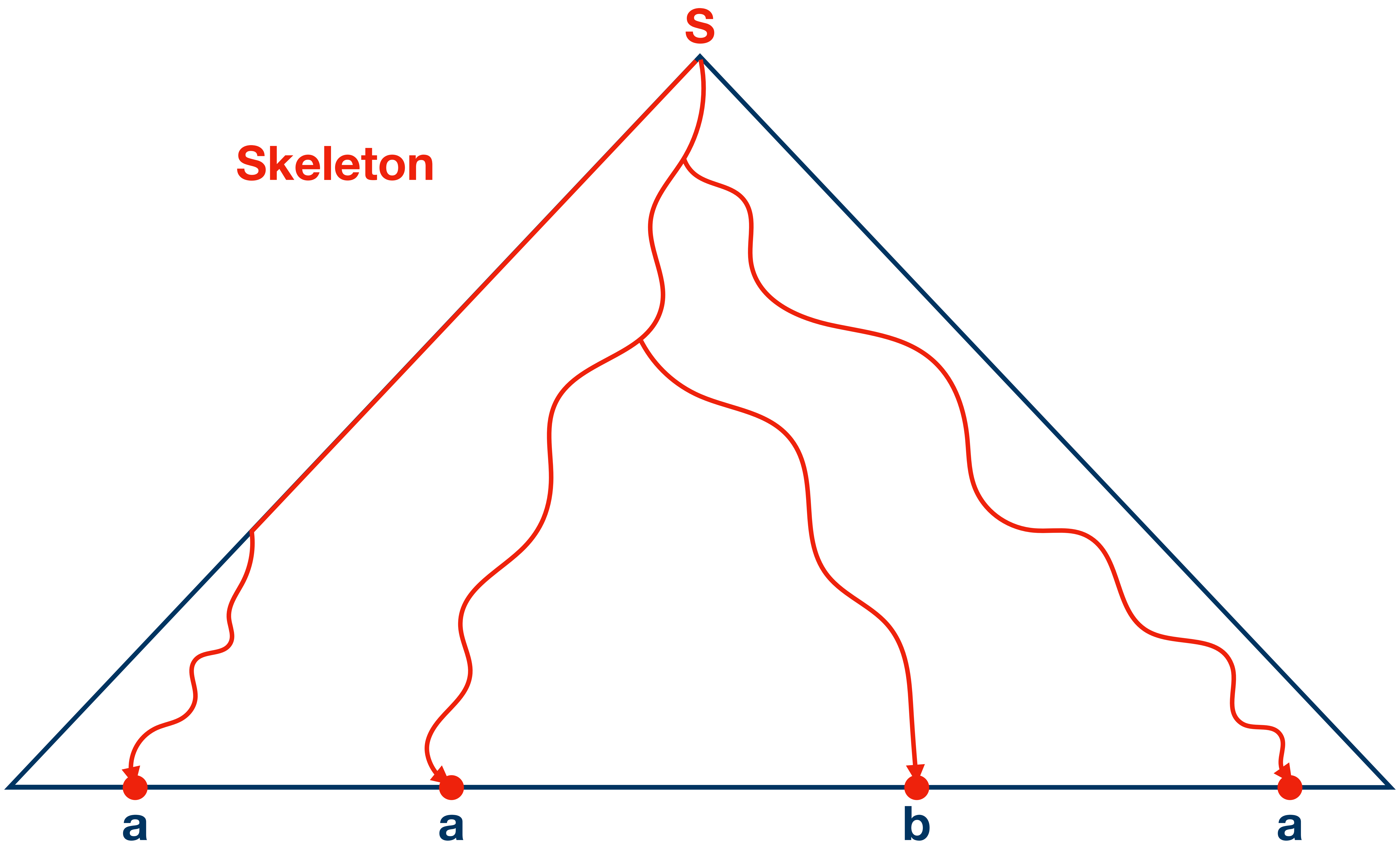
Lemma: if $w_1 \preceq w_2$ and $w_2 \in L$

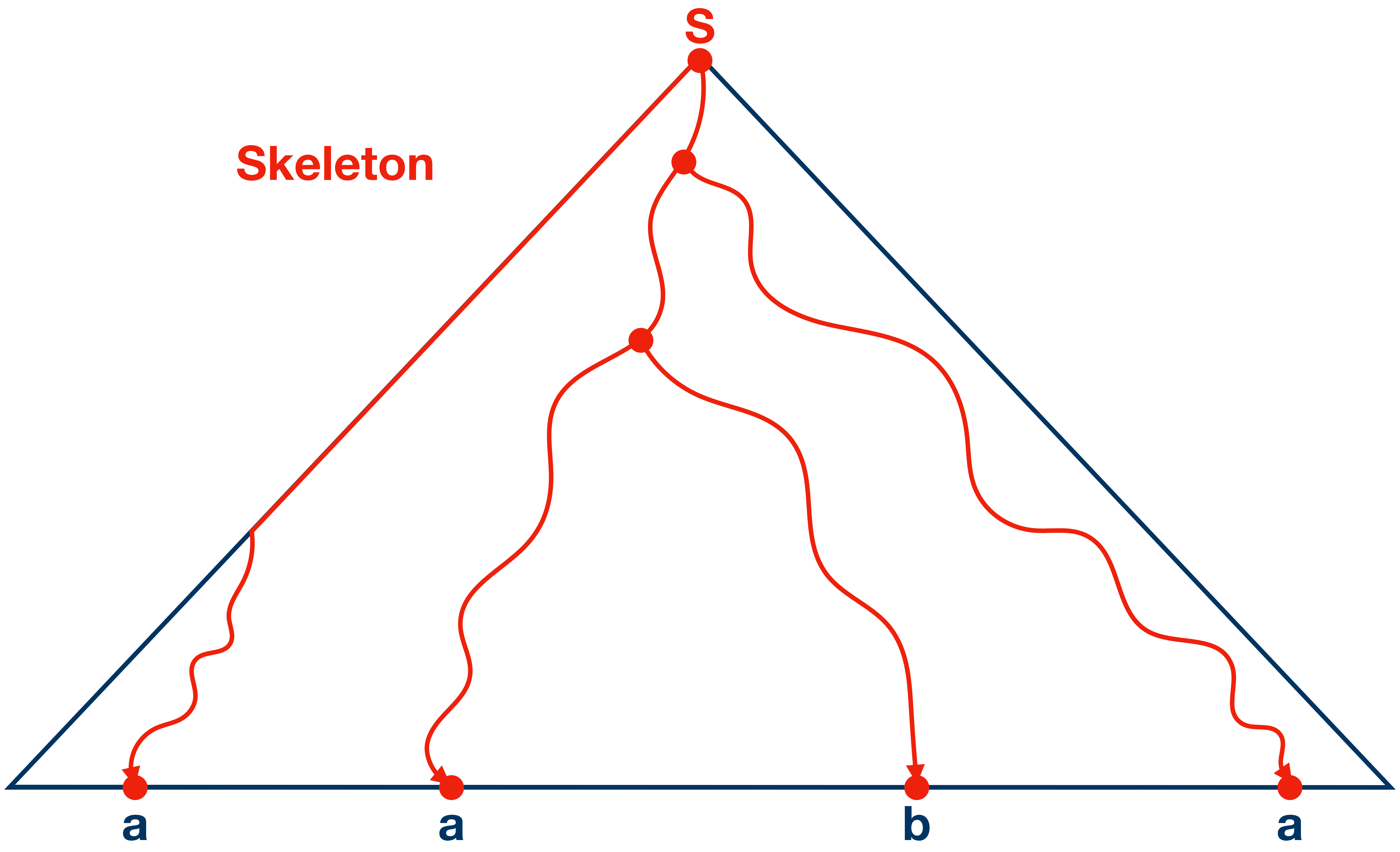
then $\exists\, w_3$: $w_1 \preceq w_3$, $w_3 \in L$ and w_3 has bounded size.

