Partial Order Reduction for Timed Systems

Frédéric Herbreteau[⋆] B Srivathsan[⋆] Igor Walukiewicz[⋆]
Govind R[⋆⋆]

LaBRI, Université de Bordeaux
Chennai Mathematical Institute

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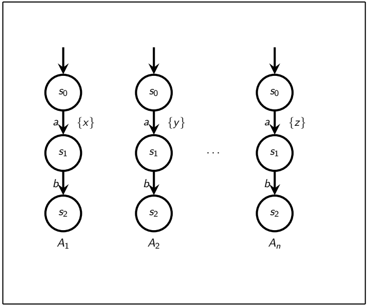
Networks of Timed automata: used to model several safety-critical systems.



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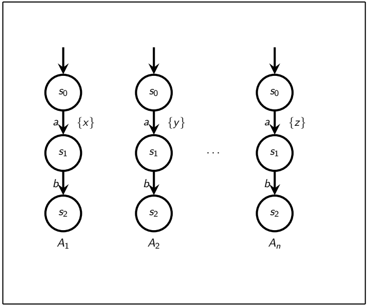
State-space explosion is a major challenge in their verification!



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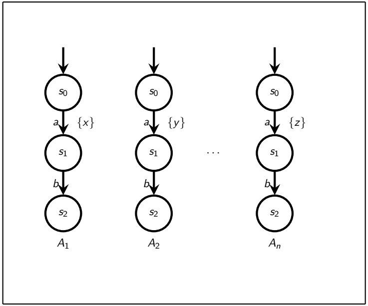

Our work

Algorithms for networks of timed automata motivated by **partial order reduction**.

Overview of the talk

Partial order reduction

Timed automata and our problem

Local time semantics

Local zone graph

Solution 1

Local time semantics and aggregated zones

Sync-subsumption

Interleavings in local sync graphs

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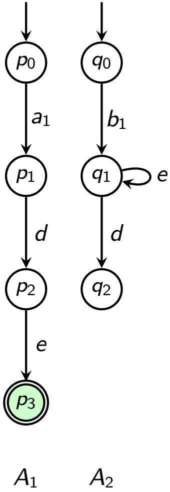
Local time semantics and POR

Local sync graphs + POR ?

No finite subsumption for LZG that allows POR

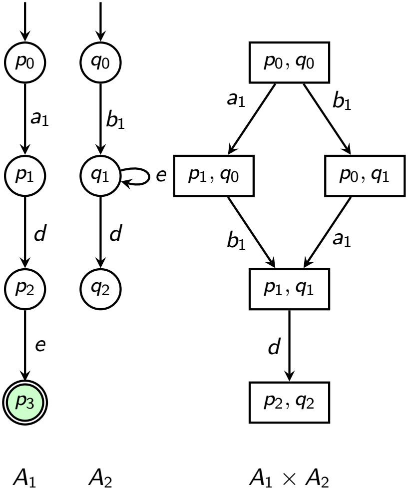
POR for spread-bounded systems

Reachability in networks of automata



Synchronises on joint actions

Reachability in networks of automata



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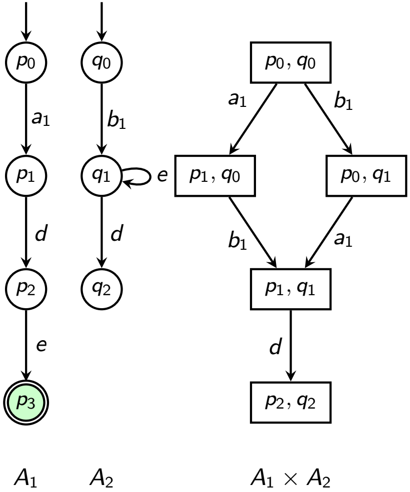

State space explosion
due to multiple interleavings
of same actions

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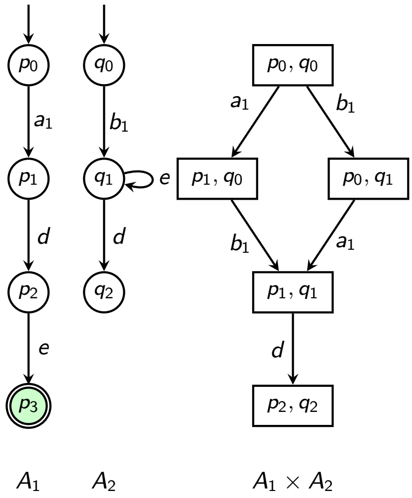

State space explosion
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Can we do better than
exhaustively explore $A_1 \times A_2$?

