Analysing String constraints using Separability

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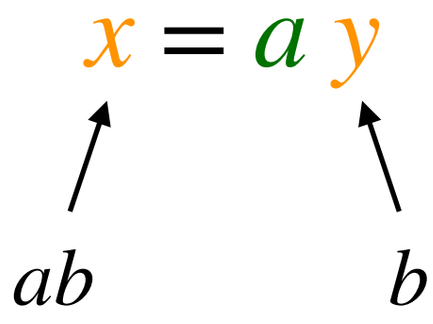
String Constraints



1. membership constraints

$x \in L$ ← regular language

$$\varphi : x \in (ab)*$$

$$(x = abab) \models \varphi \qquad (x = aba) \not\models \varphi$$

$$L(\varphi) = \{(ab)^n \mid n \in \mathbb{N}\}$$

String Constraints

1. membership constraints $x \in L$

2. relational constraints

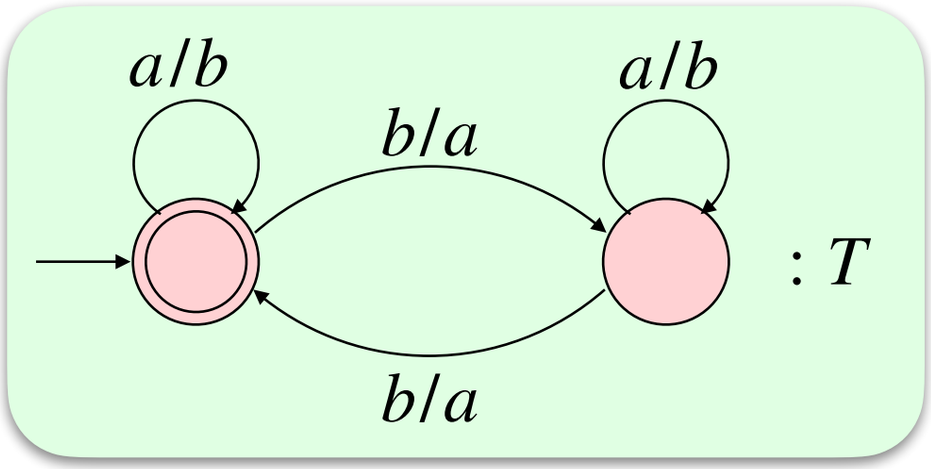
$(t, t') \in R$ ← definable by input - output automata (transducer)