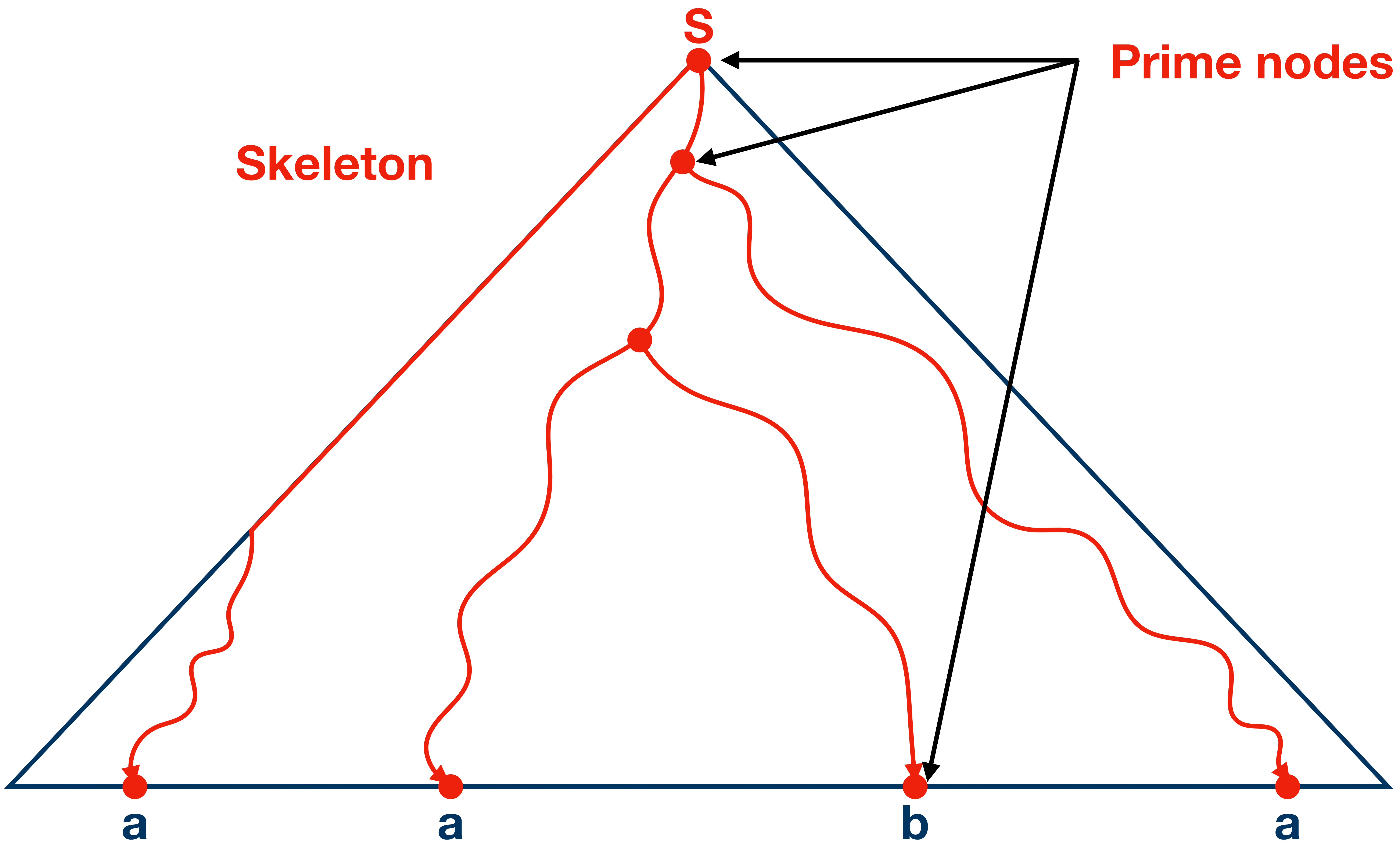

a
a
b
a

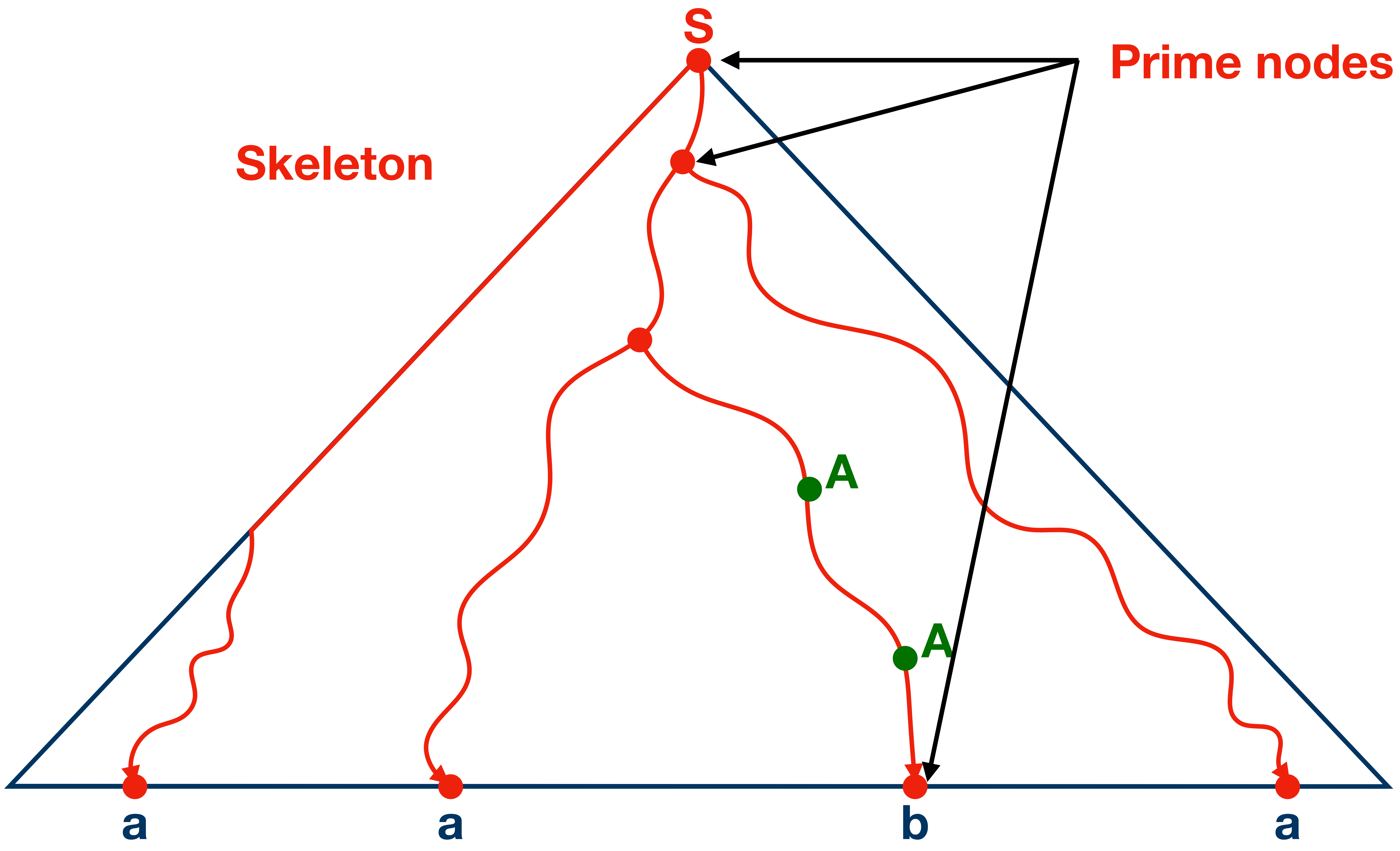

s
a
a
b
a


S
a
a
b
a

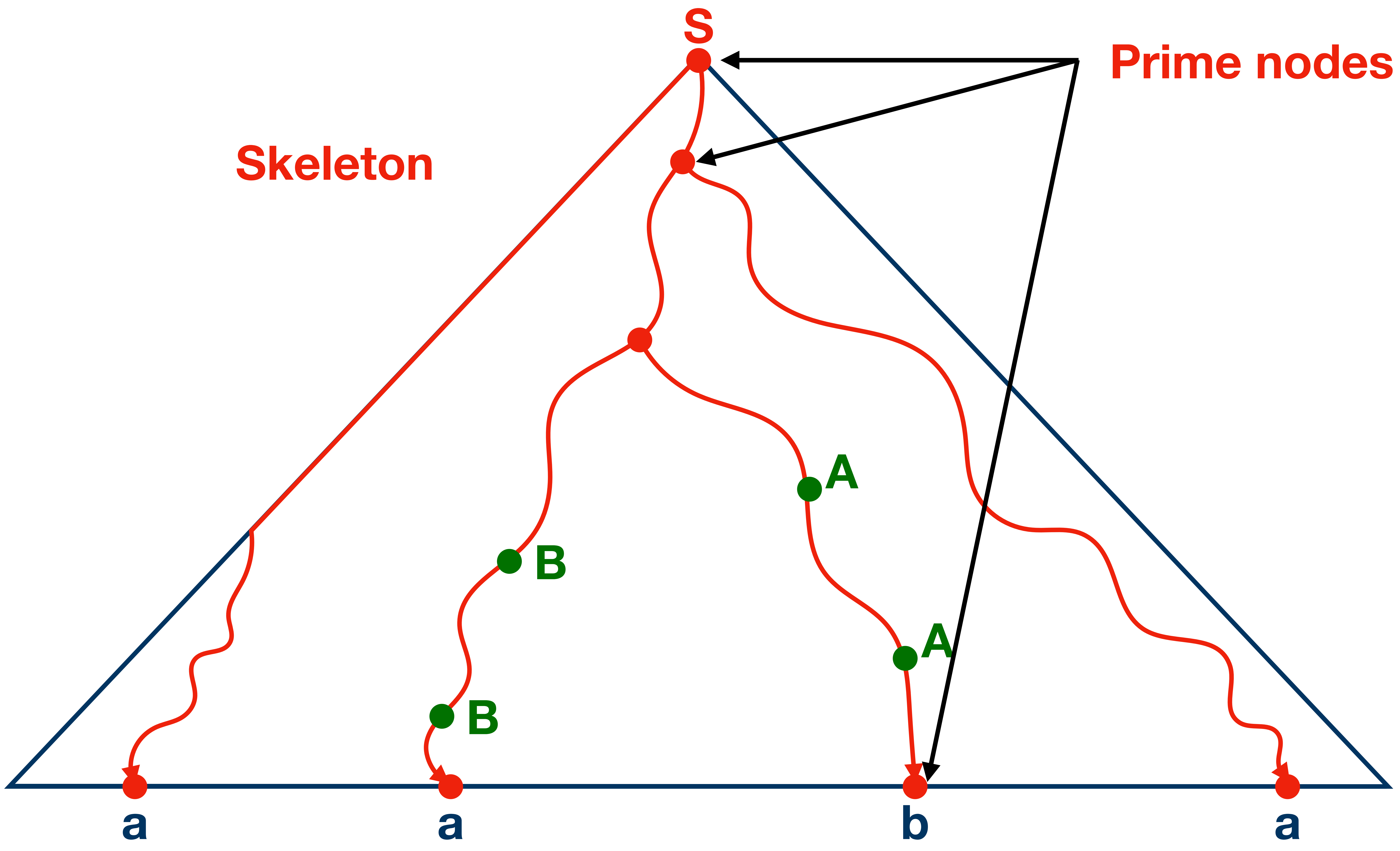

S
Skeleton
a
a
b
a

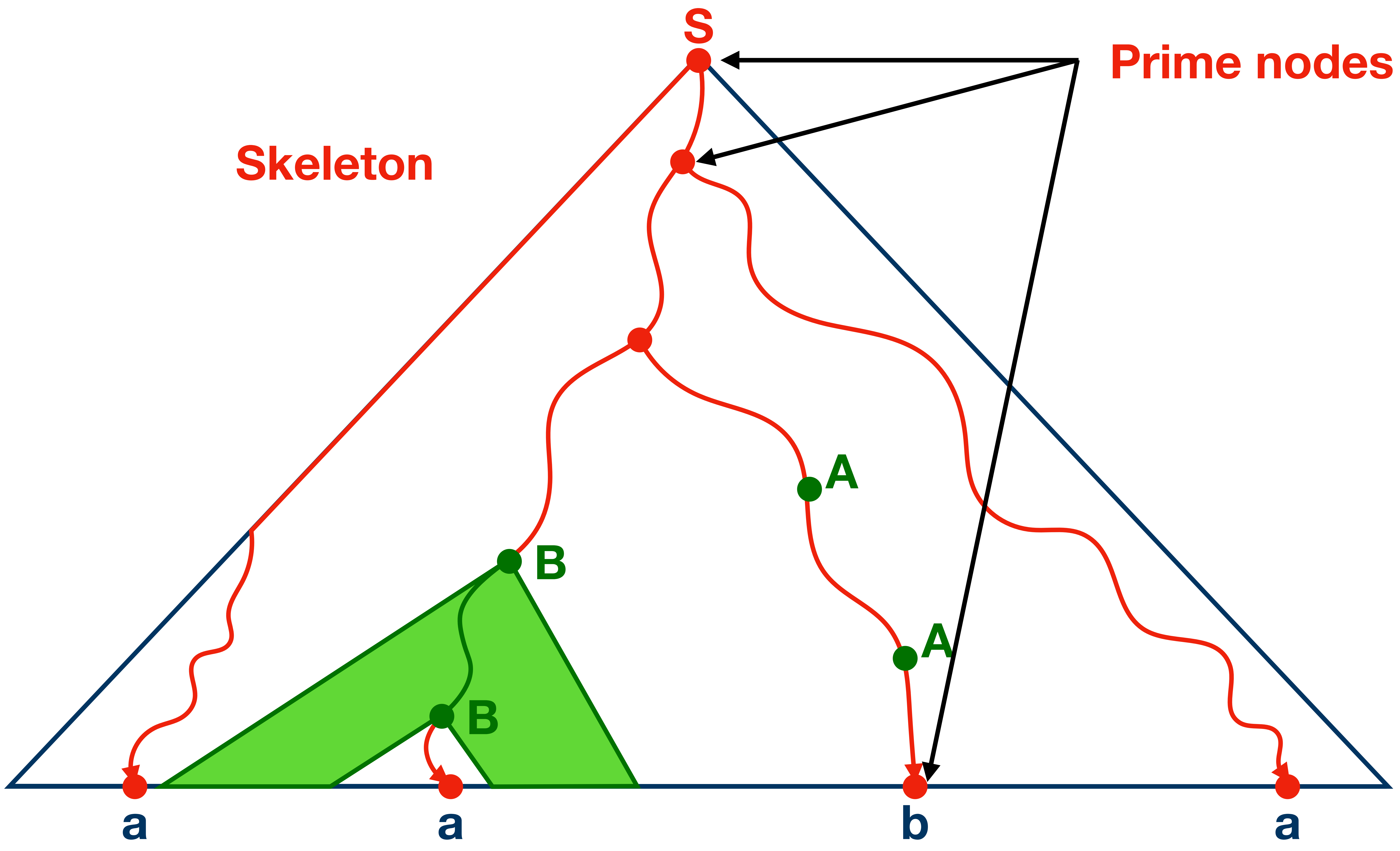

S
Skeleton
a
a
b
a

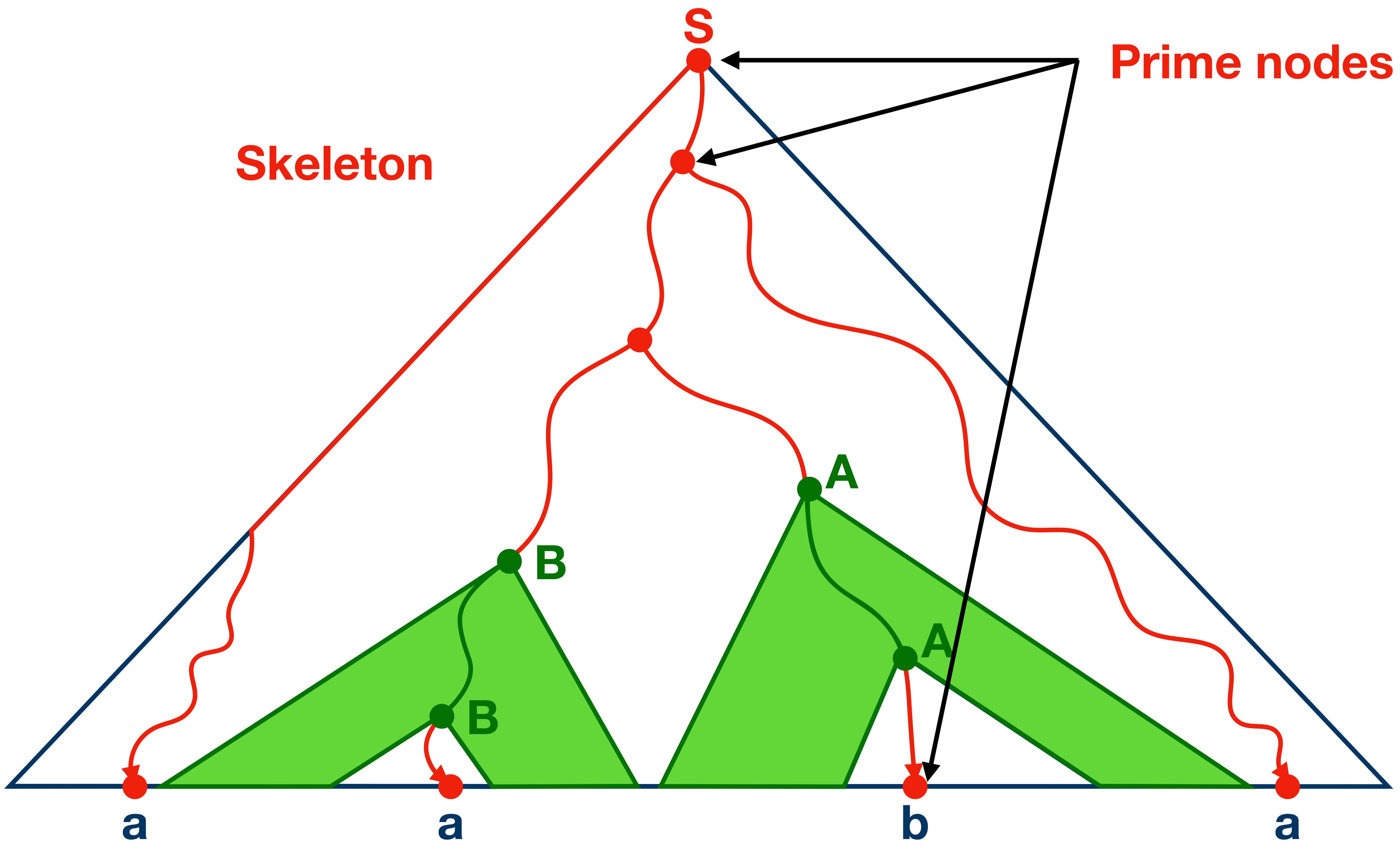

S
Prime nodes
Skeleton
a
a
b
a



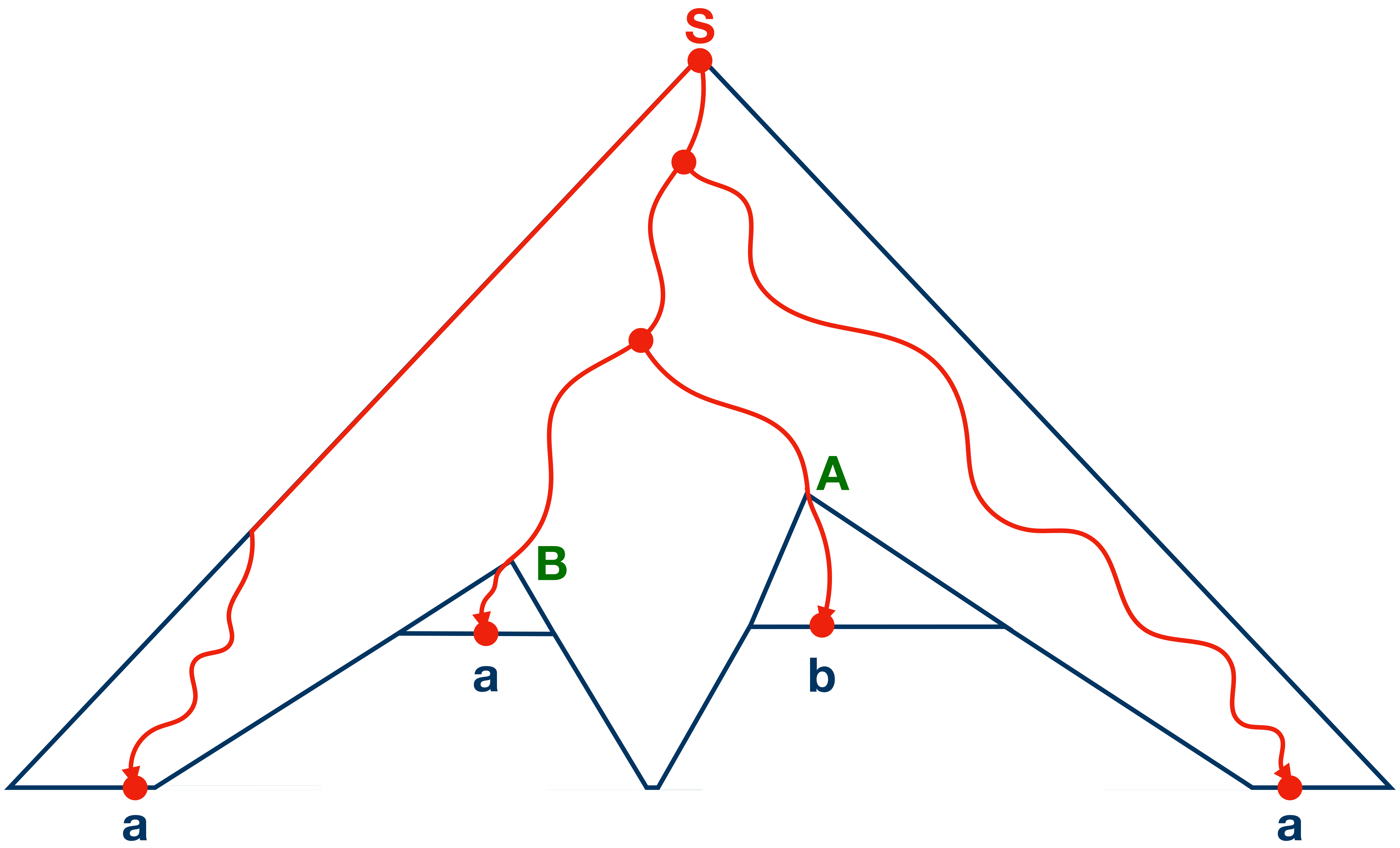
S
Prime nodes
Skeleton
A
A
a
a
b
a

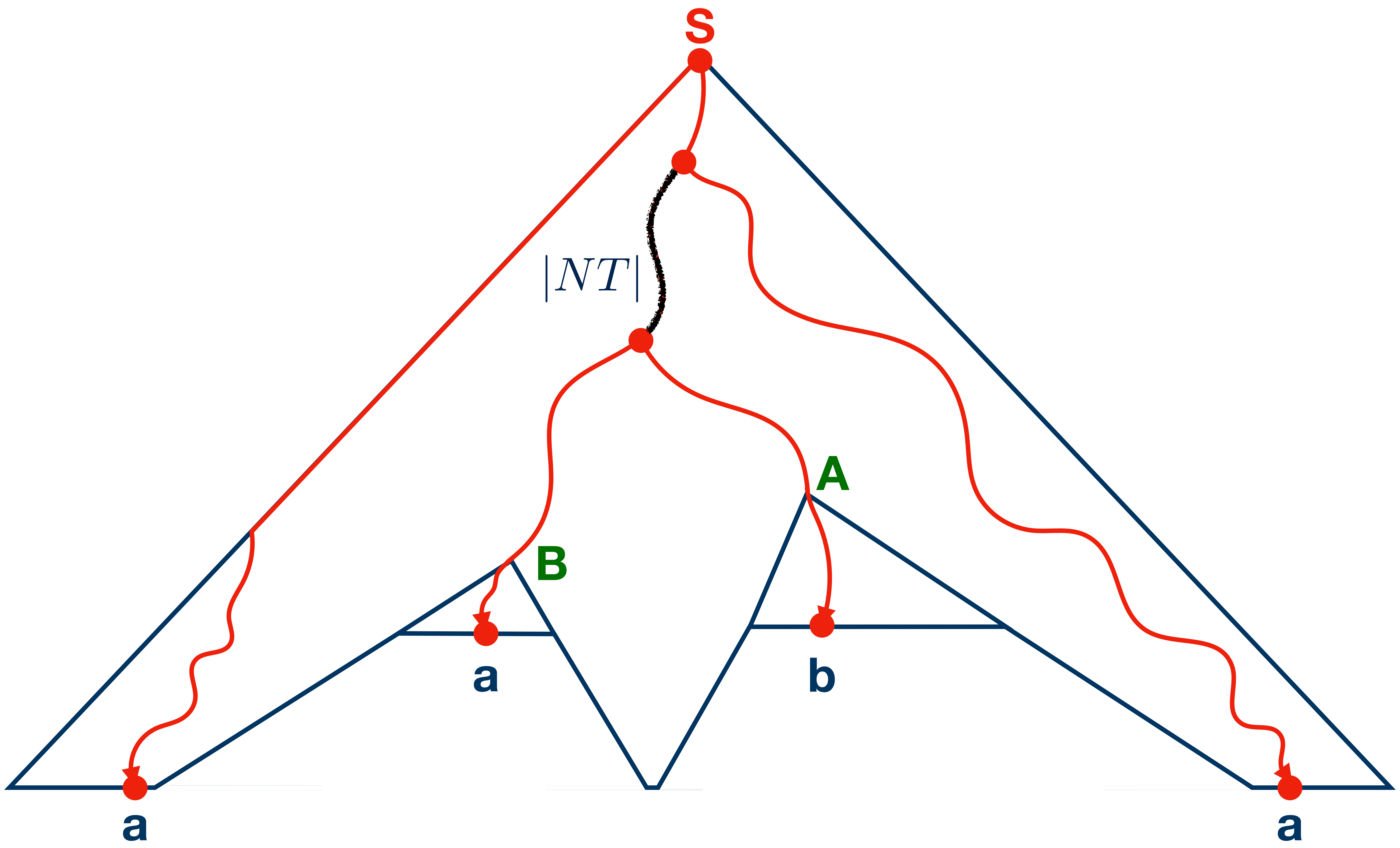

S
Prime nodes
Skeleton
A
B
A
B
a
a
b
a

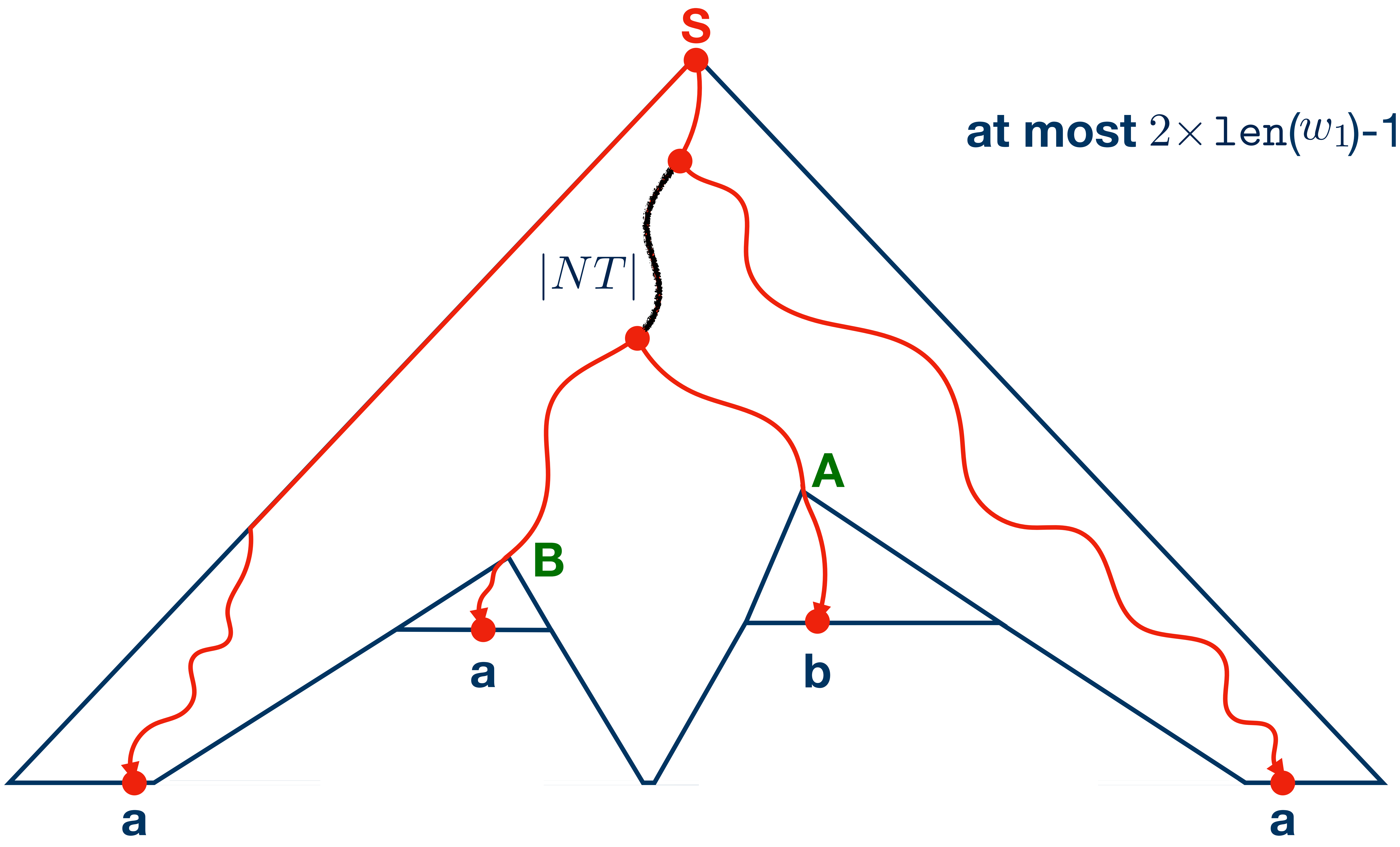

S
Prime nodes
Skeleton
A
A
B
B
a
a
b
a



S
Prime nodes
Skeleton
A
B
A
B
a
a
b
a


S
A
B
a
b
a
a


S
$|NT|$
A
B
a
b
a
a

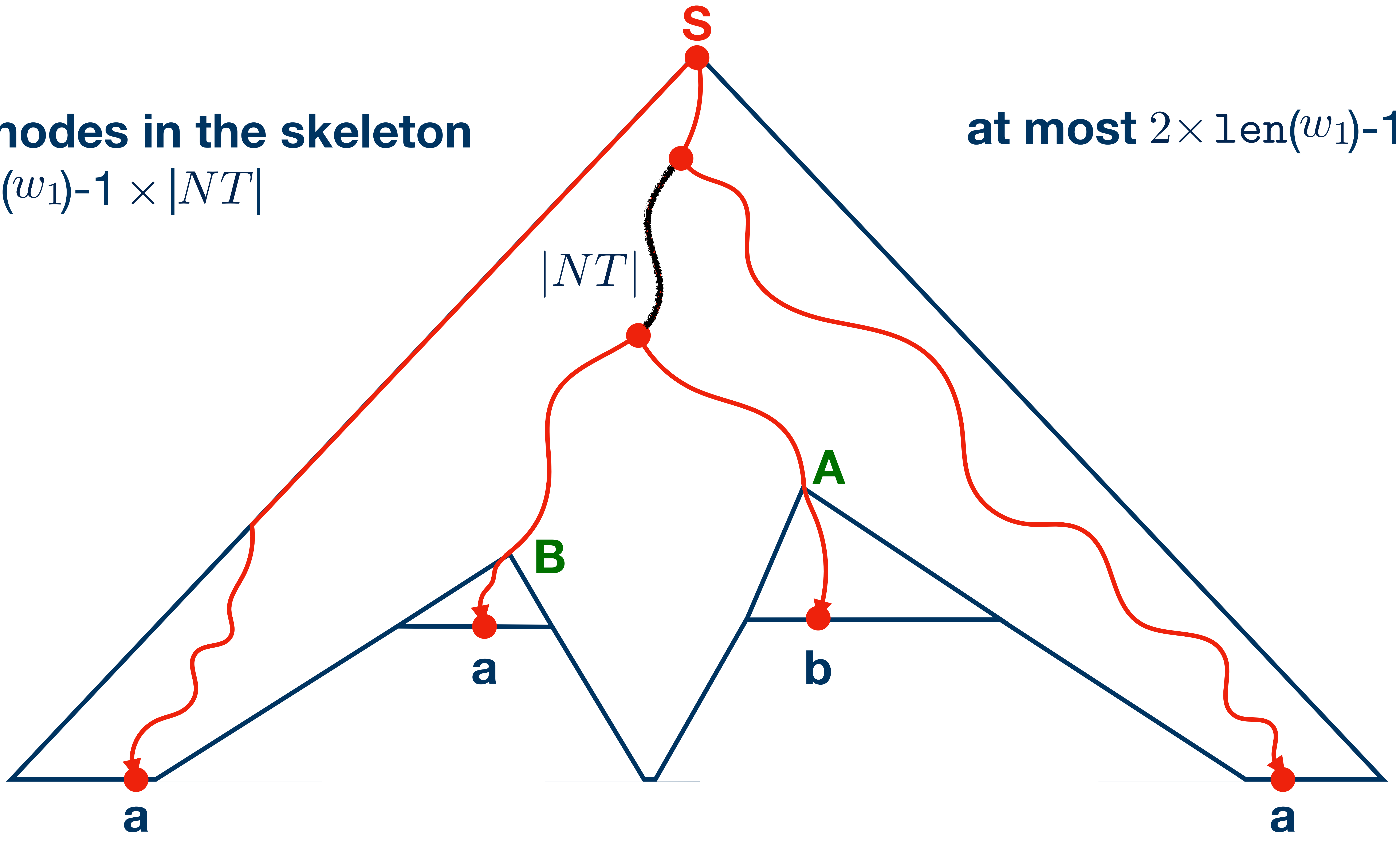


at most $2\times \texttt{len}(w_1)$-1 prime nodes

Total number of nodes in the skeleton

$= 2 \times \texttt{len}(w_1) - 1 \times |NT|$

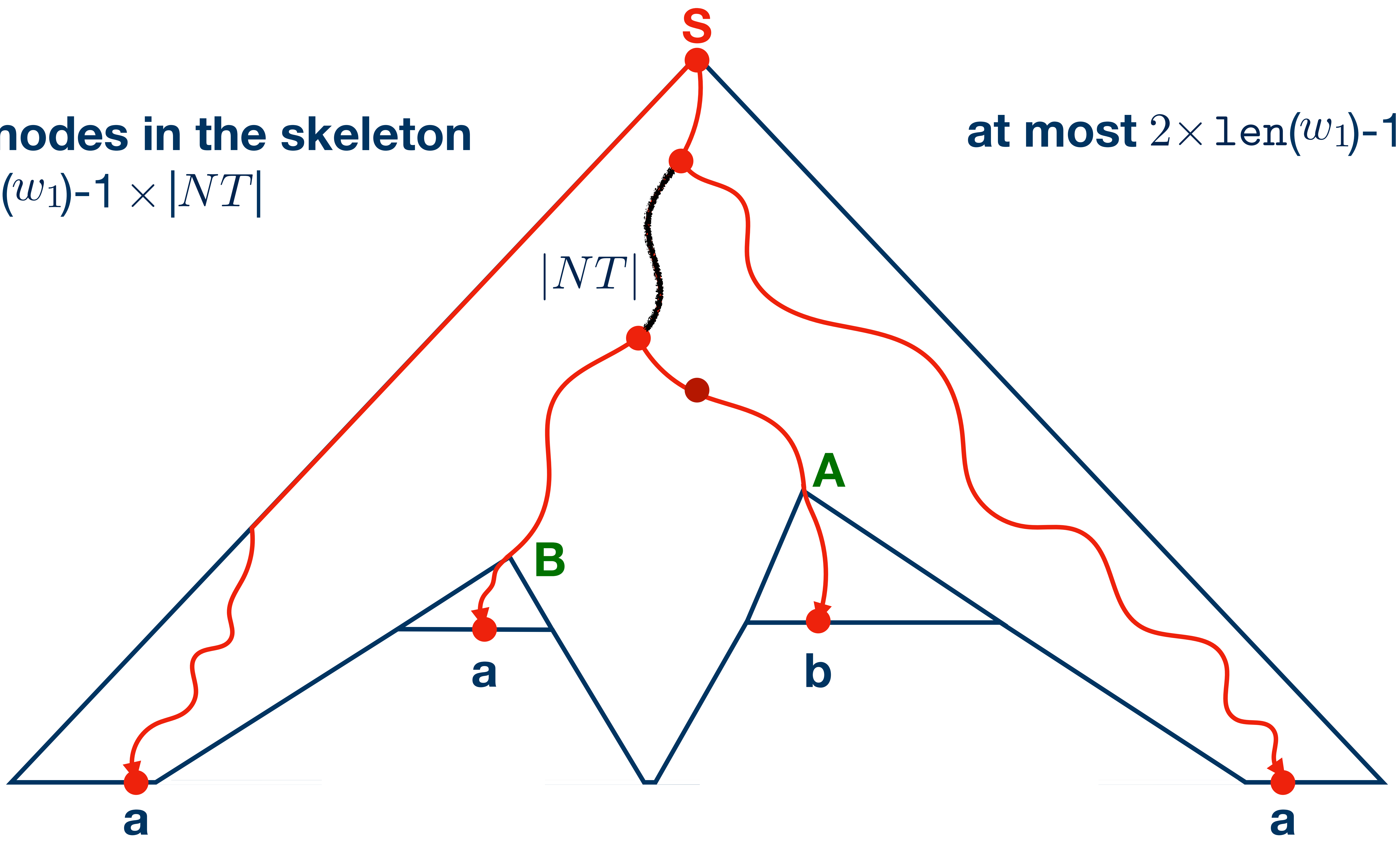
at most $2 \times \texttt{len}(w_1) - 1$ prime nodes



Total number of nodes in the skeleton

$= 2 \times \texttt{len}(w_1) - 1 \times |NT|$

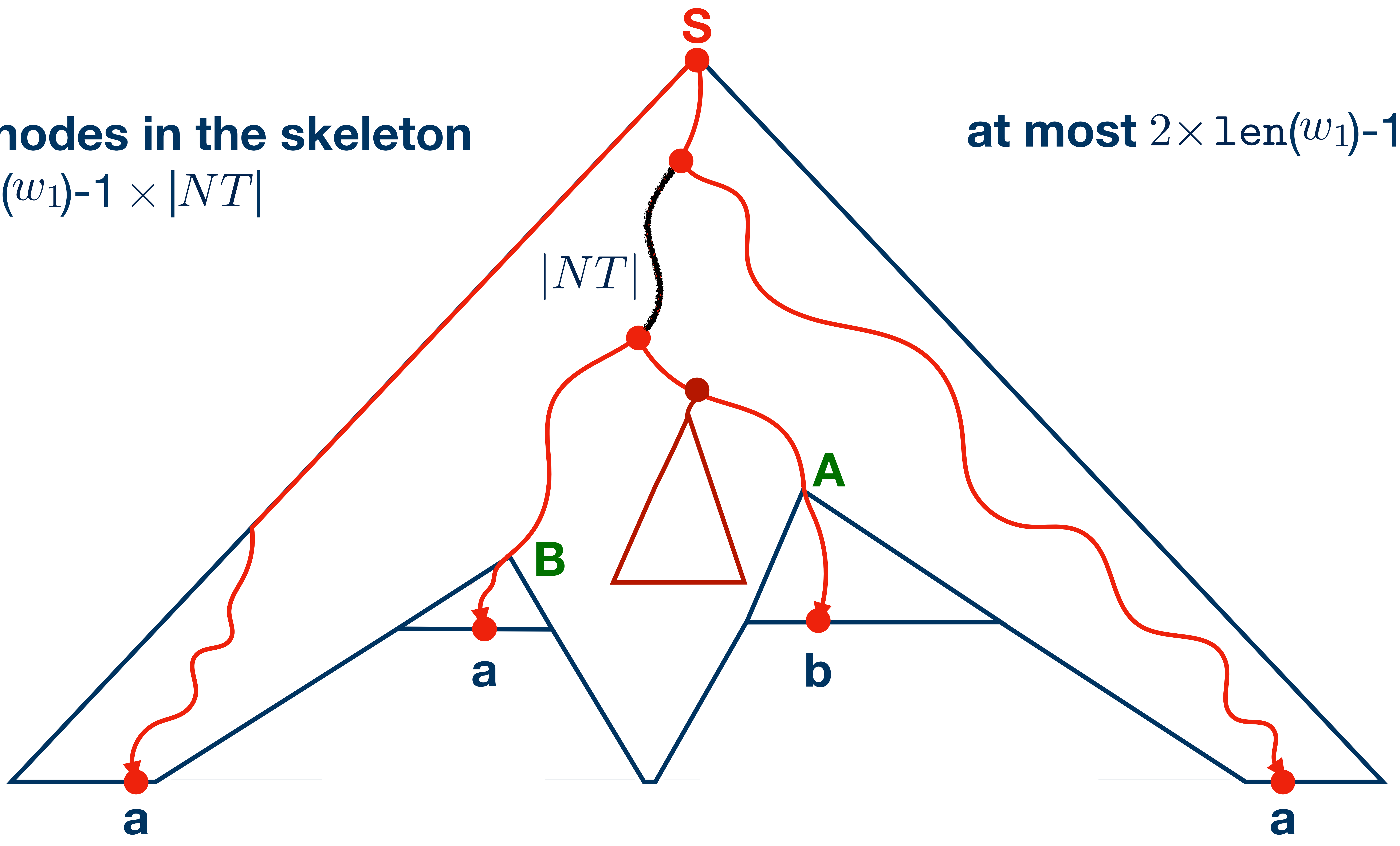
at most $2 \times \texttt{len}(w_1)$-1 prime nodes

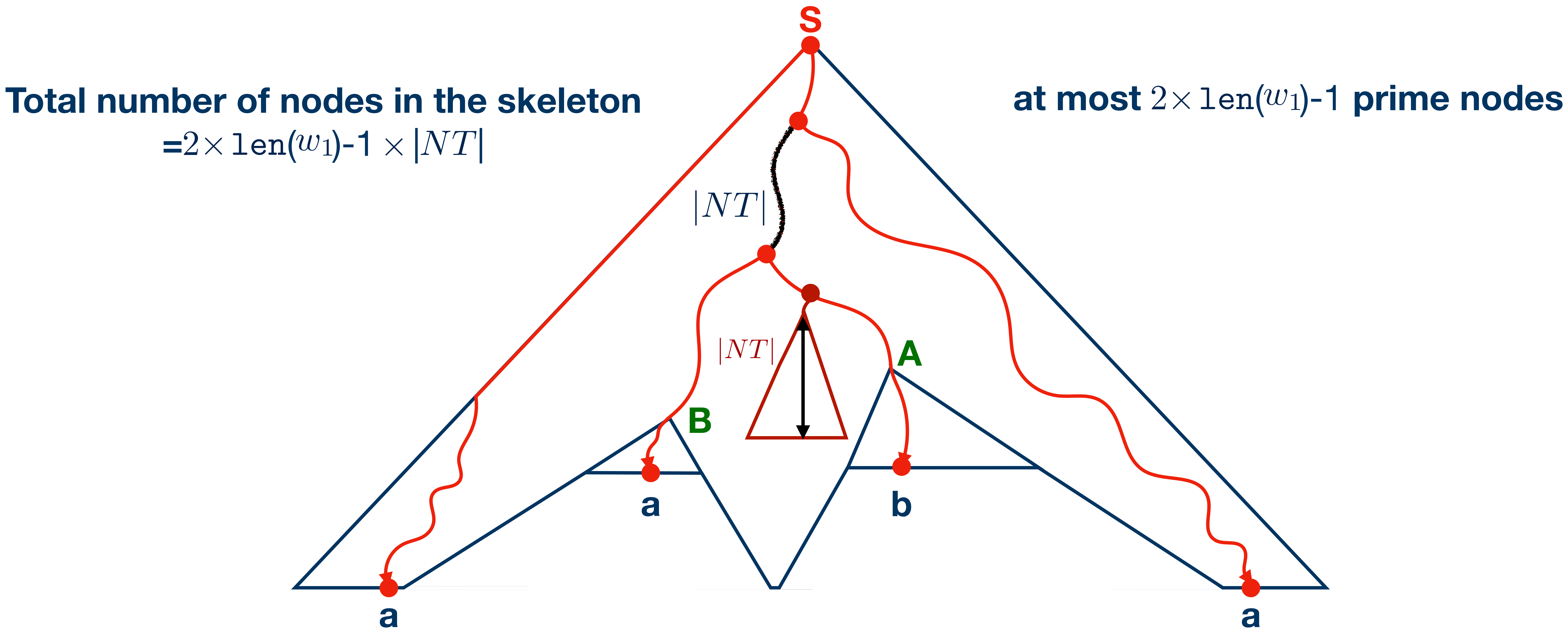


Total number of nodes in the skeleton

=$2\times \texttt{len}(w_1)$-1 $\times |NT|$

at most $2\times \texttt{len}(w_1)$-1 prime nodes




Total number of nodes in the skeleton
$=2\times \texttt{len}(w_1)-1 \times |NT|$
at most $2\times \texttt{len}(w_1)-1$ prime nodes
S
$|NT|$
$|NT|$
A
B
a
b
a
a

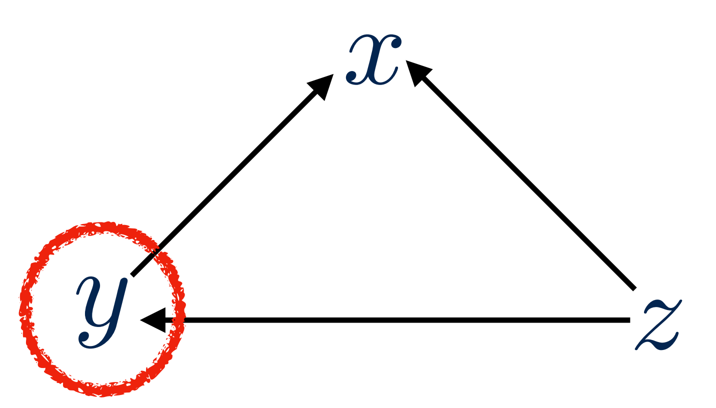