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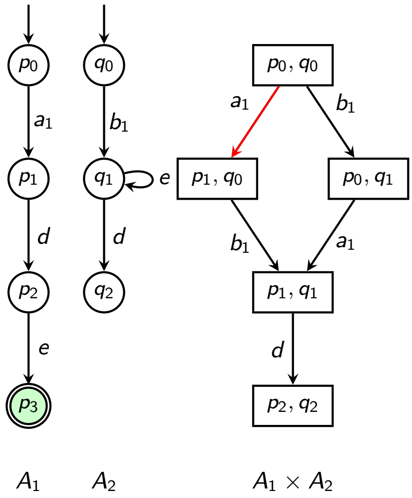

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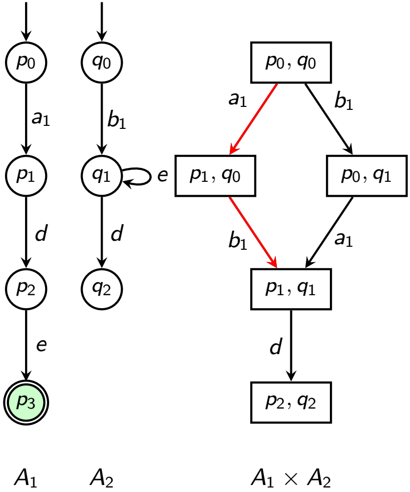

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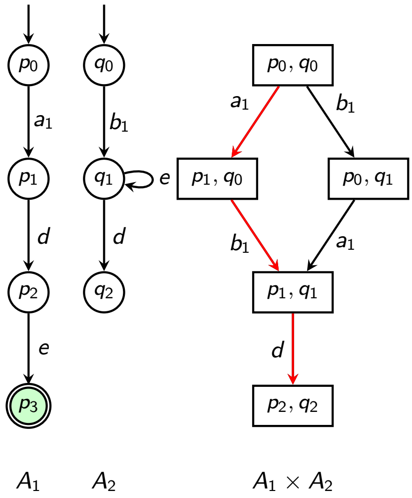

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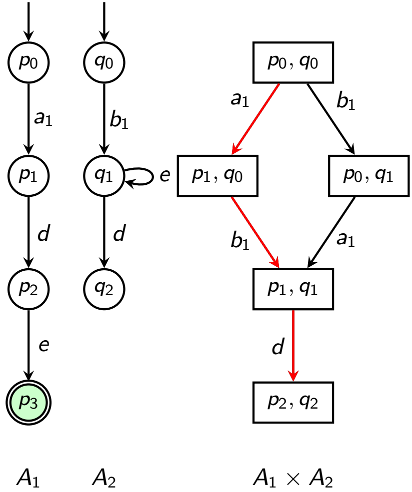

State space explosion
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Sufficient to explore a
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Synchronises on joint actions

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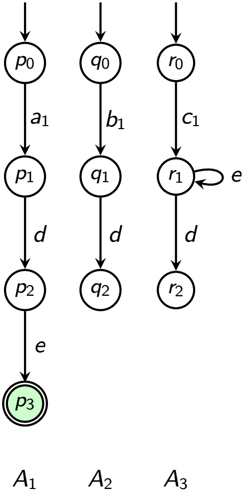

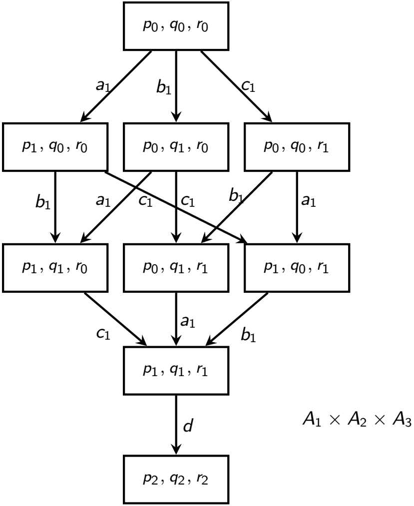

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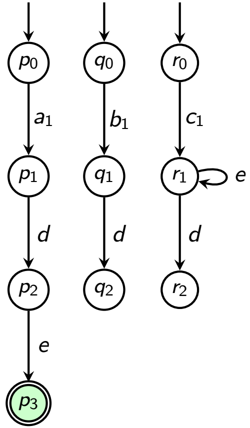
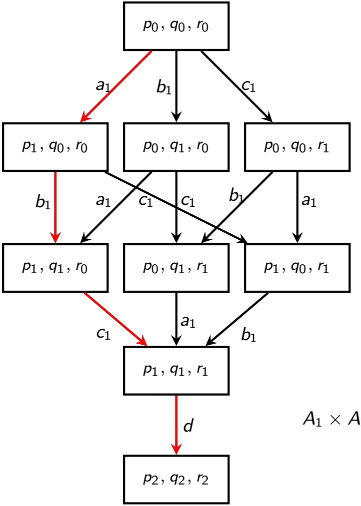


A_1 A_2 A_3



$A_1 \times A_2 \times A_3$

Reachability in networks of automata

The graphs occuring in practice are very large, in general!