$\varphi : (x, y) \in T$

$(x = abb, y = baa) \models \varphi$

$\{(abb, baa), (aa, bb)\} \subseteq \mathscr{L}(\varphi)$

$\mathscr{L}(\varphi) \subseteq (\Sigma^*)^2$



String Constraints

1. membership constraints $x \in L$

2. relational constraints $(t, t') \in R$

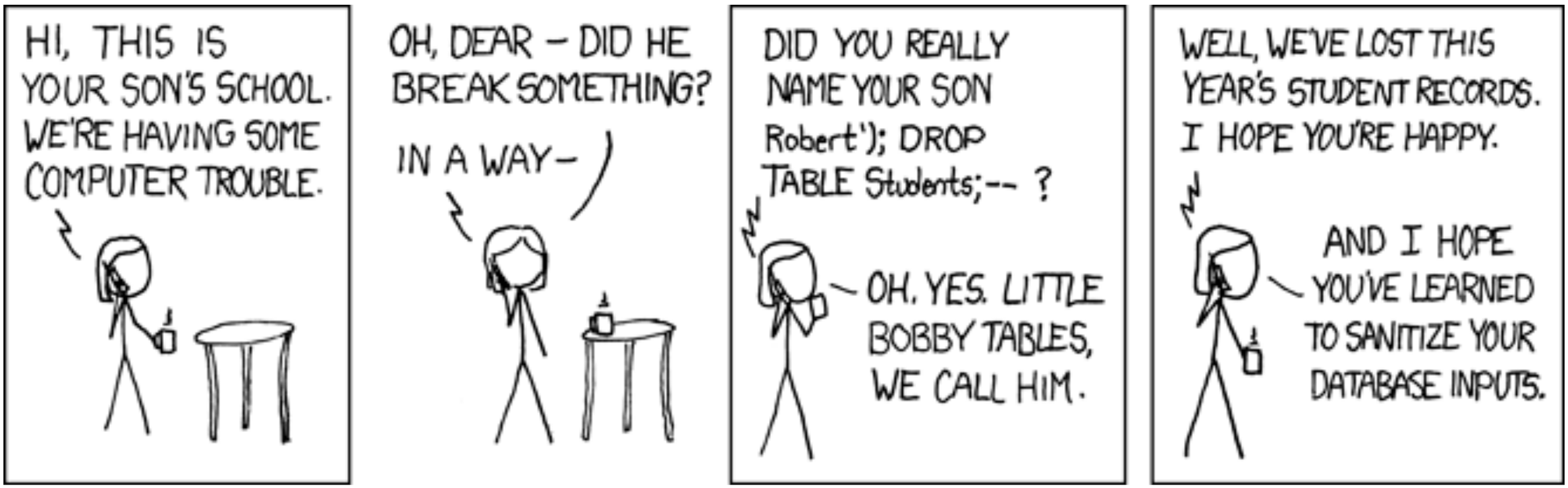
String constraints: Conjunction of atomic string constraints

$$\varphi_1 : x \in \mathsf{even}_a \wedge (x, y) \in T \wedge y \in \mathsf{even}_a$$

$$(x = abba, y = baab) \models \varphi_1$$

Satisfiability Problem

Given a string constraint φ, does there exists a satisfying assignment/solution to φ?



source: https://xkcd.com/327/

Related Work

PCP $\longrightarrow$ $(x, x) \in R$

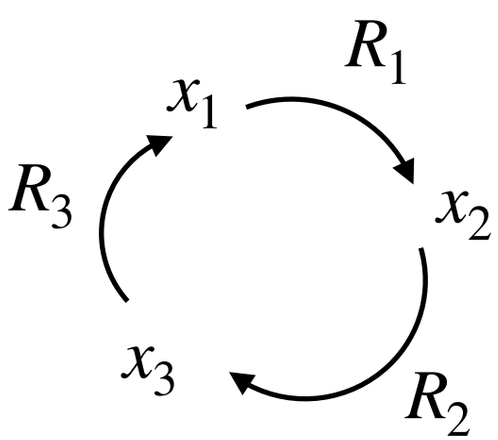
undecidable

Straight Line Fragment

- no cyclic dependency

[Anthony Lin, Pablo Barceló] (POPL16)

POPL 16, POPL 18, POPL 19



length preserving

[Abdulla et al.] (ATVA 19)

decidable

Related Work

Straight Line Fragment

- no cyclic dependency

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undecidable

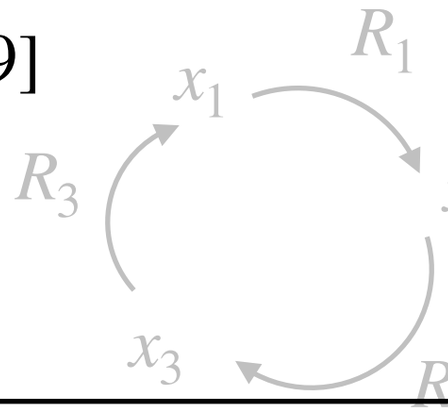
Z3-str3 [Murphy Berzish, Vijay Ganesh, and Yunhui Zheng, FMCAD 17]

CVC4 [Andrew Reynolds et al., CAV 17]

Trau [Parosh Abdulla et al., PLDI 17]

SLOTH [Anthony Lin et al., POPL 2018]

OSTRICH [Philipp Rümmer et al., POPL 2019]



length preserving

[Abdulla et al.] (ATVA 19)