Total number of nodes in the skeleton
=$2\times$ len(w_1)-1 $\times |NT|$
at most $2\times$ len(w_1)-1 prime nodes
S
$|NT|$
subtree of size atmost $2^{|NT|}$
$|NT|$
A
B
a
b
a
a

S
Total number of nodes in the skeleton
$=2\times$ len(w_1)-1 $\times |NT|$
at most $2\times$ len(w_1)-1 prime nodes
$|NT|$
Size of the parse tree
$\mathcal{O}(Len(w_1) \times |NT| \times 2^{|NT|})$
subtree of size atmost $2^{|NT|}$
$|NT|$
A
B
a
b
a
a

NEXPTIME Upper bound

$x \preceq \texttt{Shuffle}(yz)$

$y \preceq z$



Acyclic!

small model property.

Size of the parse tree

$\mathcal{O}(Len(x) \times |NT| \times 2^{|NT|})$

NEXPTIME Upper bound

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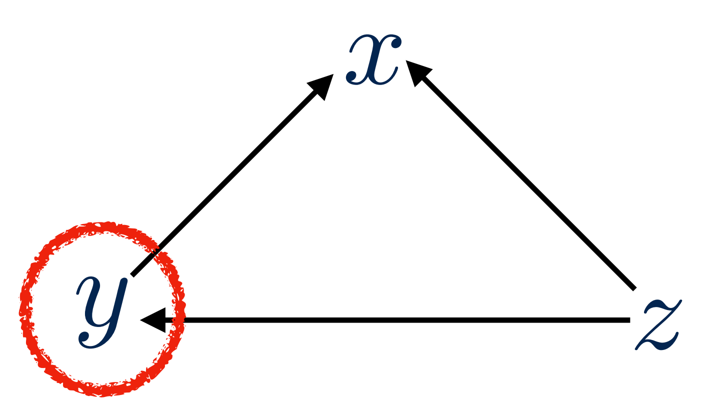

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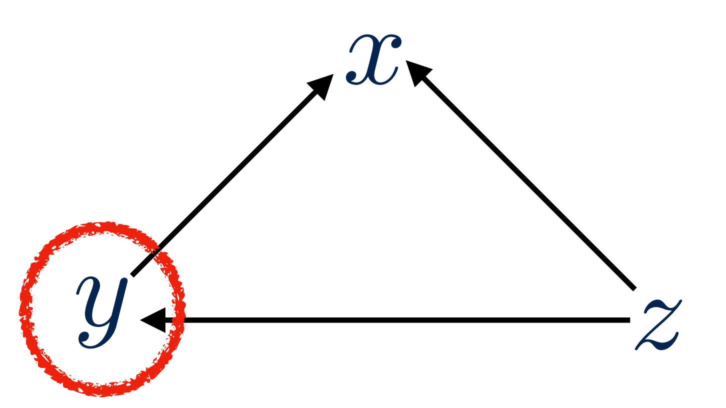

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Size of the parse tree

$\mathcal{O}(Len(x) \times |NT| \times 2^{|NT|})$

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y is bounded by $\mathcal{O}(2^{2^{2 \times |NT|}} \times |NT|)$

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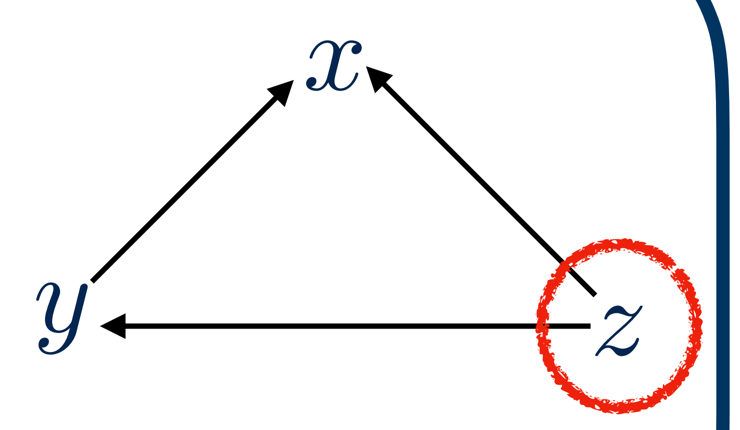

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$\mathcal{O}(\texttt{Len}(y) \times |NT| \times 2^{|NT|})$

y is bounded by $\mathcal{O}(2^{2\times|NT|} \times |NT|)$

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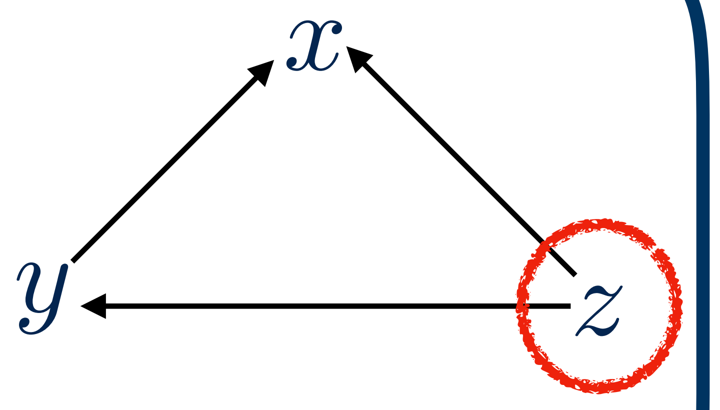


Acyclic!

small model property.