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A network with 10 processes
5 states per process $\quad \rightarrow \quad 5^{10} \approx 10$ million states.

Partial Order Reduction (POR)

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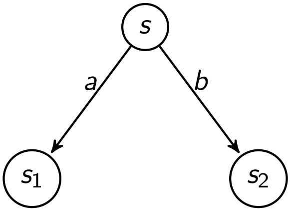
Actions a and b are *independent*, if for every state s from which they are enabled:



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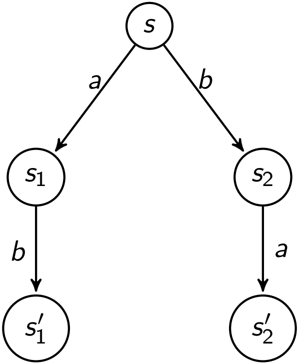
Forward Diamond : If $s \xrightarrow{a} s_1$ and $s \xrightarrow{b} s_2$, then b is enabled from s_1 and a is enabled from s_2.



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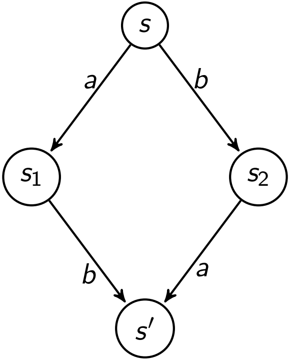

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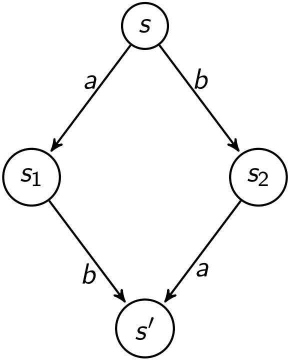
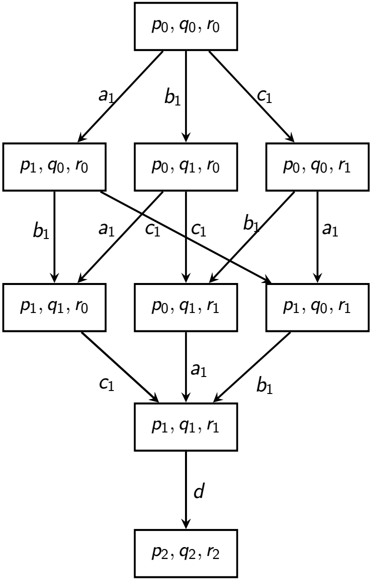


Actions a, b, are independent if they belong to different processes.

Partial Order Reduction (POR)



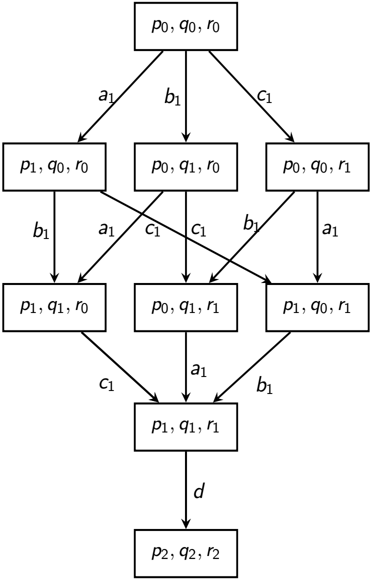
Partial Order Reduction (POR)

Equivalence relation between paths

$u \sim w$ - when w can be obtained from u by permuting adjacent independent actions.

$a_1 b_1 c_1 d$

$b_1 a_1 c_1 d$



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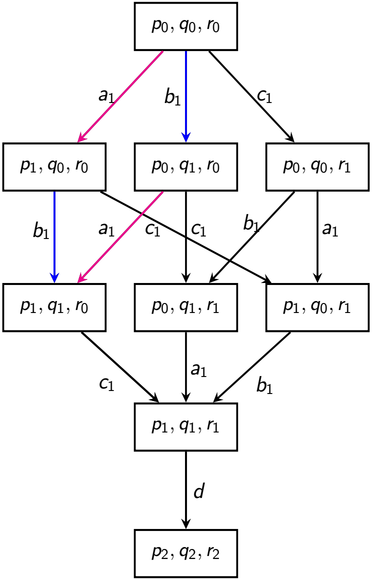