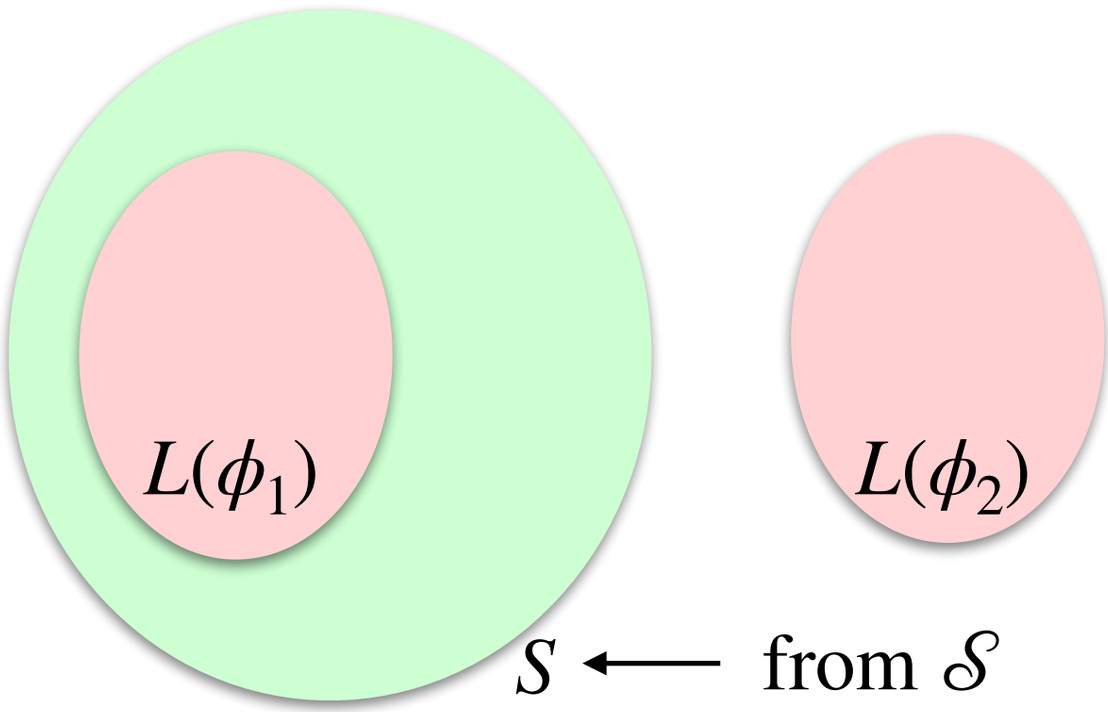
decidable

Separability Problem of String Constraints

Input: Two constraints ϕ_1 and ϕ_2 ⟵ from $\mathscr{C}$

Question: Does there exists a language S *separating* $L(\phi_1)$ and $L(\phi_2)$?

$$\exists S,\ L(\phi_1) \subseteq S \text{ and } L(\phi_2) \cap S = \emptyset?$$



$\mathcal{S}$ *separability* of class $\mathscr{C}$

Related Work

undecidable

PCP $\longrightarrow$ $(x, x) \in R$

$\phi_1 = (x, y) \in R_1$

$\phi_2 = (y, x) \in R_2$

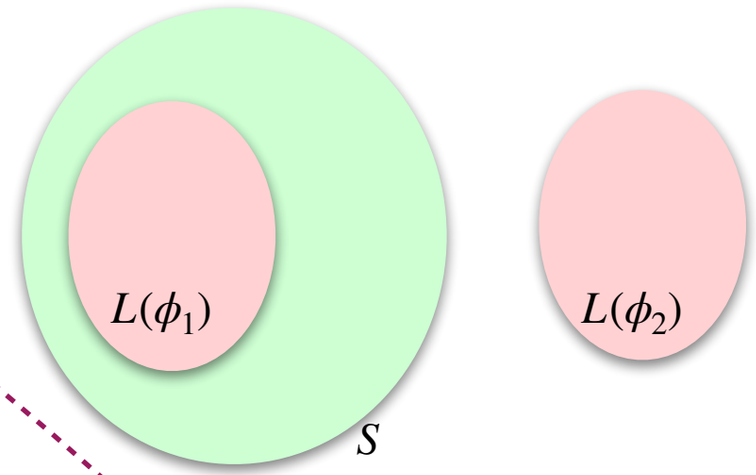
SL $\wedge$ SL

Straight Line (SL) Fragment

- no cyclic dependency

[Anthony Lin, Pablo Barceló] (POPL16)