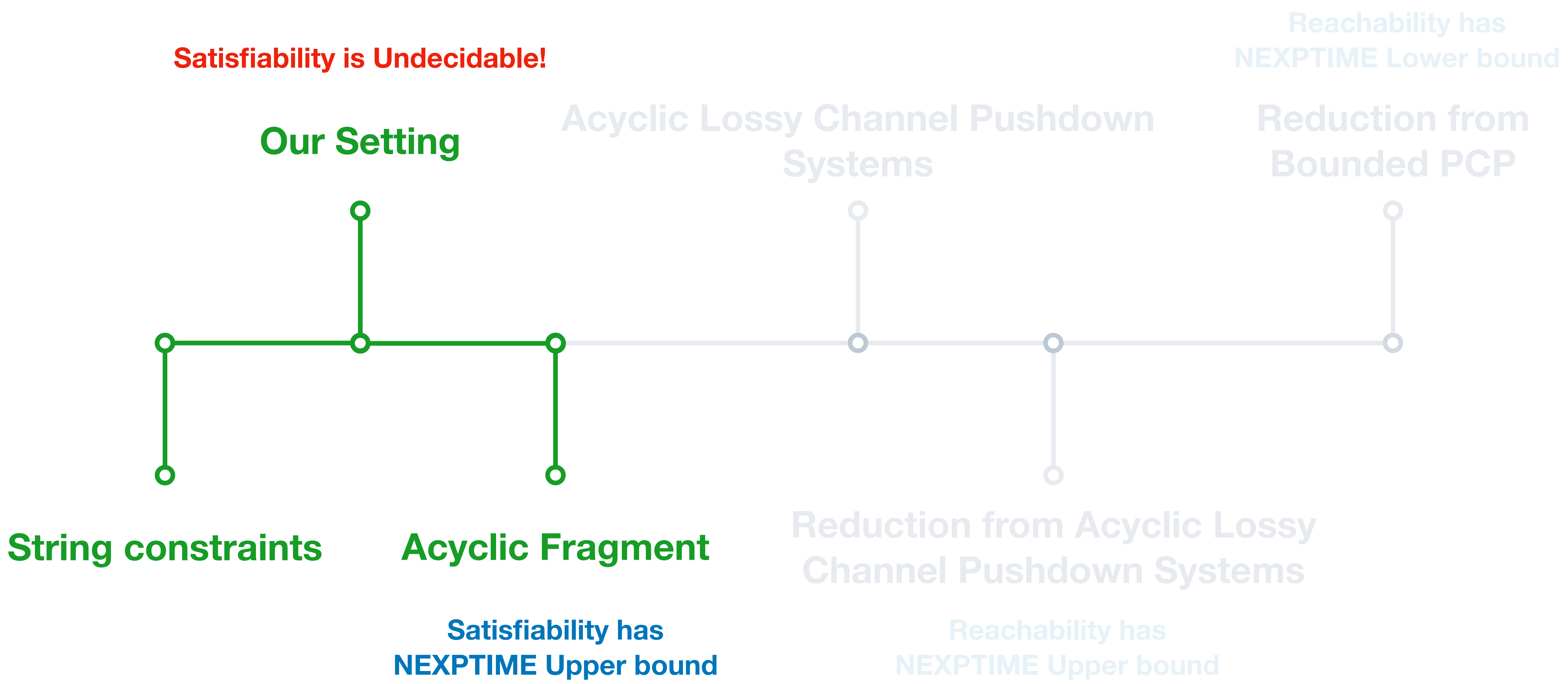
Size of the parse tree

$\mathcal{O}(\texttt{Len}(y) \times |NT| \times 2^{|NT|})$

y is bounded by $\mathcal{O}(2^{2\times|NT|} \times |NT|)$

z is bounded by $\mathcal{O}(2 \times 2^{3\times|NT|} \times |NT|^2)$

Outline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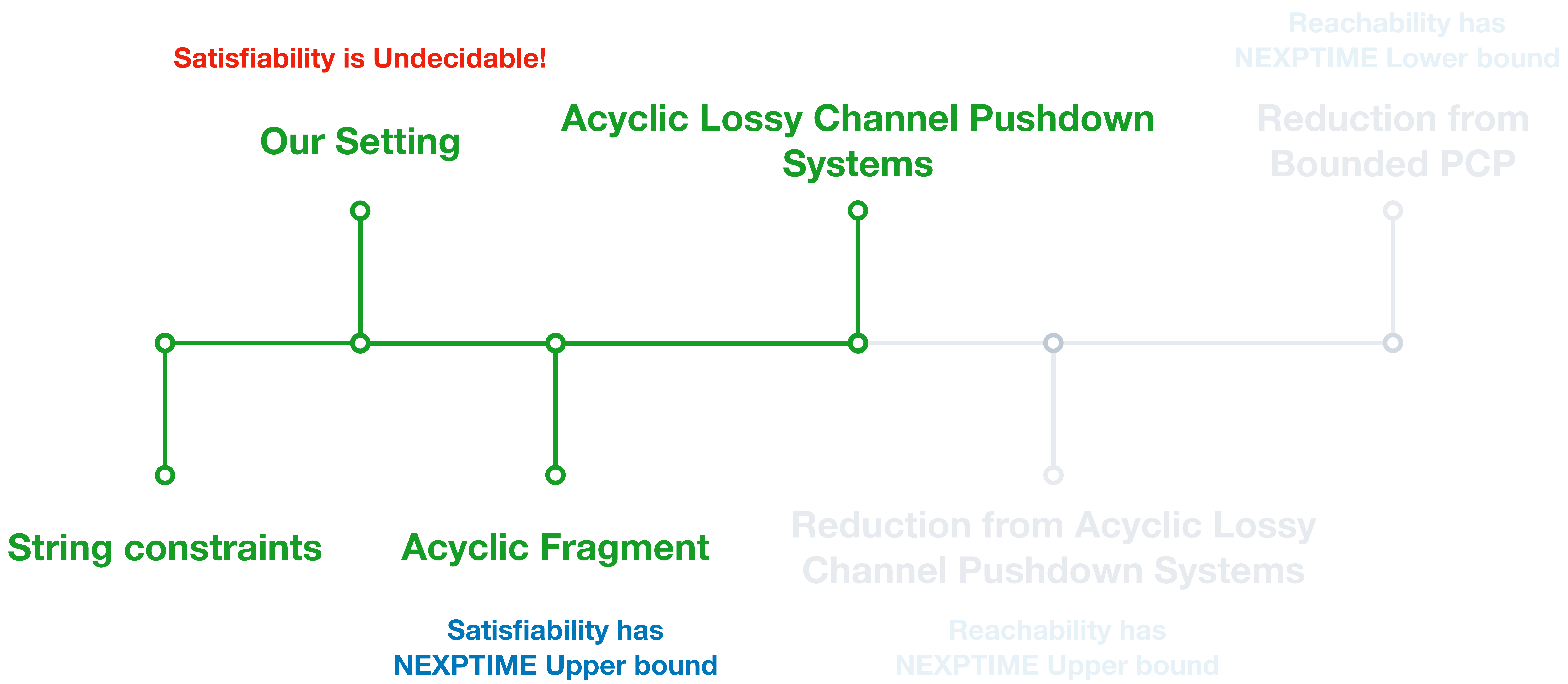

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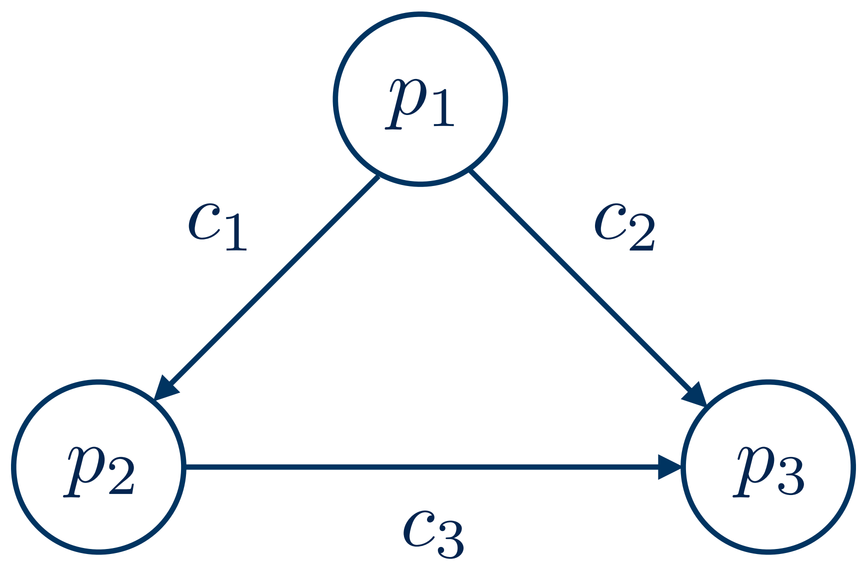
Acyclic Lossy Channel Pushdown Systems



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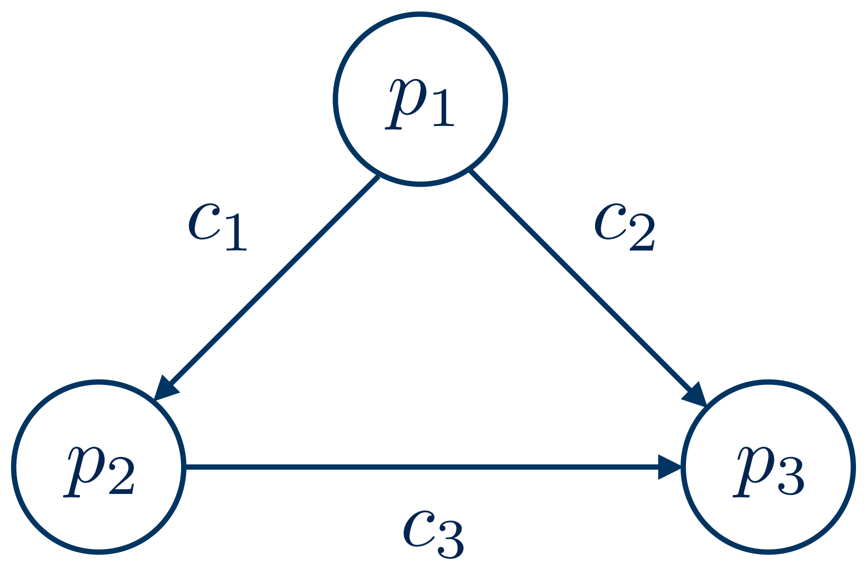

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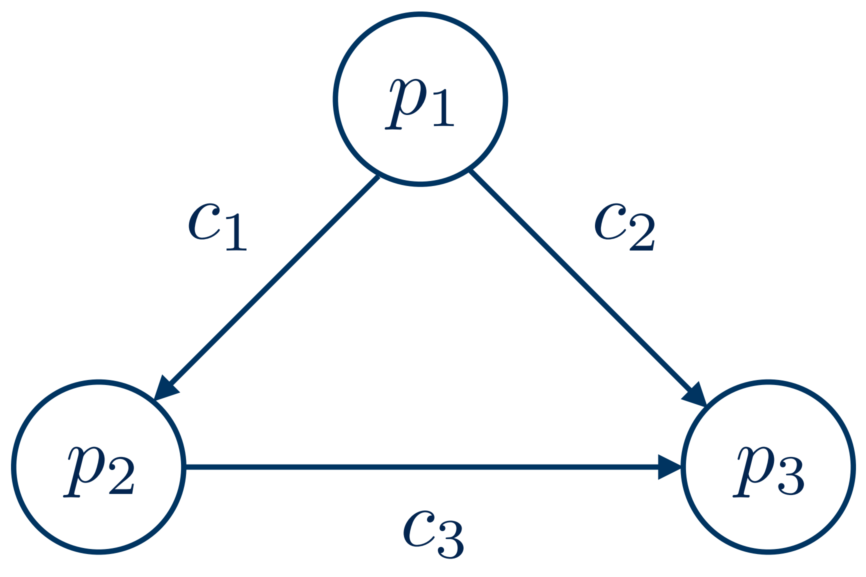
Processes - Pushdown Systems



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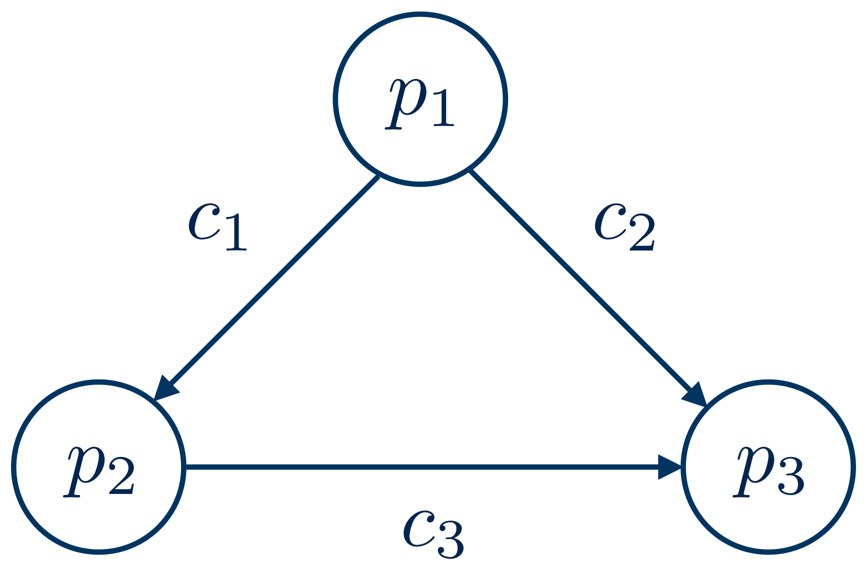
Processes - Pushdown Systems

Lossy FIFO channels



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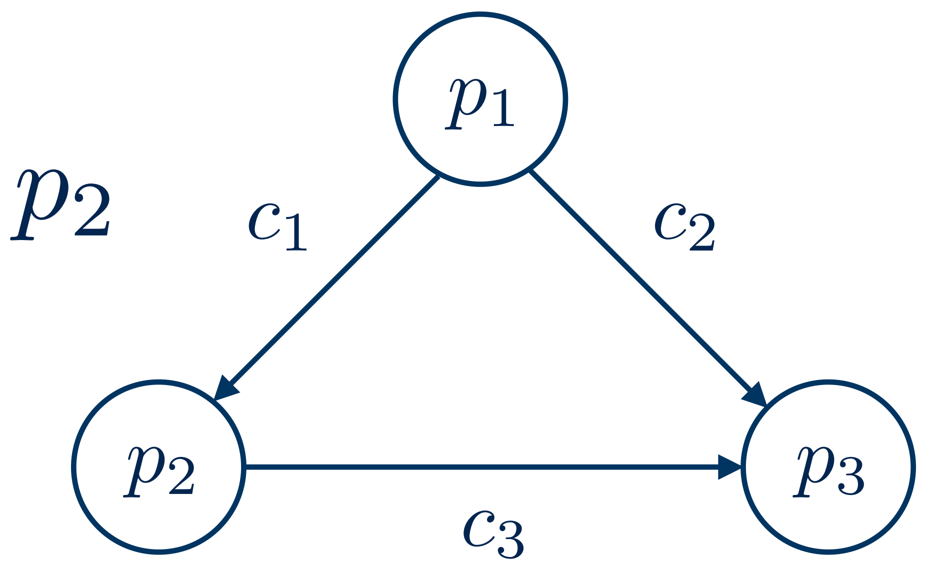
$$\texttt{Reader}(c_1) = p_2$$



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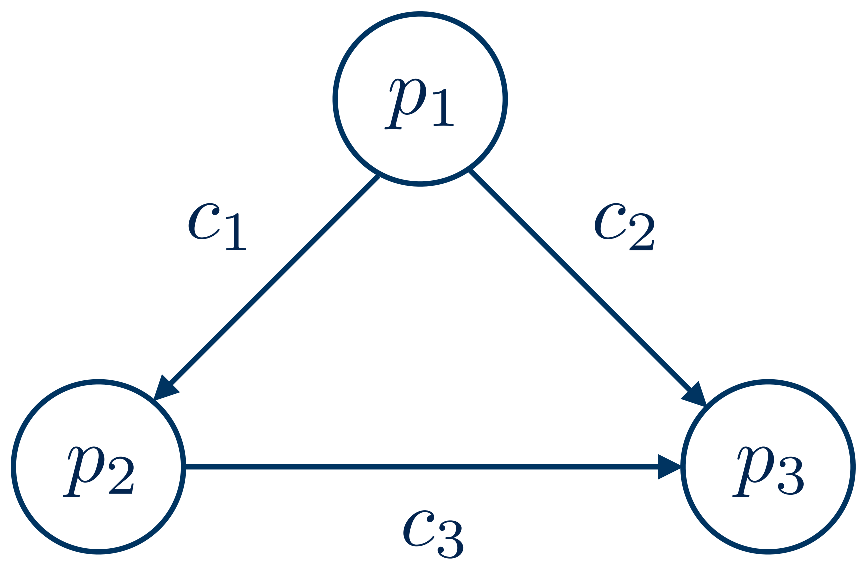

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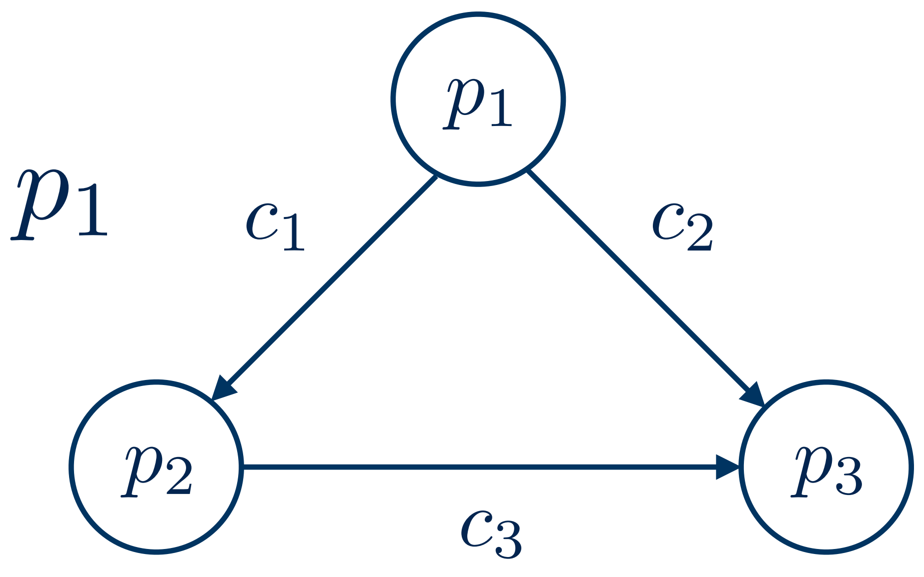
$$\texttt{Writer}(c_1) = p_1$$



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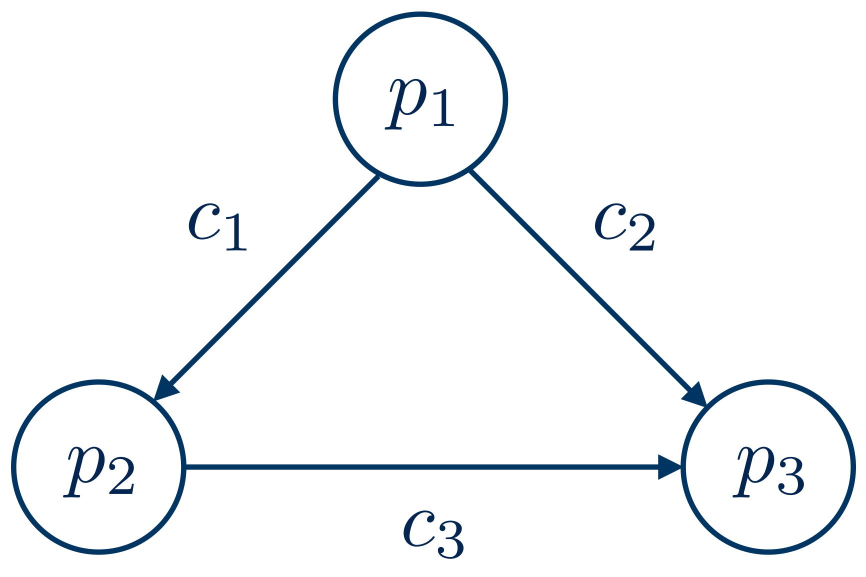

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Acyclic Lossy Channel Pushdown Systems

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Control State Reachability Problem

Acyclic Lossy Channel Pushdown Systems

[Atig et. al. 2008]

Control State Reachability Problem

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Acyclic Lossy Channel Pushdown Systems

[Atig et. al. 2008]