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Equivalence relation between paths

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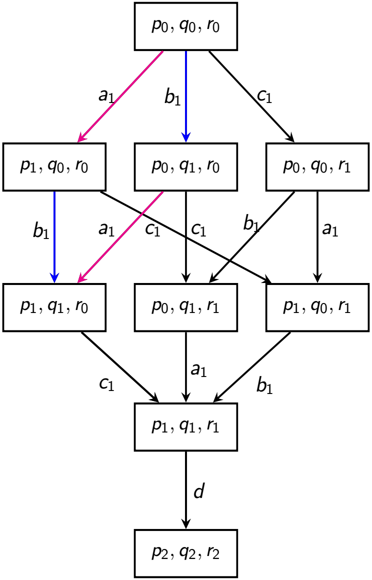
Strategy

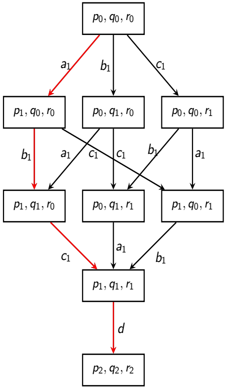
- Classify paths into equivalence classes and explore only one path from each equivalence class.
- Avoid exploring multiple interleavings of independent actions.



Partial Order Reduction (POR)

Goal

To compute a subset of successors from each state seen during exploration.



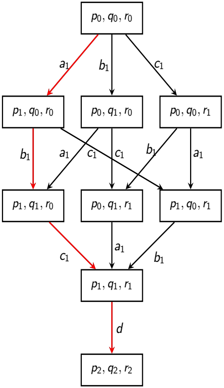
Partial Order Reduction (POR)

Goal

To compute a subset of successors from each state seen during exploration.

Challenges

- Completeness.
- Computing subsets should be "much more efficient" than testing reachability.
- Subset should be as small as possible.



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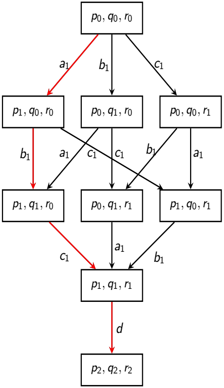
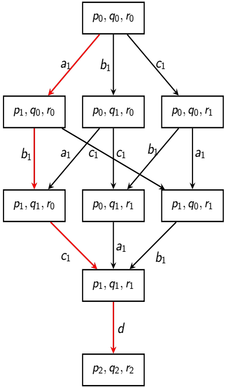


Partial order reduction methods

- Extensively studied.
- Still an active research field.
- Several known variants.

Partial Order Reduction (POR)



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Variants

- Stubborn sets [Valmari '89]
- Ample sets [Godefroid '90]
- Persistent sets [Peled '93]
- Source sets [Abdulla et al. '16]

Similar ideas, but differ in the choice of the subset of actions to be played.

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✓ Partial order reduction

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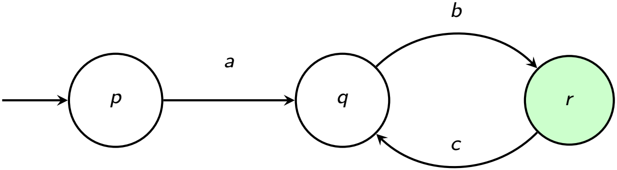
Our focus

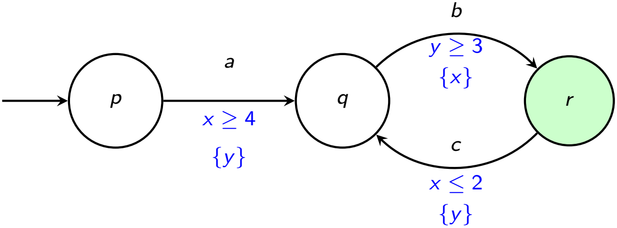
Reachability in networks of timed automata

Timed automaton [Alur Dill '94]



Timed automaton [Alur Dill '94]

Clocks: $X = \{x, y\}$



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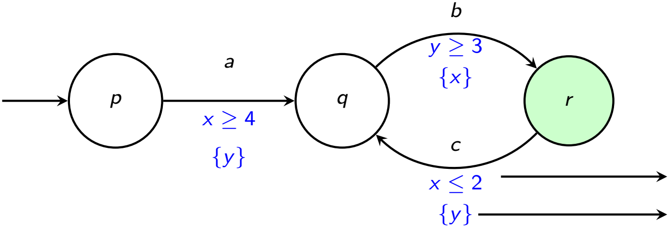

Guard

Reset

Timed automaton [Alur Dill '94]