POPL 16, POPL 18, POPL 19



For any string constraint φ, we can construct two SL φ_1 and φ_2

φ satisfiable $\iff$ $\varphi_1 \wedge \varphi_2$ satisfiable

decidable

Separability Problem of String Constraints

Regular separability of String Constraints — undecidable ✗

Regular separability of Straight Line Constraints — undecidable ✗

$$\phi = \bigwedge_{i=1}^{n} x_i \in L_i \quad \wedge \quad \bigwedge_{i=1}^{k} (x_i, t_i) \in R_i$$

Separability Problem of String Constraints

Regular separability of String Constraints — undecidable ❌

Regular separability of Straight Line Constraints — undecidable ❌

PTL separability of Straight Line Constraints — decidable ✅

$\mathbb{B}(\Sigma^* a_1 \Sigma^* a_2 \Sigma^* \cdots a_n \Sigma^*)$ — complexity open 😕

PosPTL separability of Straight Line Constraints — decidable ✅

+ve $\mathbb{B}(\Sigma^* a_1 \Sigma^* a_2 \Sigma^* \cdots a_n \Sigma^*)$ — complexity open 😕

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PosPTL separability of *Right sided SL constraints* $(x_1, x_2) \in R_1 \wedge (x_2, x_3) \in R_2$

variables - - - - ▸ independent

can implement all SL equations with functional transducers

PosPTL separability

Languages L_1 and L_2 are PosPTL separable

iff

$$L_1{\uparrow} \cap L_2 = \emptyset$$

$\uparrow$ wrt subword relation

$\{ab\}\uparrow = \Sigma^* a \Sigma^* b \Sigma^*$

PosPTL separability of constraints

Languages $L(\psi_1)$ and $L(\psi_2)$ are PosPTL separable

iff

$$L(\psi_1)\uparrow \cap L(\psi_2) = \emptyset$$

$\uparrow$ wrt subword relation

$\{ab\}\uparrow = \Sigma^* a \Sigma^* b \Sigma^*$

PosPTL separability of constraints

$$L(\psi_1)\uparrow \cap L(\psi_2) = \emptyset$$

$L(\psi_i)$ ⟶ formal model ?

Logic?

automata?

1. $L(\psi_1)\uparrow$ is *computable*

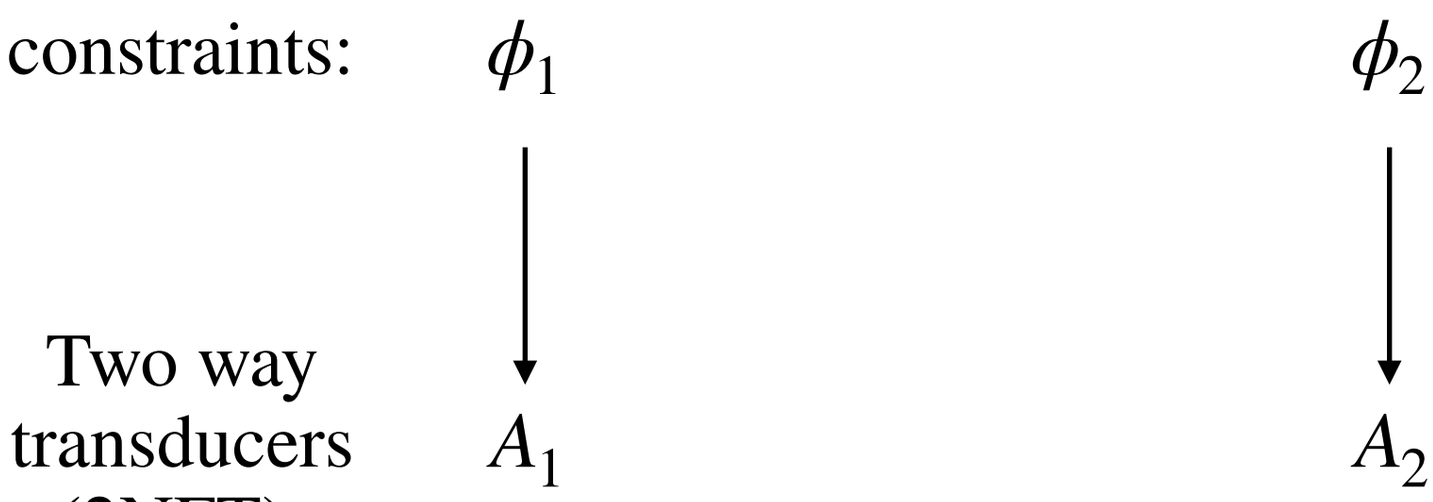
yes !!