Control State Reachability Problem

NEXPTIME Upper bound

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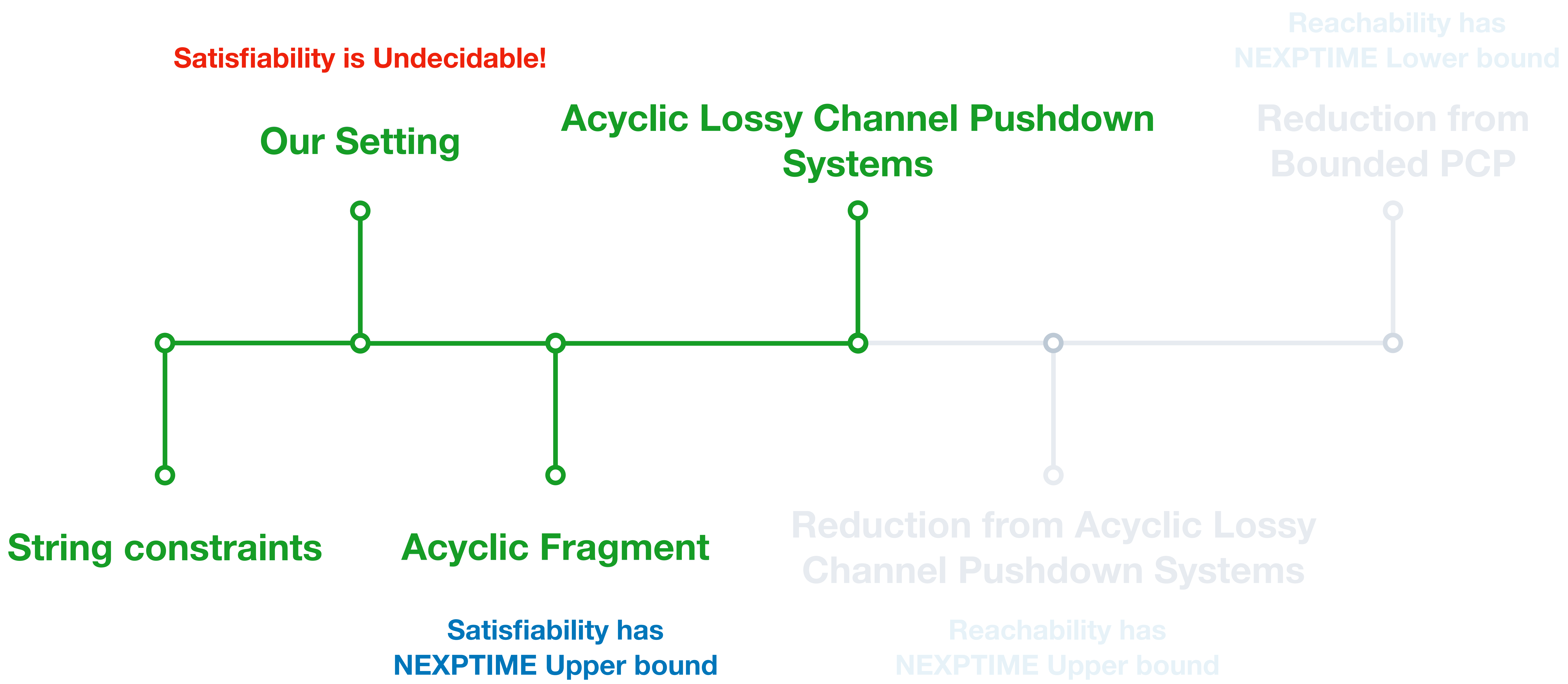

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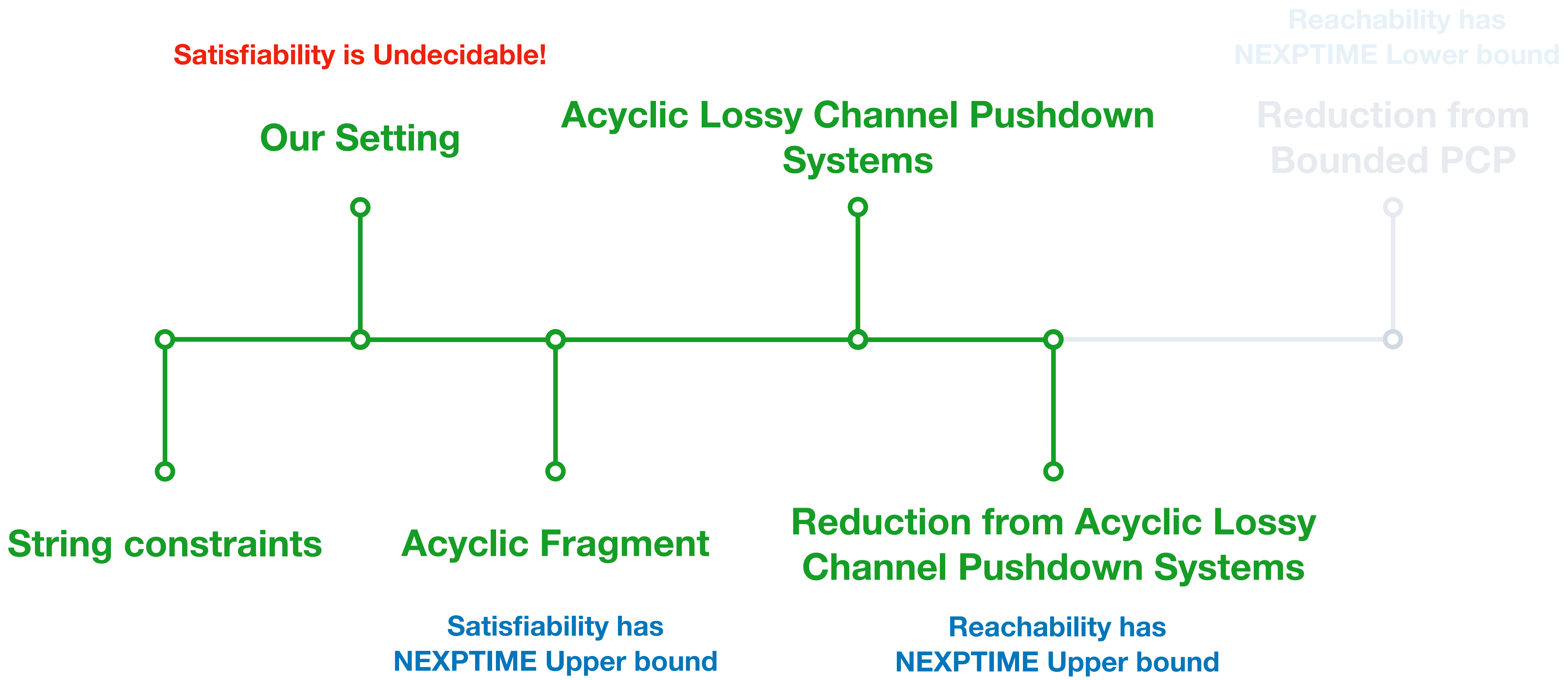
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From

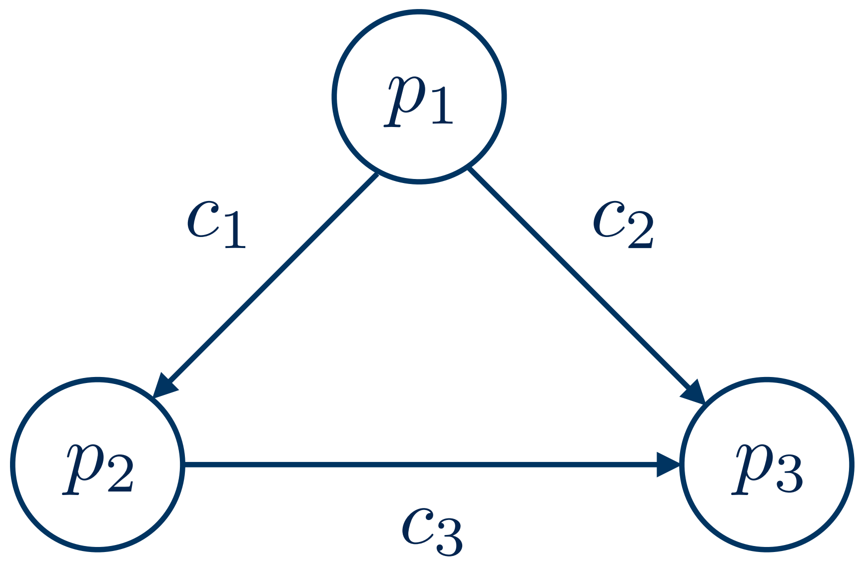
Reachability problem of acyclic LCS

to

Satisfiability problem of acyclic String Constraints

NEXPTIME Upper bound

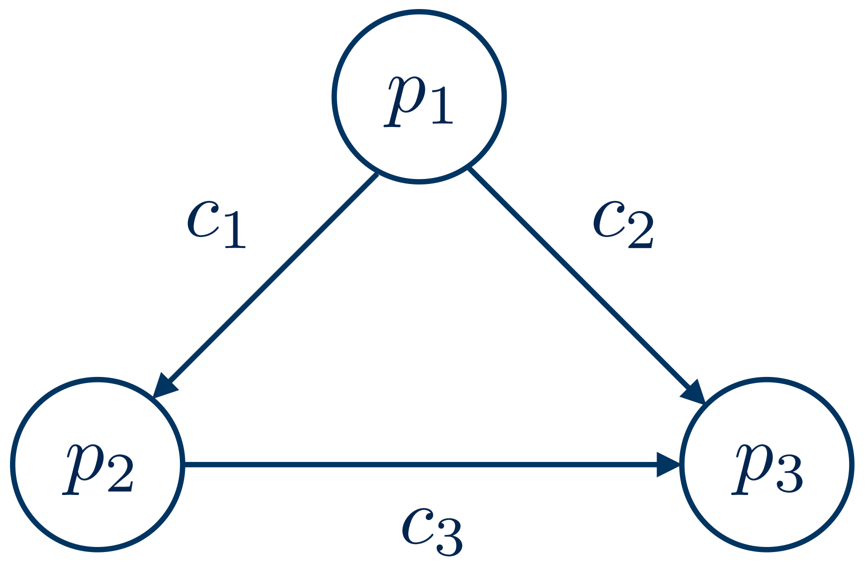
NEXPTIME Upper bound



An Acyclic LCS

NEXPTIME Upper bound

Equivalent Acyclic String constraints

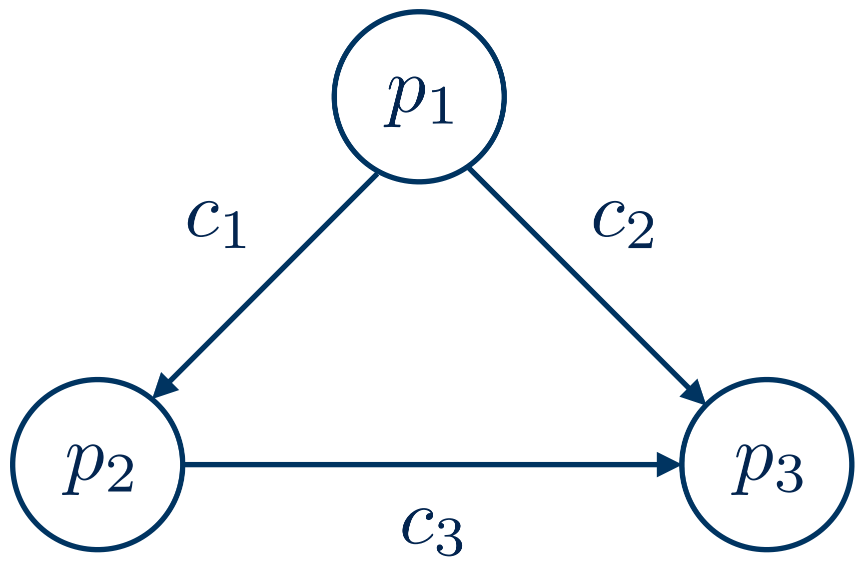


An Acyclic LCS

NEXTPTIME Upper bound

Equivalent Acyclic String constraints

Variable set V = $\{\text{one variable for each process}\}$



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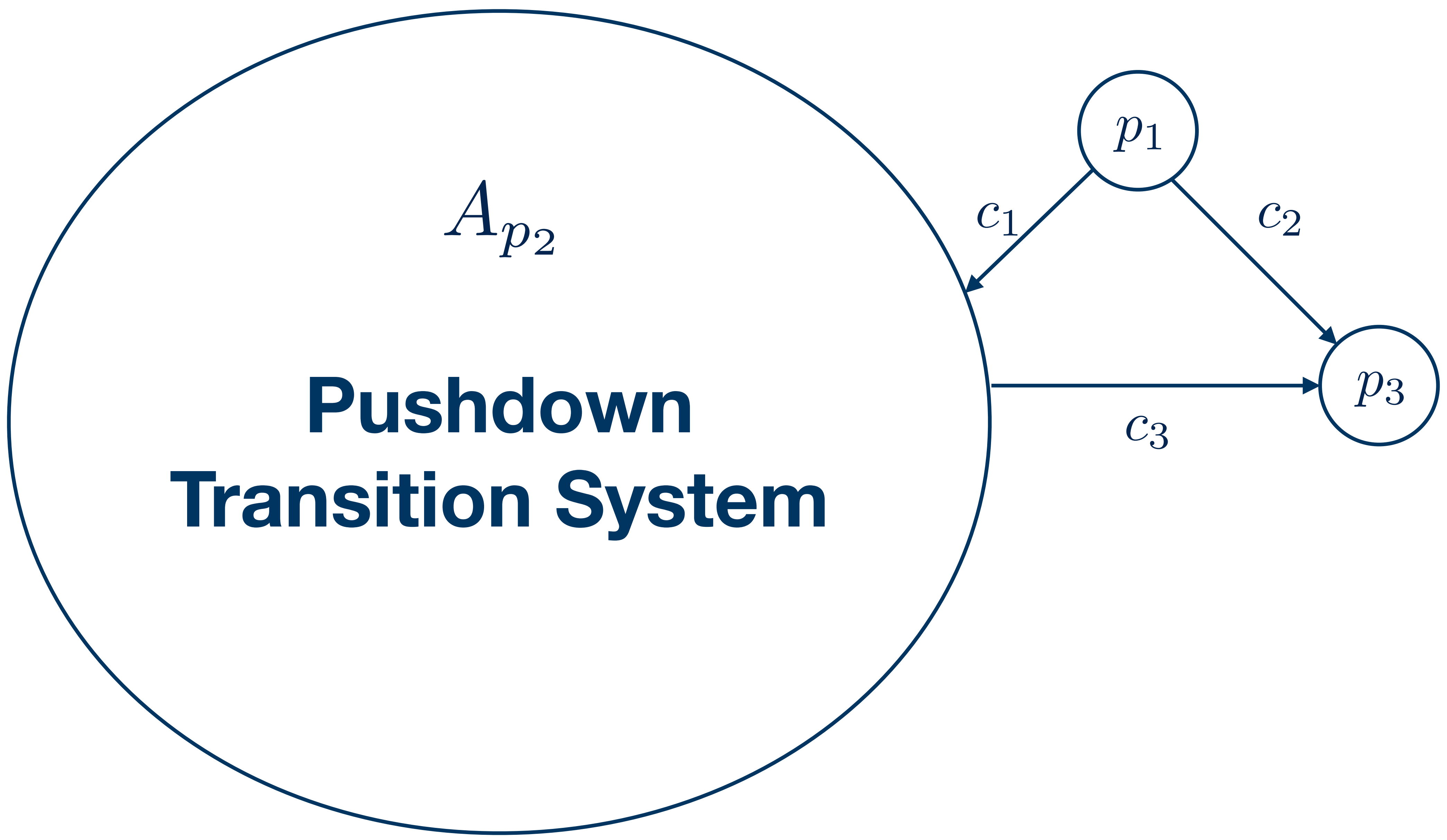

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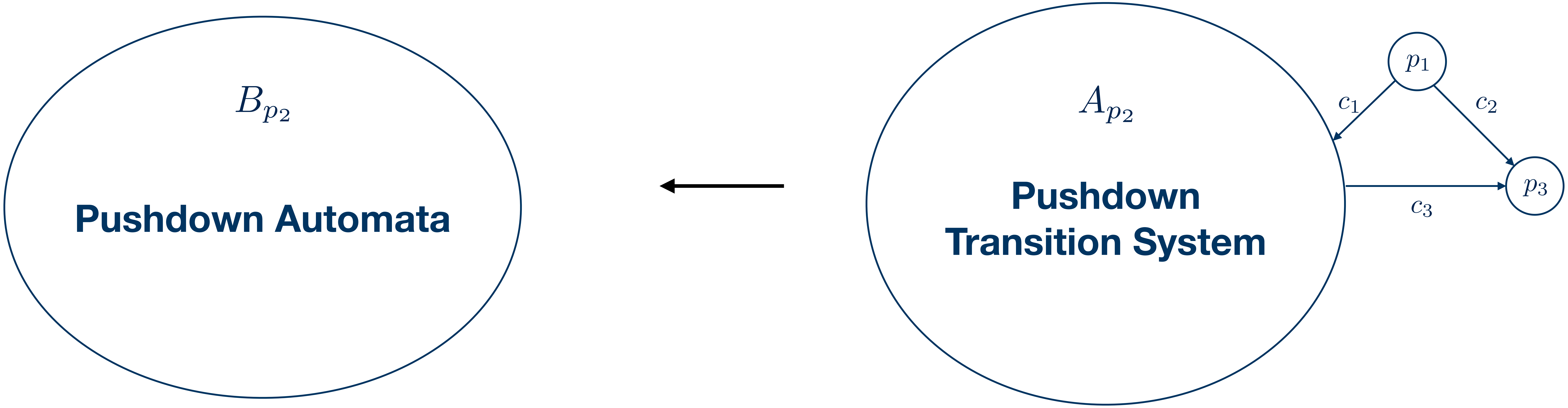

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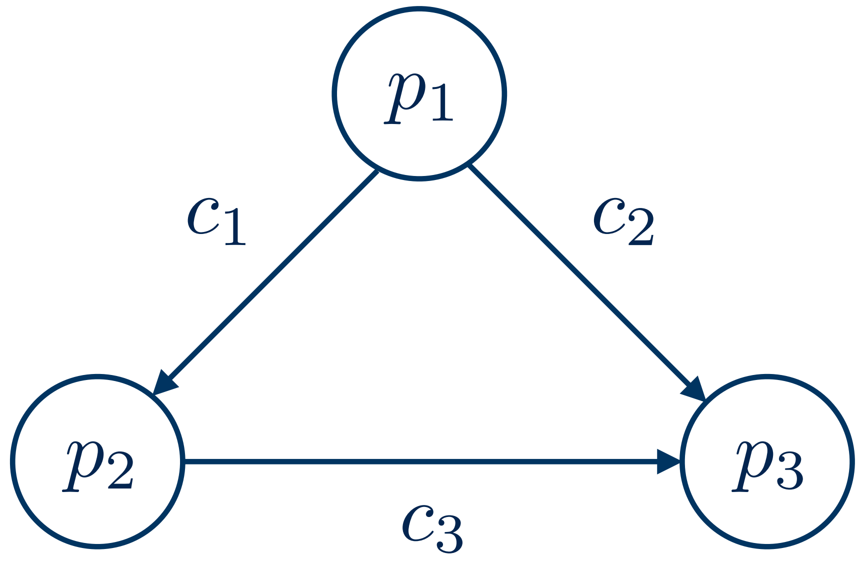
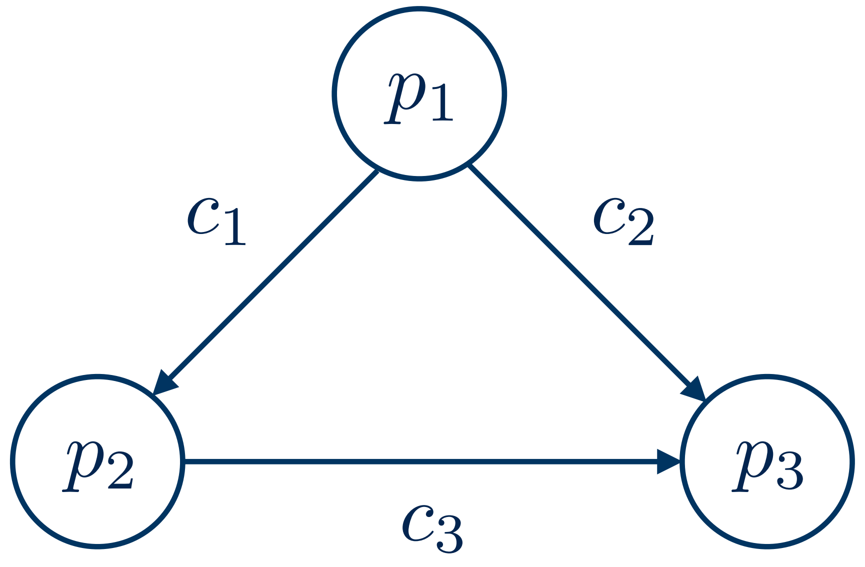


An Acyclic LCS

NEXPTIME Upper bound

Subword ordered Constraints:

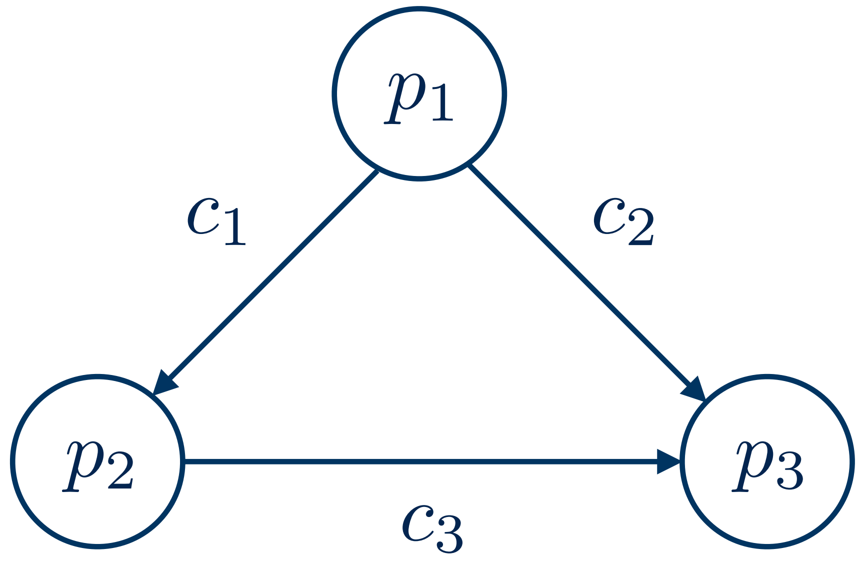


An Acyclic LCS

NEXPTIME Upper bound

Subword ordered Constraints:

For each channel c, $\texttt{Reader}(c) = p$ $\texttt{Writer}(c) = q$



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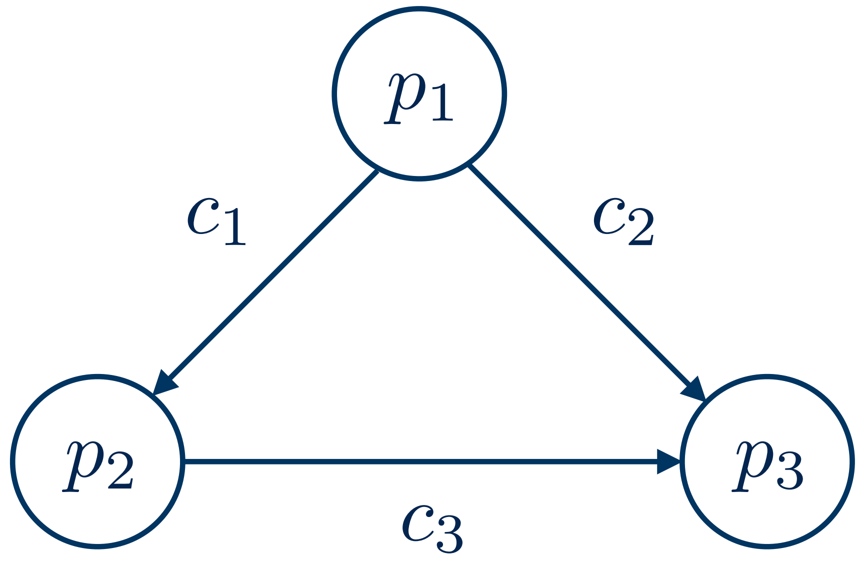

An Acyclic LCS

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Equivalent Acyclic String constraints

Variable set V = $\{\text{one variable for each process}\}$

Membership constraints :

$$x_p = \mathcal{L}(B_p)$$

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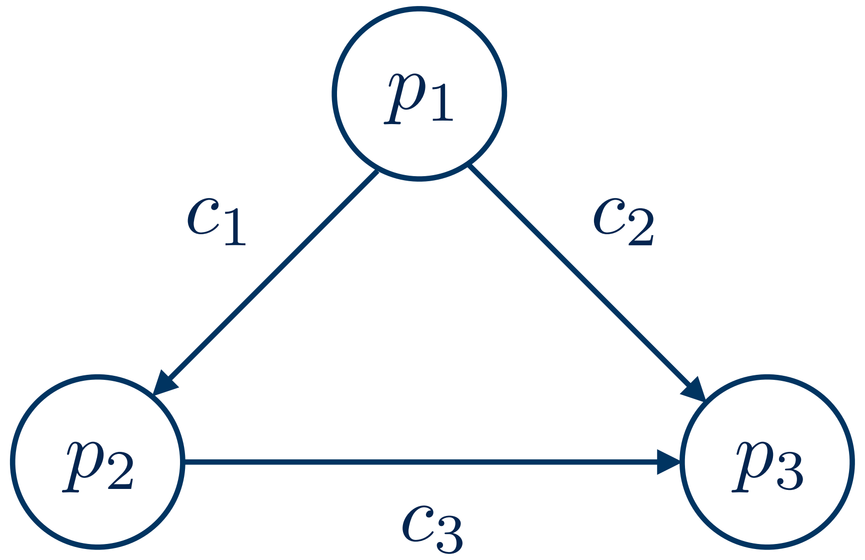

An Acyclic LCS

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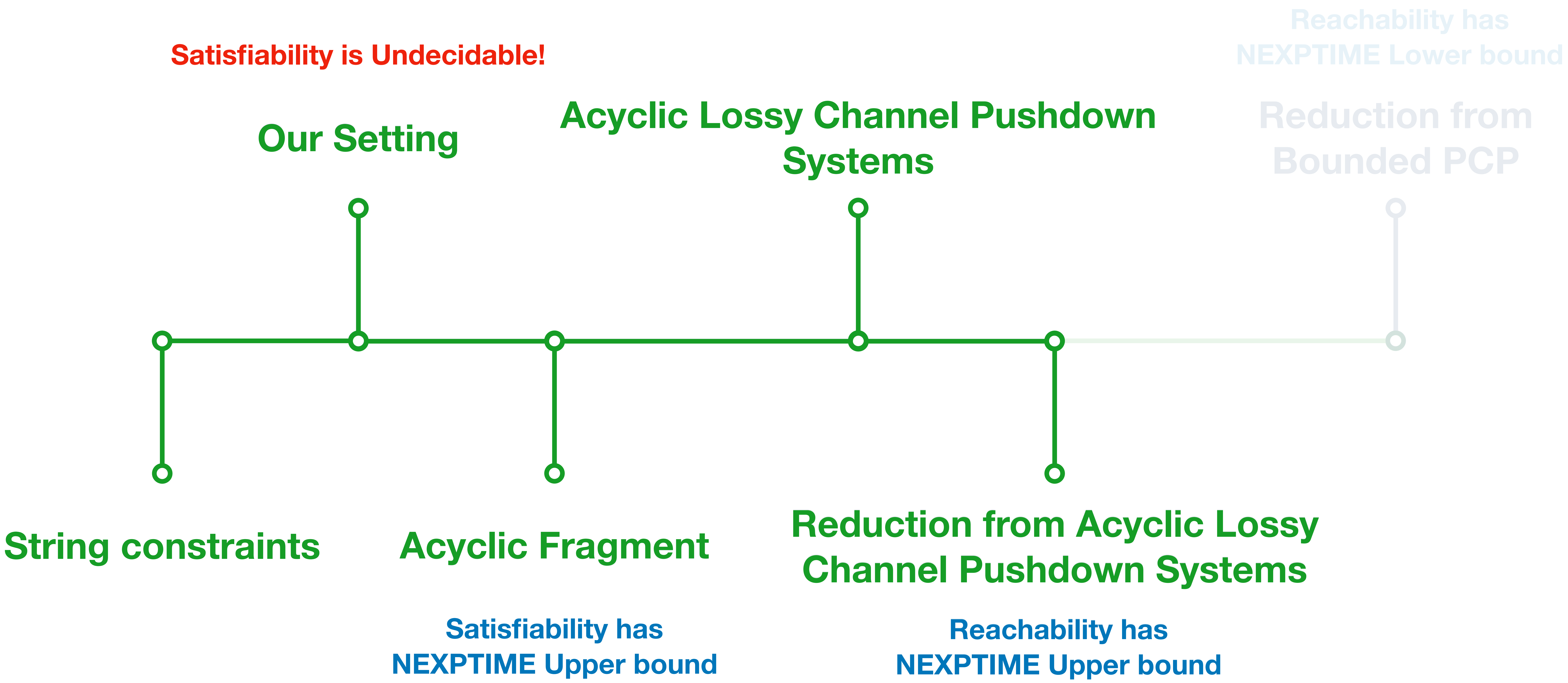
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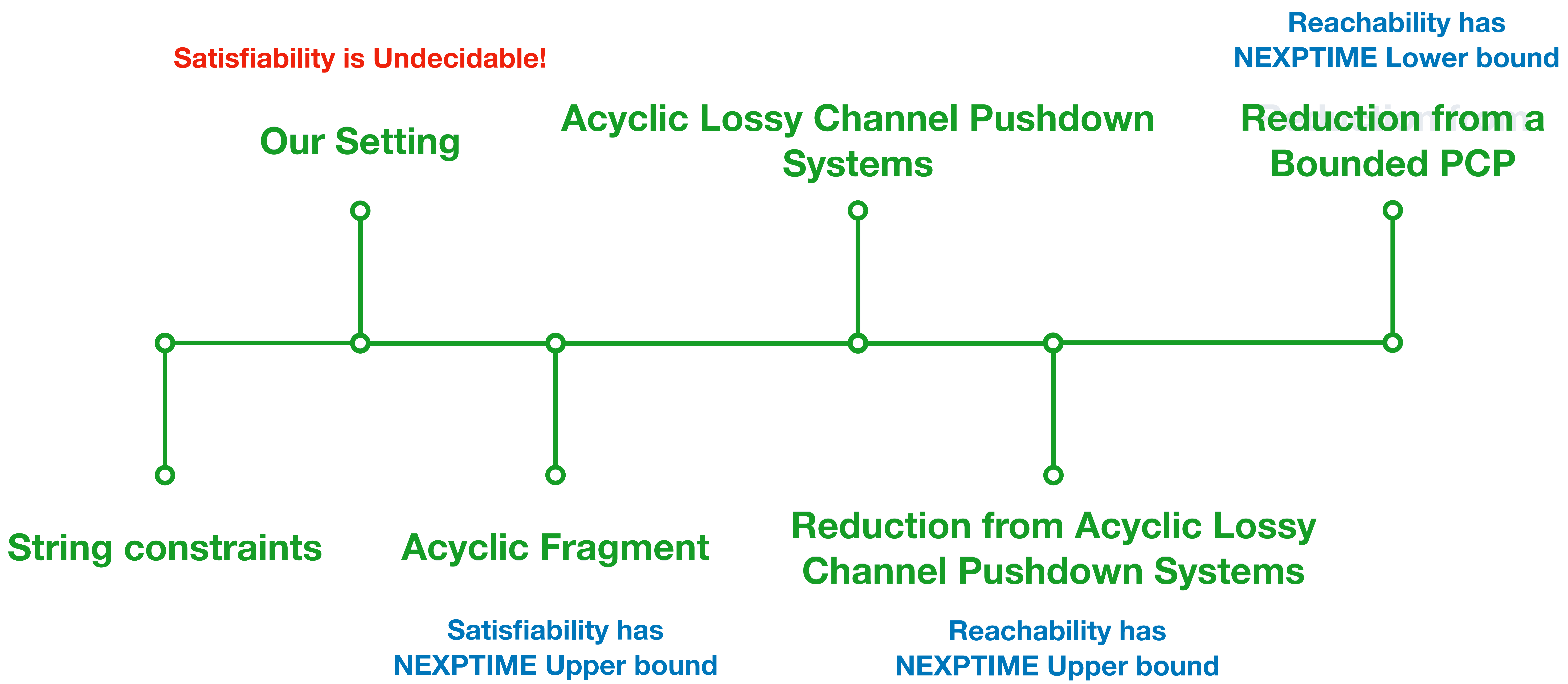
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NEXPTIME Lower bound

Bounded variant of PCP problem - NEXPTIME complete.

NEXPTIME Lower bound

Bounded variant of PCP problem - NEXPTIME complete.

Reduction from it to the Reachability problem for acyclic LCS

NEXPTIME Lower bound

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NEXPTIME Lower bound

Bounded PCP instance over alphabet Σ:

NEXPTIME Lower bound

Bounded PCP instance over alphabet Σ:

Given two equi-dimensional vector of strings U and V:

NEXPTIME Lower bound

Bounded PCP instance over alphabet Σ:

Given two equi-dimensional vector of strings U and V:

	1	2	...	n
U	u_1	u_2	...	u_n
V	v_1	v_2	...	v_n

NEXPTIME Lower bound

Bounded PCP instance over alphabet Σ:

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and an integer ℓ given in binary

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and an integer ℓ given in binary

sequence of indices

$\exists \quad i_1, i_2, \ldots, i_k$

$$u_{i_1} \cdot u_{i_2} \cdots u_{i_k}$$
$$=$$
$$v_{i_1} \cdot v_{i_2} \cdots v_{i_k}$$

NEXPTIME Lower bound

Bounded PCP instance over alphabet Σ:

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$\exists \quad i_1, i_2, \ldots, i_k$

$$u_{i_1} \cdot u_{i_2} \cdots u_{i_k}$$

=

$$v_{i_1} \cdot v_{i_2} \cdots v_{i_k}$$

AND

$$\texttt{len}(u_{i_1} \cdot u_{i_2} \cdots u_{i_k}) = \texttt{len}(v_{i_1} \cdot v_{i_2} \cdots v_{i_k}) = 2^\ell$$

?

NEXPTIME lower bound

To give a reduction, we look at the following gadgets:

Gadget to count at most 2^ℓ

NEXPTIME lower bound

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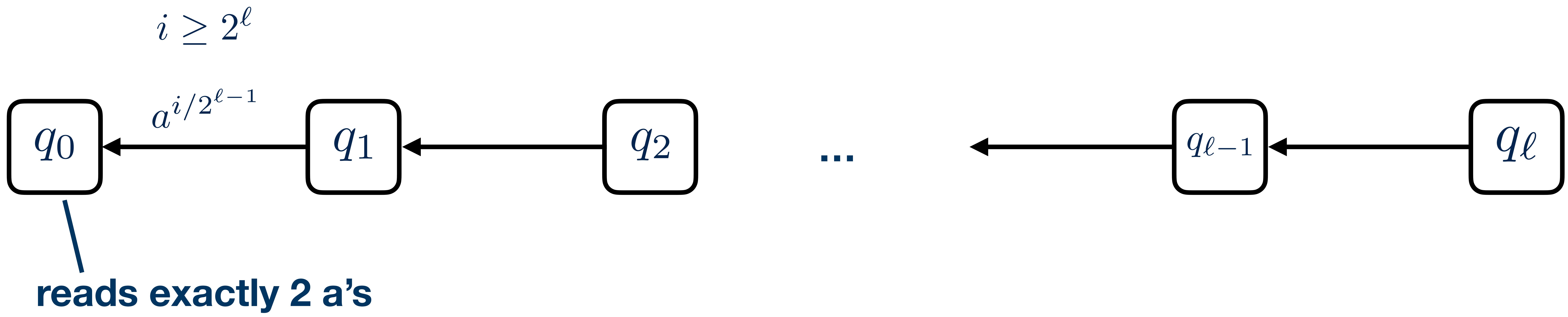

Gadget to count at least 2^ℓ

NEXPTIME lower bound

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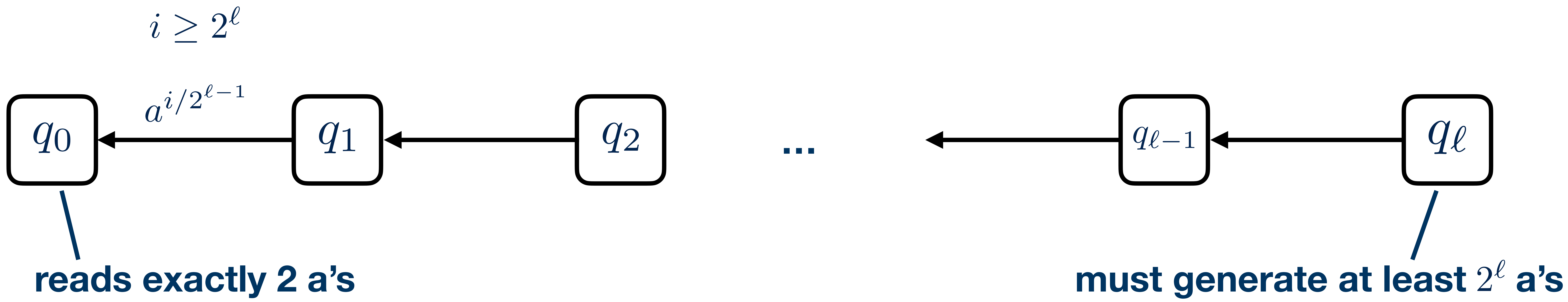

Gadget to count at least 2^ℓ

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Gadget to count exactly 2^ℓ

NEXPTIME lower bound

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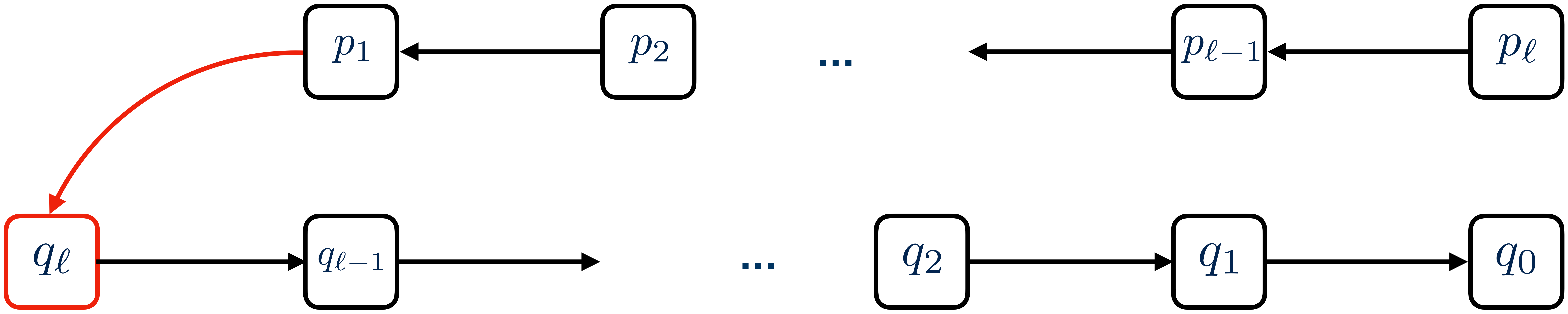

Gadget to count exactly 2^ℓ

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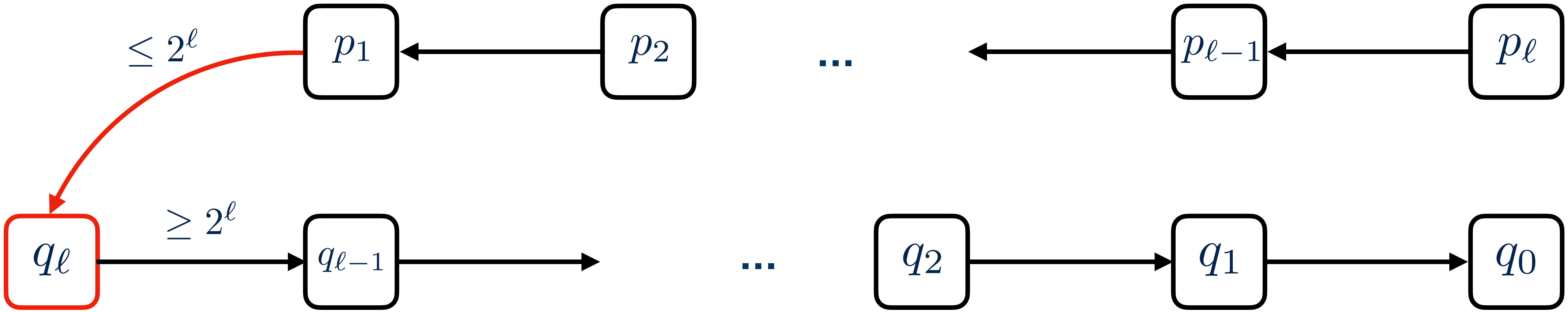

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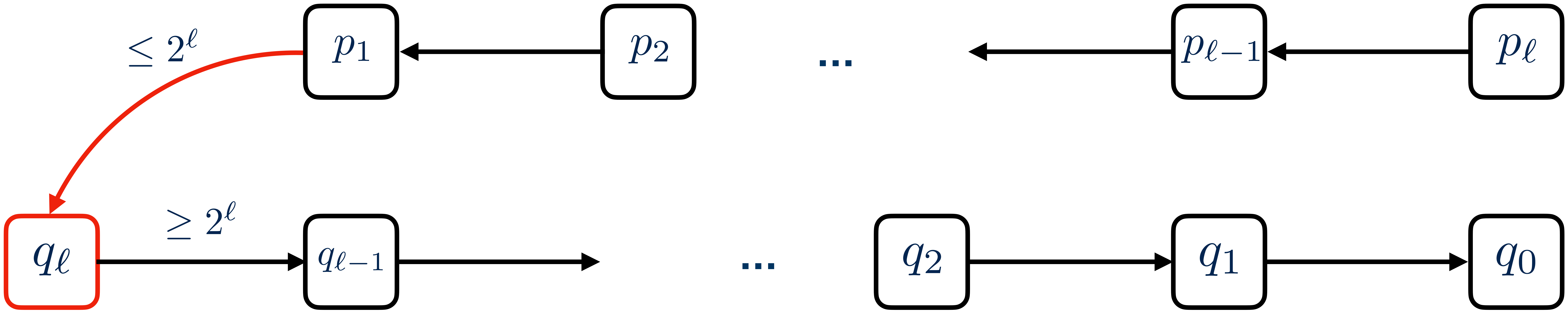