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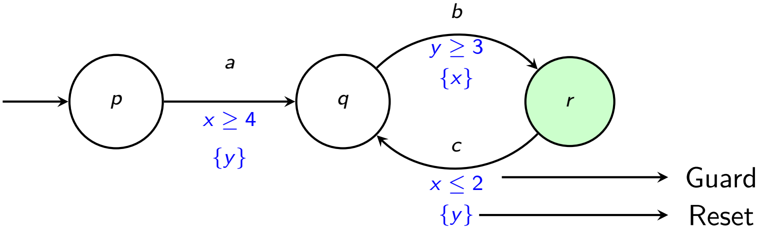


Configuration: (q, v)

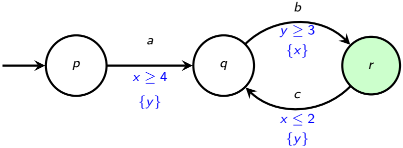
State

Valuation

$v : X \mapsto \mathbb{R}_{\geq 0}$

Timed automaton [Alur Dill '94]

Timed automaton



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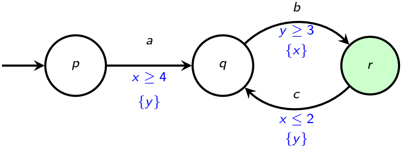

Run of the timed automaton

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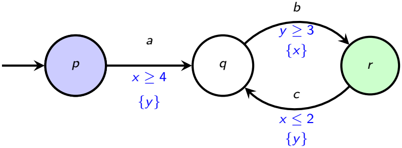

Run of the timed automaton

$(p, x = 0, y = 0)$

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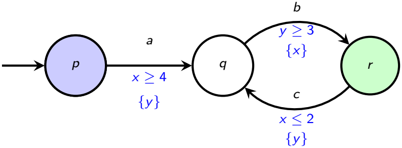


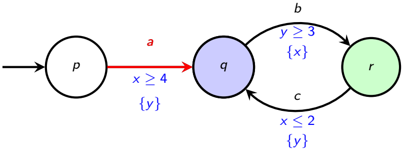
Run of the timed automaton

Delay

$$(p, x = 0, y = 0) \xrightarrow{5} (p, x = 5, y = 5)$$

Timed automaton [Alur Dill '94]

Timed automaton



Run of the timed automaton

Delay

Action

$$(p, x = 0, y = 0) \xrightarrow{5} (p, x = 5, y = 5) \xrightarrow{a} (q, x = 5, y = 0)$$

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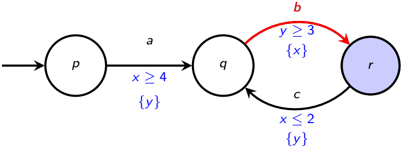
Timed automaton



Run of the timed automaton

Delay ↑ Action ↑

$$(p, x=0, y=0) \xrightarrow{5} (p, x=5, y=5) \xrightarrow{a} (q, x=5, y=0) \xrightarrow{3} \xrightarrow{b} (r, x=0, y=3)$$

Timed automaton [Alur Dill '94]

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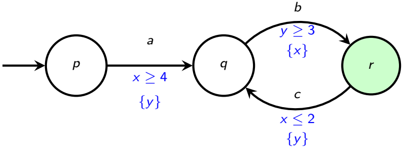

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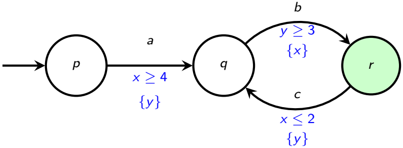
Reachability Problem

Decide if a given timed automaton has a run reaching a green state.

Timed automaton [Alur Dill '94]

Timed automaton



Run of the timed automaton

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Reachability Problem

Decide if a given timed automaton has a run reaching a green state.

This problem is PSPACE-complete
[Alur Dill '94]

Standard Reachability Algorithm [Daws Tripakis '98]

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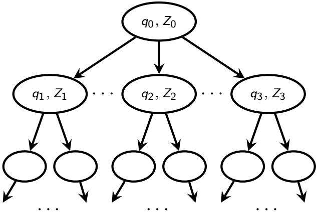
Zone graph

Directed graph with nodes of the form (state, zone).



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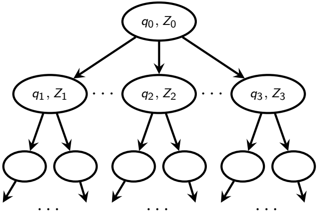


Used in tools like
UPPAAL, TChecker, PAT, KRONOS

Standard Reachability Algorithm

Zone graphs are infinite in general!