2. *intersection* with regular language

3. Emptiness is *decidable*

Two way transducers

PosPTL separability of Right sided SL constraints



constraints PosPTL separable $\iff$ 2NFTs 2-PosPTL separable

PosPTL separability of Right sided SL constraints

$$L(\psi_1)\uparrow \cap L(\psi_2) = \emptyset$$

0. *Encoding* of n-tuple words

$$(w_1, w_2, \ldots, w_n) \mapsto w_1\#w_2\#\ldots\#w_n$$

$$(x = ab, y = ac) \mapsto ab\#ac$$

1. Construct *two way transducer* which outputs *encoding* of $L(\psi_i)$

$$\psi : x = ay \qquad L(\psi) = \{aw\#w \mid w \in \Sigma^*\}$$

PosPTL separability of Right sided SL constraints

$$L(\psi_1) \uparrow \cap\, L(\psi_2) = \emptyset$$

1. Construct *two way transducer* which outputs encoding of $L(\psi_i)$

independent variables: $x_{k+1}, \ldots, x_n$ variables: $x_1, \ldots, x_k, x_{k+1}, \ldots, x_n$

$$u_1 \# u_2 \# \ \ldots \ \# u_k \# u_{k+1} \# \ \ldots \ \# u_n$$

$\mathcal{A}_i$

$$(w_1, \ldots, w_k, u_{k+1}, \ldots, u_n) \models \psi_i$$

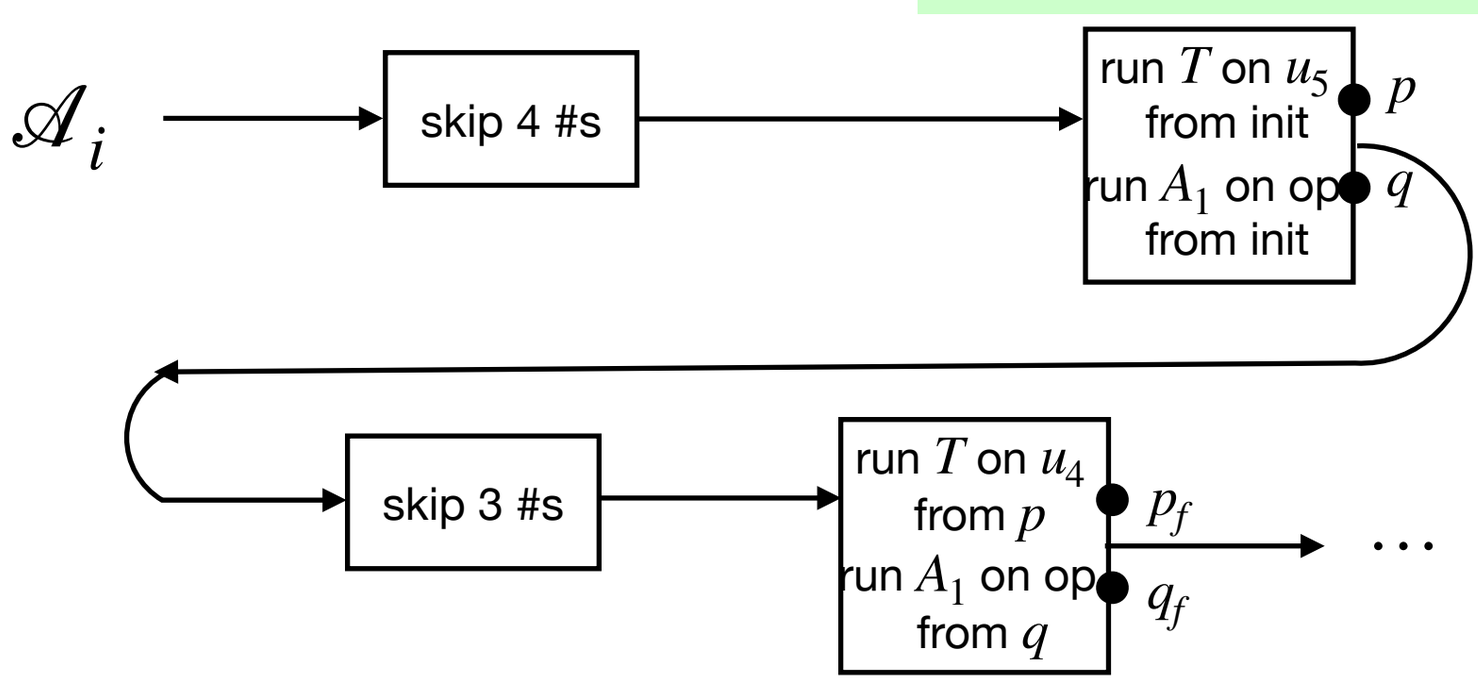
$$w_1 \# w_2 \# \ \ldots \ \# w_k \# u_{k+1} \# \ \ldots \ \# u_n$$

PosPTL separability of Right sided SL constraints

$$L(\psi_1) \uparrow \cap L(\psi_2) = \emptyset$$

1. Construct *two way transducer* which outputs encoding of $L(\psi_i)$

$$u_1 \quad \# \quad u_2 \quad \# \quad u_3 \quad \# \quad u_4 \quad \# \quad u_5 \quad \# \quad u_6$$