Topology for the PCP simulation

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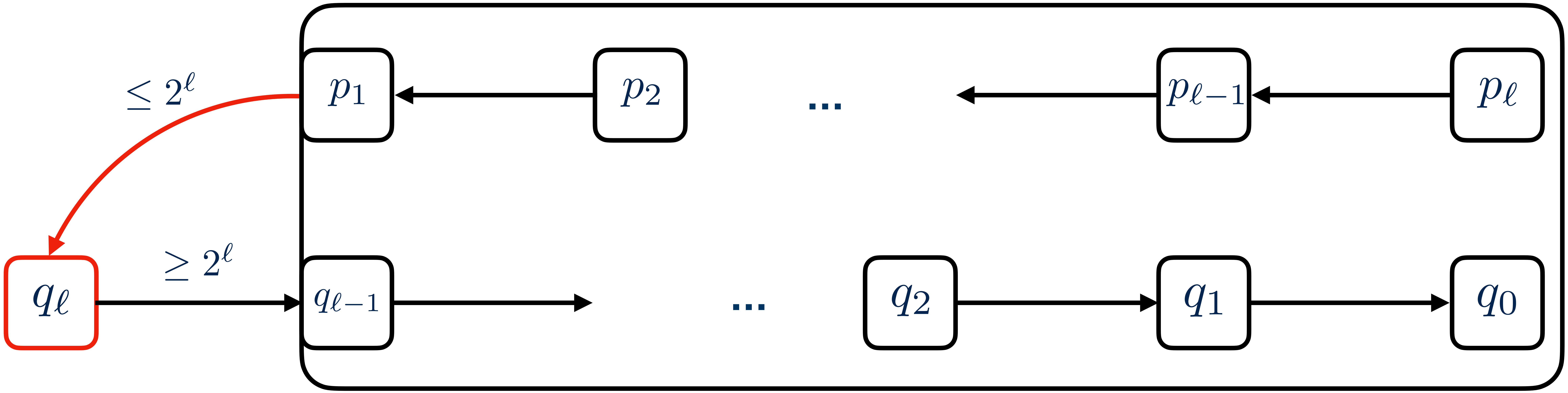

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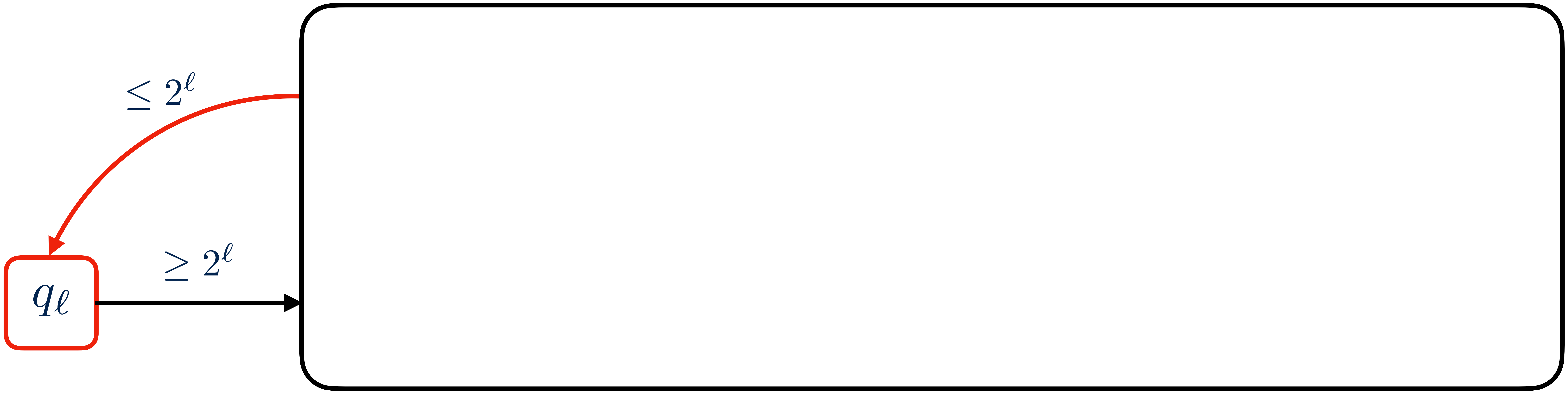

Topology for the PCP simulation

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PCP instance over alphabet Σ:

Given two vector of strings U and V, each having n elements:

	1	2	...	n
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sequence of indices

$\exists \quad i_1, i_2, \ldots, i_k \qquad u_{i_1} \cdot u_{i_2} \cdots u_{i_k} = v_{i_1} \cdot v_{i_2} \cdots v_{i_k}$

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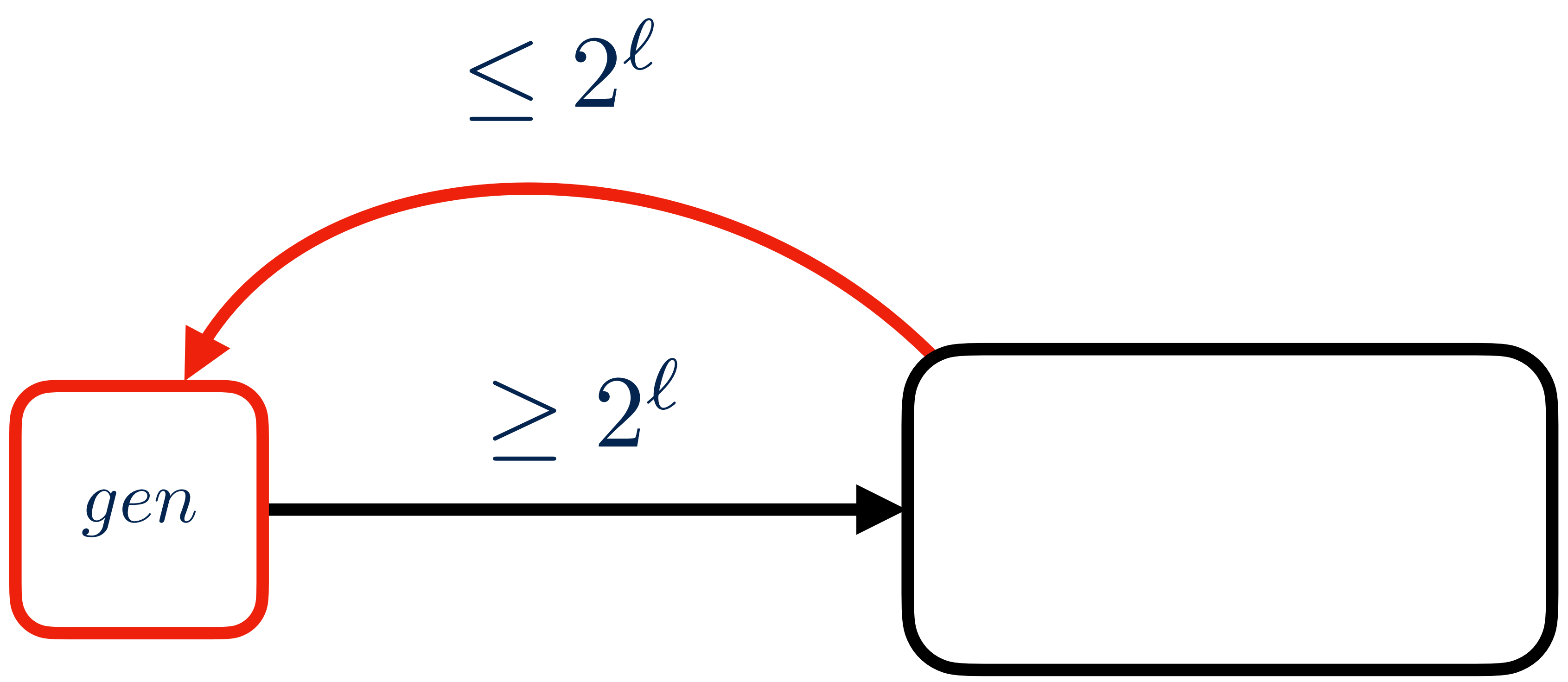

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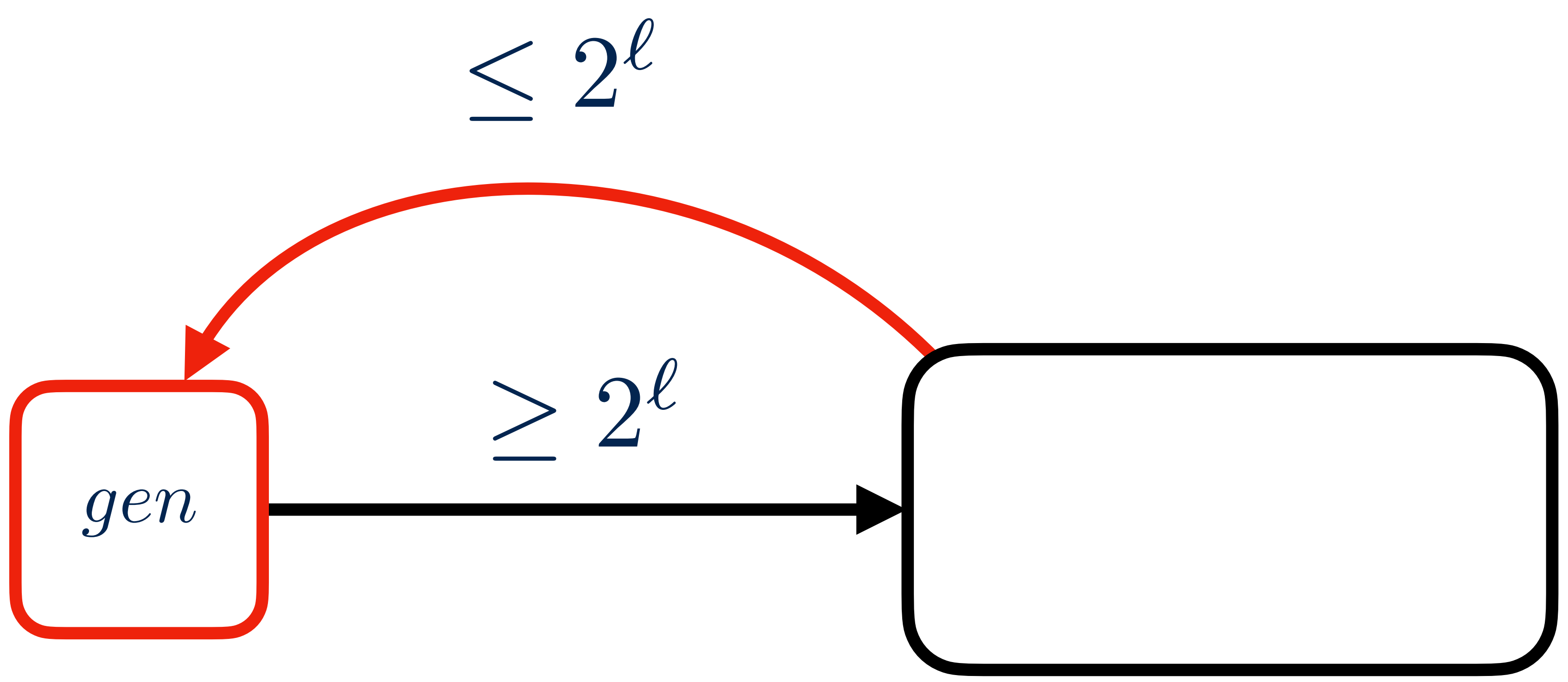

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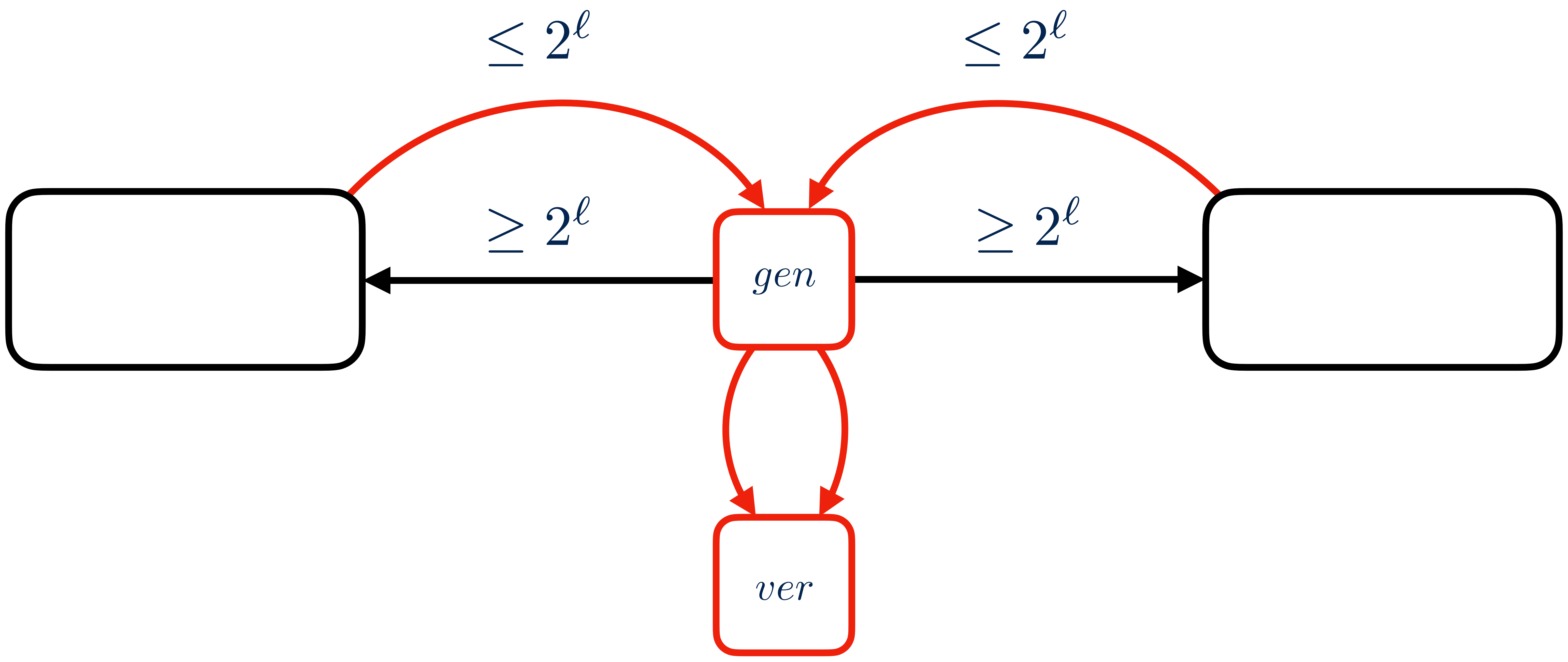

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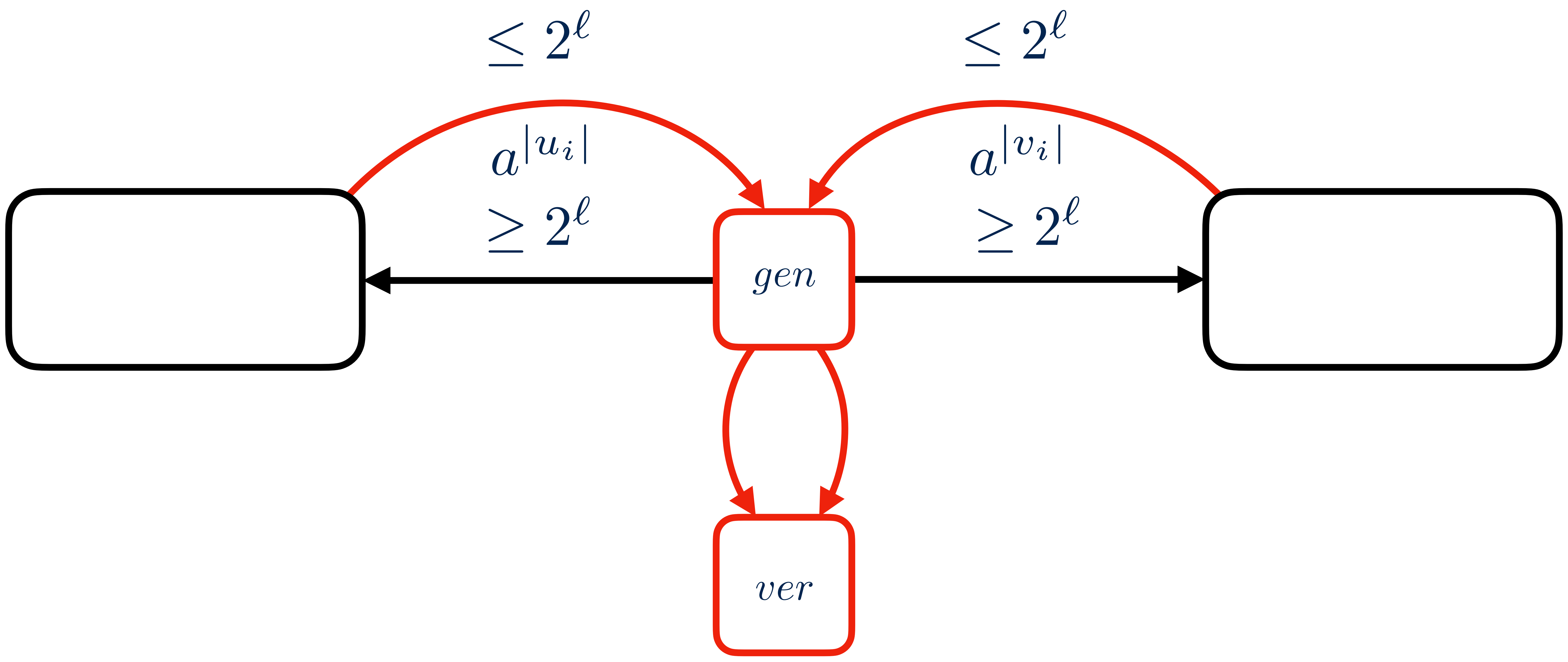

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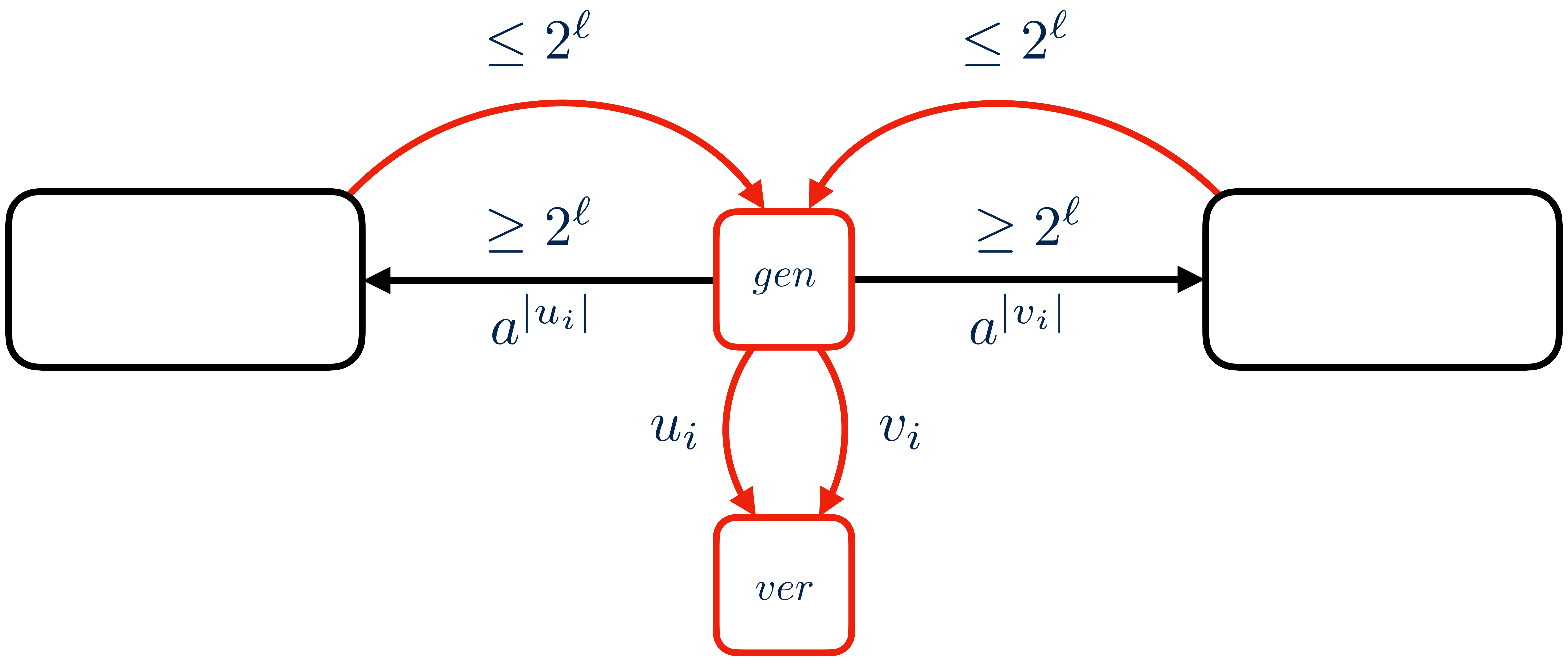