Standard Reachability Algorithm

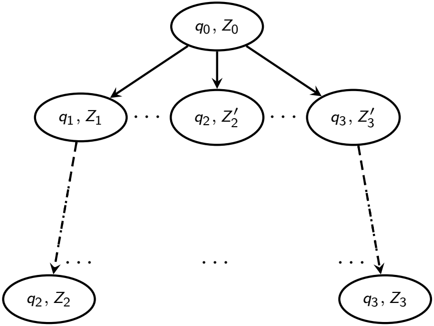
Zone graphs are infinite in general!

How do we ensure termination?

Standard Reachability Algorithm

Idea

For each newly seen node (q, Z), check if there exists an already visited node (q, Z') such that Z is subsumed by Z'.



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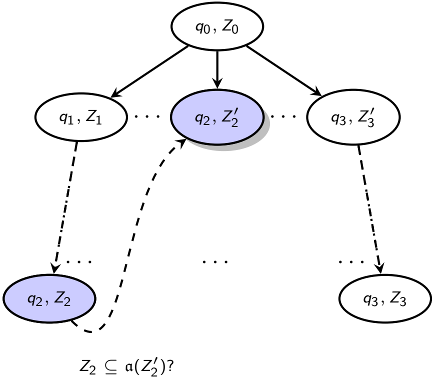

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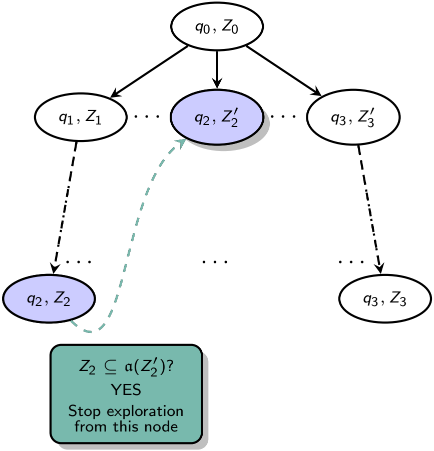

$Z_2 \subseteq \mathfrak{a}(Z'_2)$?

YES

Stop exploration from this node

Standard Reachability Algorithm

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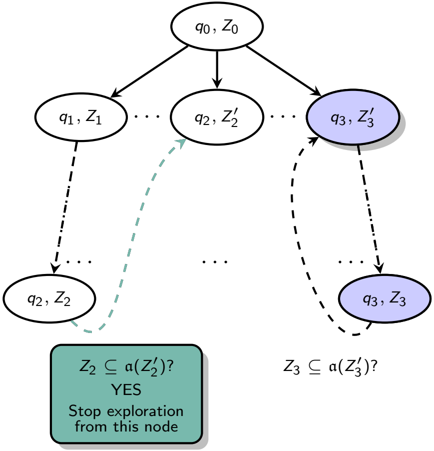

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$Z_3 \subseteq \mathfrak{a}(Z'_3)$?

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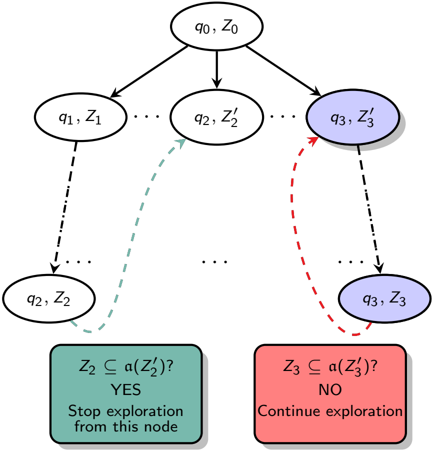

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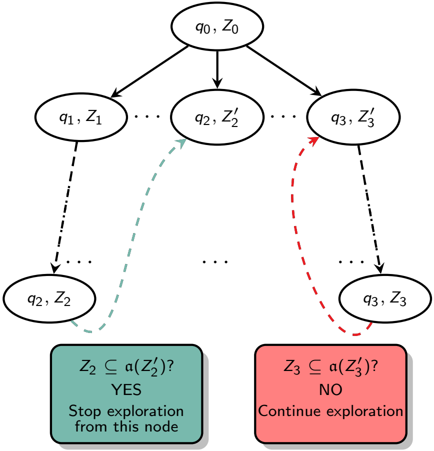
- If yes, no need to explore further from that node.
- If not, add (q, Z) to the set of visited nodes and continue exploration.