$$w_1 \quad \# \quad w_2 \quad \# \quad w_3 \quad \# \quad u_4 \quad \# \quad u_5 \quad \# \quad u_6$$

$$\psi_i : x_1 = T(x_5 x_4)$$

$$w_1 = T(u_5 u_4)$$

$$x_1 \in A_1$$

PosPTL separability of Right sided SL constraints

$$L(\psi_1) \uparrow \cap L(\psi_2) = \emptyset$$

2. upward closure for two way transducer

finite basis / minimal words in upward closure

Claim: minimal words in $L(\psi_1) \uparrow$ are of length at most exponential

$$\{(ab)^i \mid i > 0\} \uparrow \; = \; \Sigma^* a \Sigma^* b \Sigma^* \; = \; \{ab\} \uparrow$$

PosPTL separability of Right sided SL constraints

$$L(\psi_1)\uparrow \cap\, L(\psi_2) = \emptyset \iff A_{\psi_1}\uparrow \cap\, A_{\psi_2} = \emptyset$$

decision procedure:

minimal word (u, v) of $A_{\psi_1}\uparrow$ size no more than exponential

NExpSpace algorithm:

guess (u, v) - a minimal word of $A_{\psi_1}\uparrow$

check if $(u, v)\uparrow \cap\, A_{\psi_2} = \emptyset$

can be checked in ExpSpace in size of A_{ψ_1}, A_{ψ_2}

$\mathscr{B}$:

1. input has a subword u?
2. simulate A_{ψ_2} on input and $v\uparrow$ on output - product construction

poly state machine in size of $u\uparrow$, $v\uparrow$, and A_{ψ_2}

Emptiness of 2NFA is PSpace-complete

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PosPTL separability of *Right sided SL constraints*	NExpSpace

Thank you