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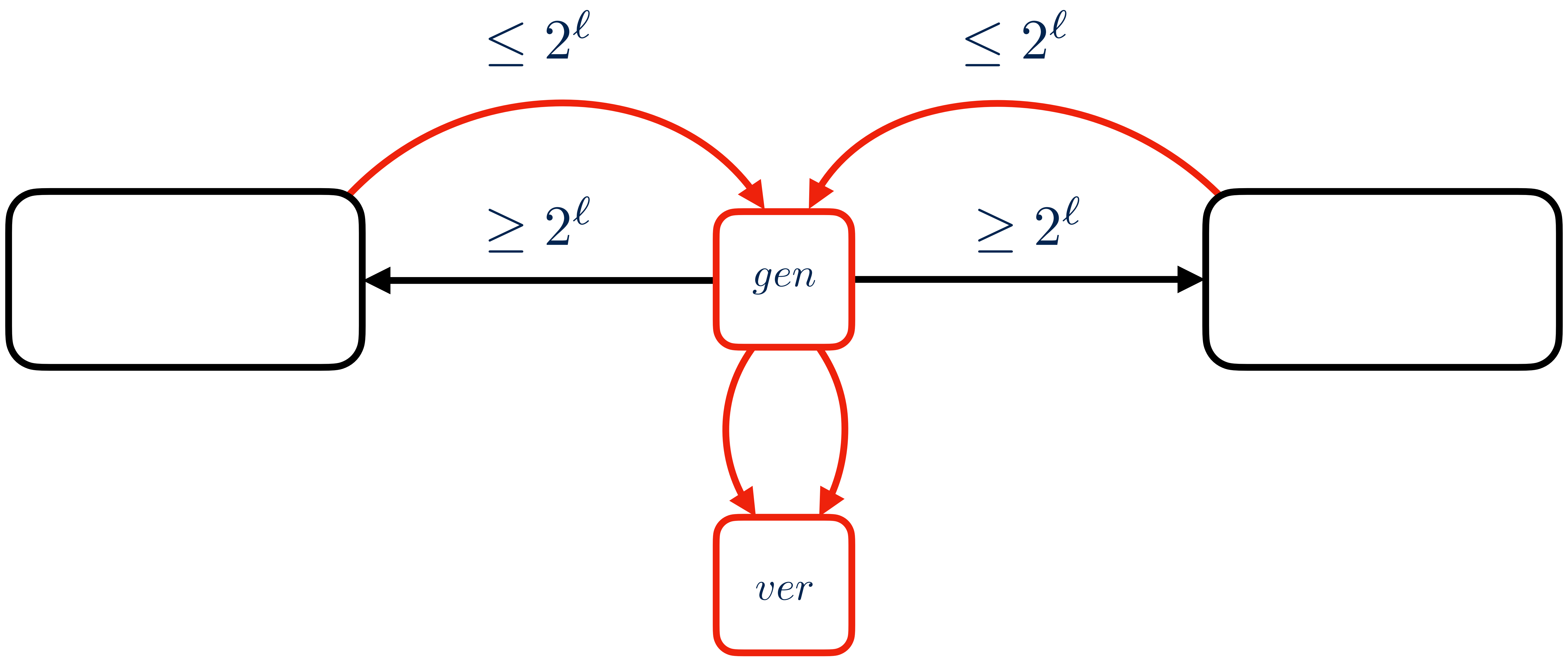

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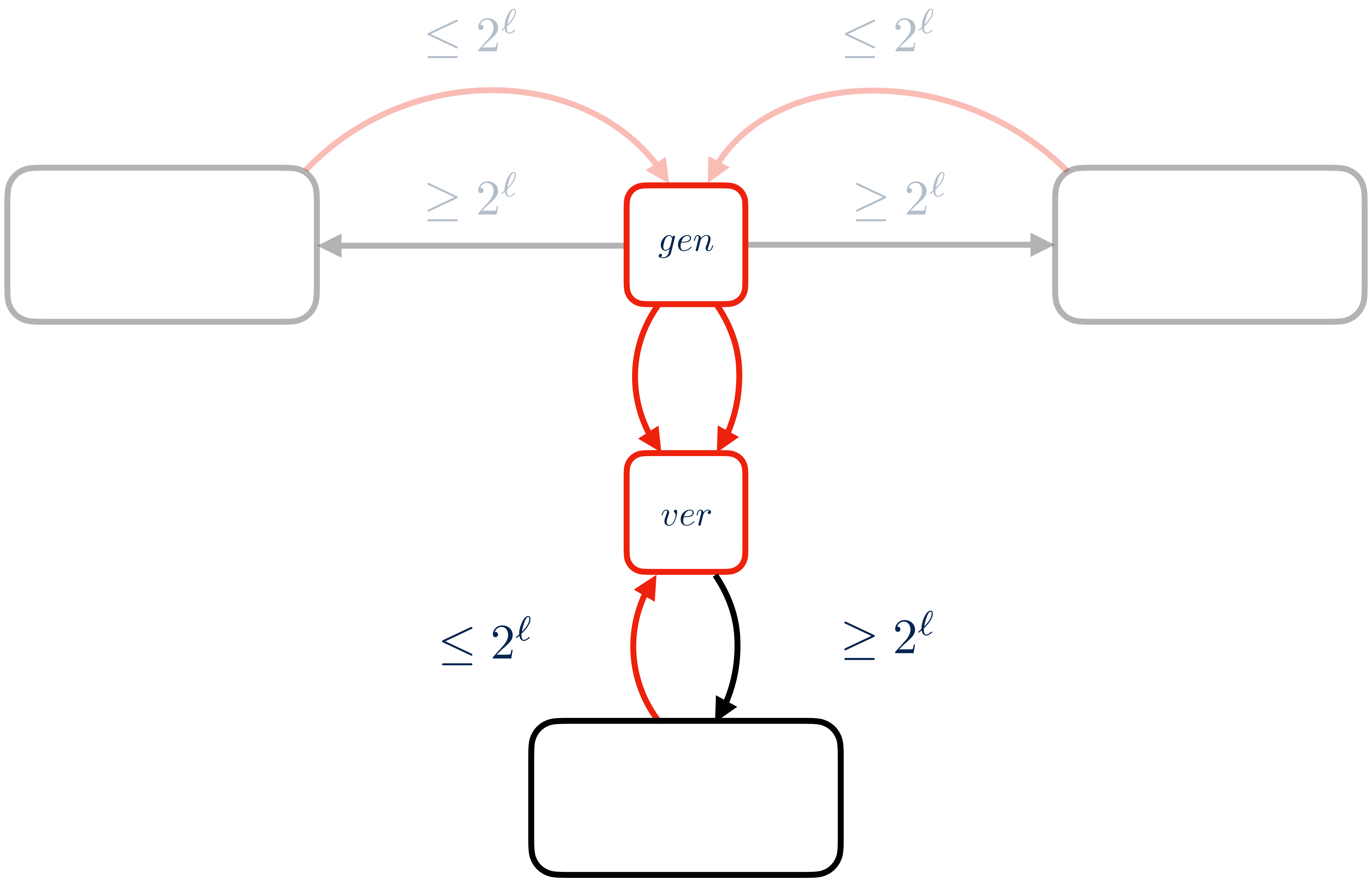

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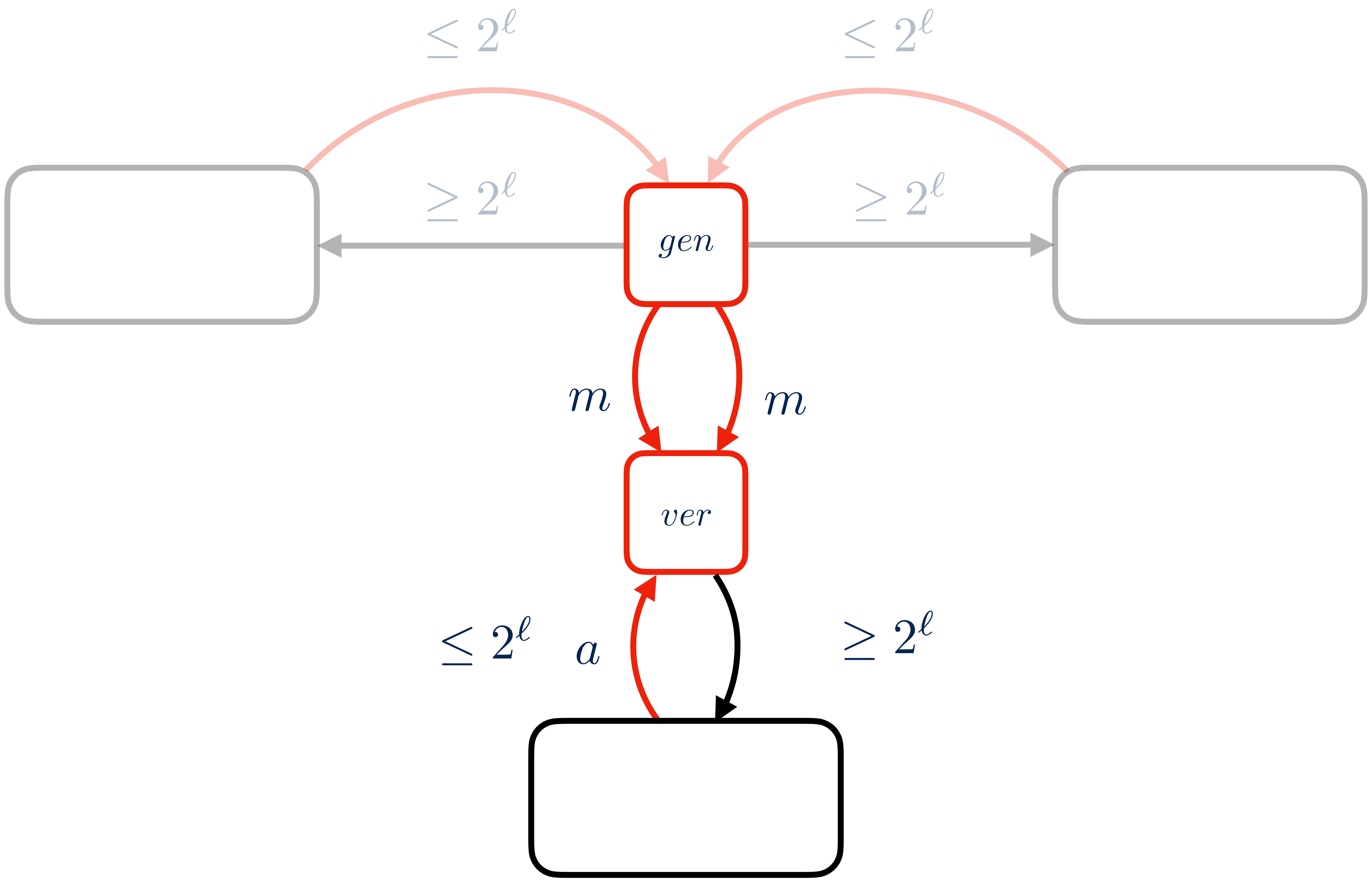

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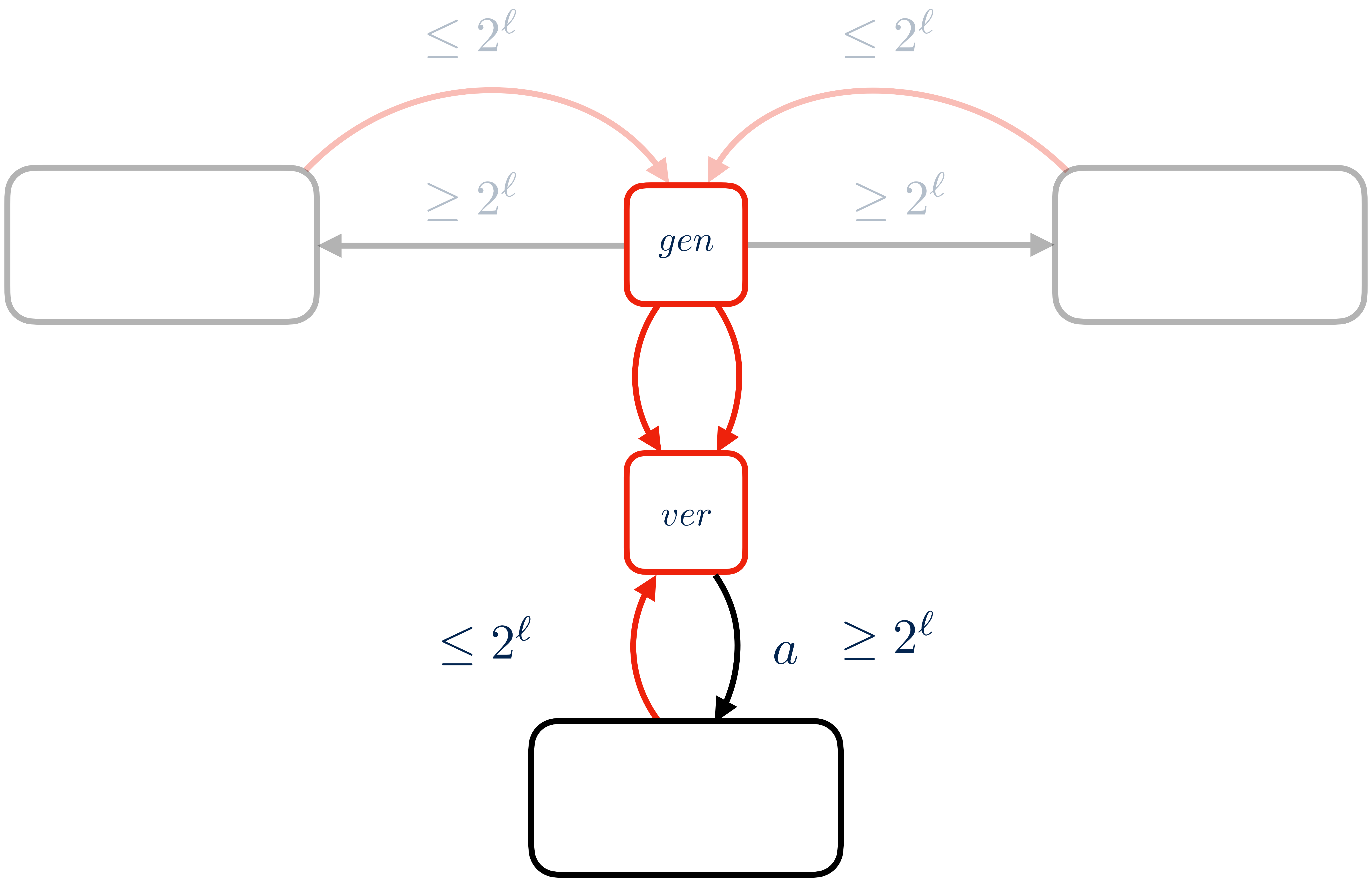

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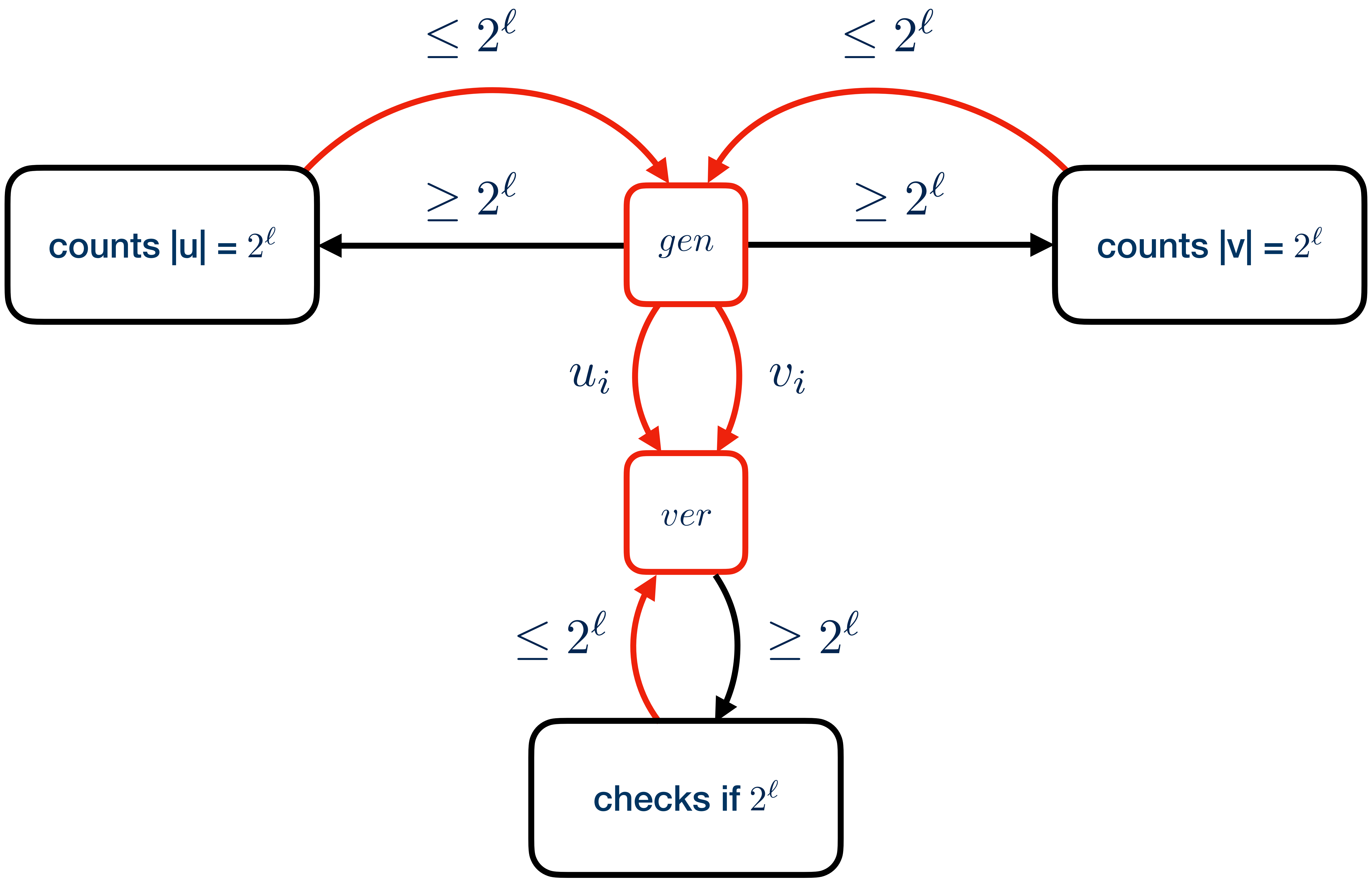

Topology for the PCP simulation

Summary

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4. Acyclic Lossy Channel Pushdown Systems

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5. From Acyclic Lossy Channel Pushdown Systems to Acyclic String constraints — Reachability has NEXPTIME Upper bound

Summary

1.	String constraints	
2.	Context-free String Constraints with Subword Order	Satisfiability is Undecidable!
3.	Acyclic Fragment	Satisfiability has NEXPTIME Upper bound
4.	Acyclic Lossy Channel Pushdown Systems	
5.	From Acyclic Lossy Channel Pushdown Systems to Acyclic String constraints	Reachability has NEXPTIME Upper bound
6.	From Bounded PCP to Acyclic Lossy Channel Pushdown Systems	$\Longrightarrow$ Both Reachability for Acyclic LCS and Satisfiability for Acyclic String constraints are NEXPTIME complete!

Thank You!