$Z_2 \subseteq \mathfrak{a}(Z'_2)$?
YES
Stop exploration from this node

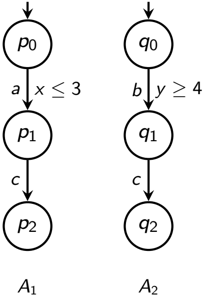
$Z_3 \subseteq \mathfrak{a}(Z'_3)$?
NO
Continue exploration

Subsumptions used to ensure finiteness for zone graphs

Well-studied subsumptions: $\mathfrak{a}_{LU}$, $\mathfrak{a}_M$ [BBLP 06, HSW12]

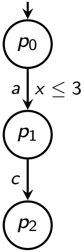
Network of timed automata

Network of timed automata

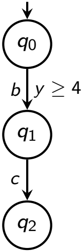


No shared clocks

Network of timed automata

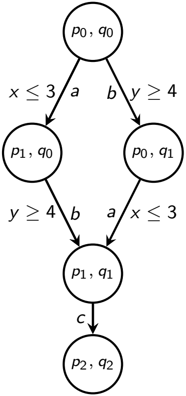


A_1



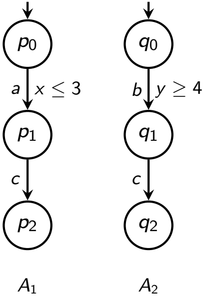
A_2

No shared clocks



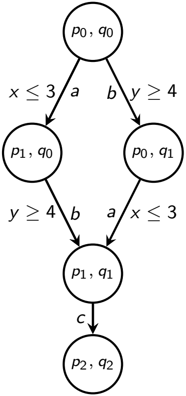
Synchronized
product

Network of timed automata



A_1 A_2

No shared clocks



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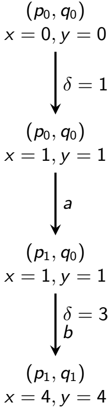
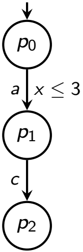


(p_0, q_0)
$x = 0, y = 0$
$\delta = 1$
(p_0, q_0)
$x = 1, y = 1$
a
(p_1, q_0)
$x = 1, y = 1$
$\delta = 3$
b
(p_1, q_1)
$x = 4, y = 4$

Run of the network

Network of timed automata



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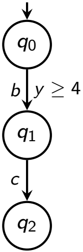
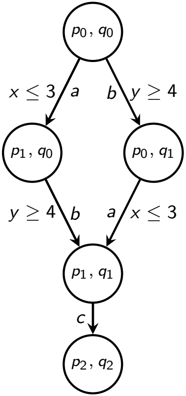


A_2

No shared clocks



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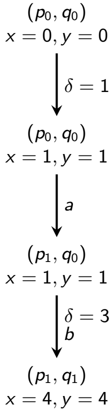
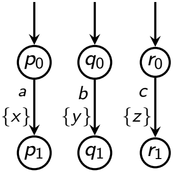


Run of the network

Time elapses synchronously for all processes

Zone Graph of a network of timed automata



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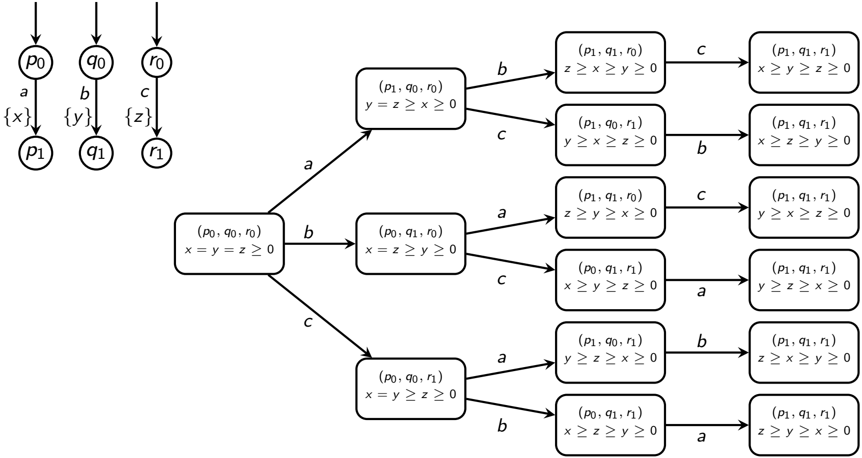

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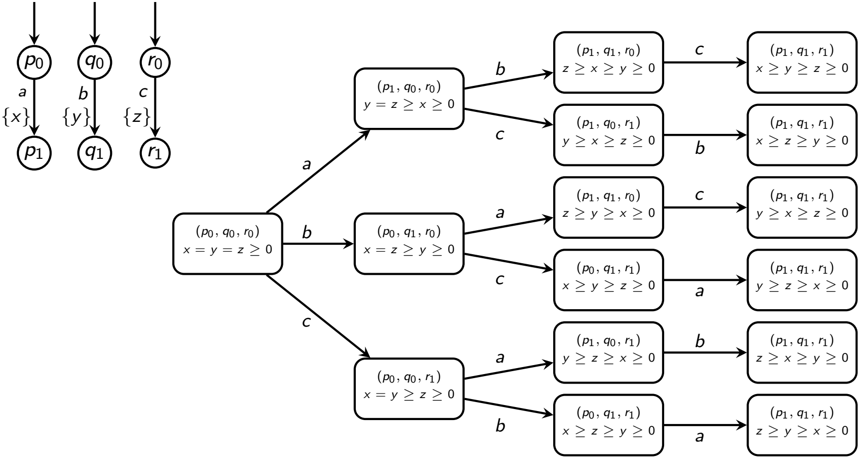

Different interleavings of independent actions lead to different nodes!

Our problem: State-space explosion due to interleavings for networks of timed automata

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Main idea: **local time semantics**

Overview of the talk

✓ Partial order reduction

✓ Timed automata and our problem

Local time semantics

Local zone graph

Solution 1
Local time semantics and aggregated zones

Sync-subsumption

Interleavings in local sync graphs

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Local sync graphs + POR ?

No finite subsumption for LZG
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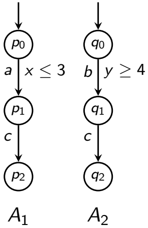
POR for spread-bounded systems

Local time semantics [Bengtsson et al. '98]

Idea: Allow each process to elapse time independently



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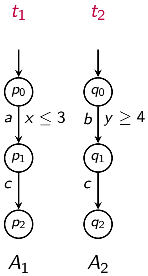

Reference clocks to measure time elapsed in each process

Local time semantics [Bengtsson et al. '98]

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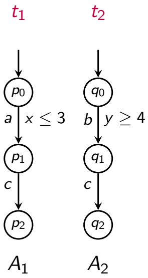

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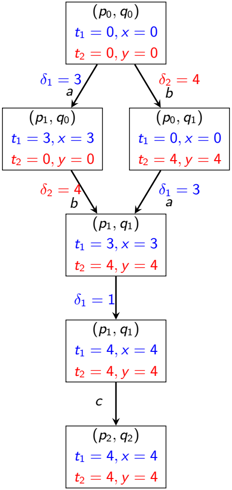

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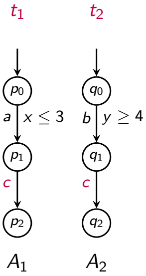
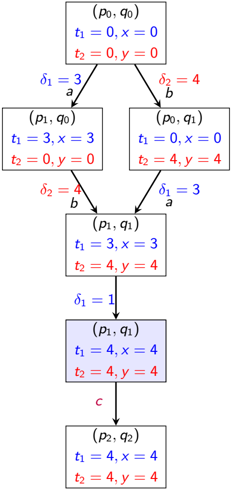


Shared actions only executed from valuations in which processes agree on the value of reference clocks



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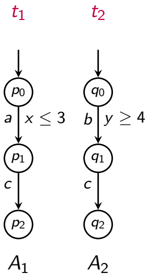
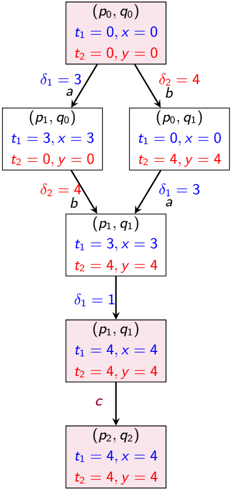


Reference clocks to measure time elapsed in each process

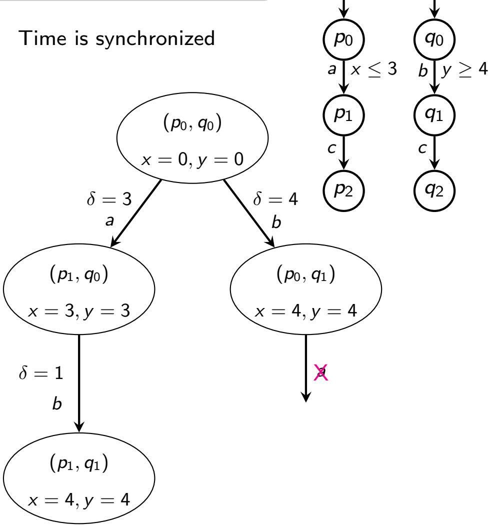
Shared actions only executed from valuations in which processes agree on the value of reference clocks

Synchronized valuations: reference clocks of all the processes have the same value



Global time semantics (Standard)

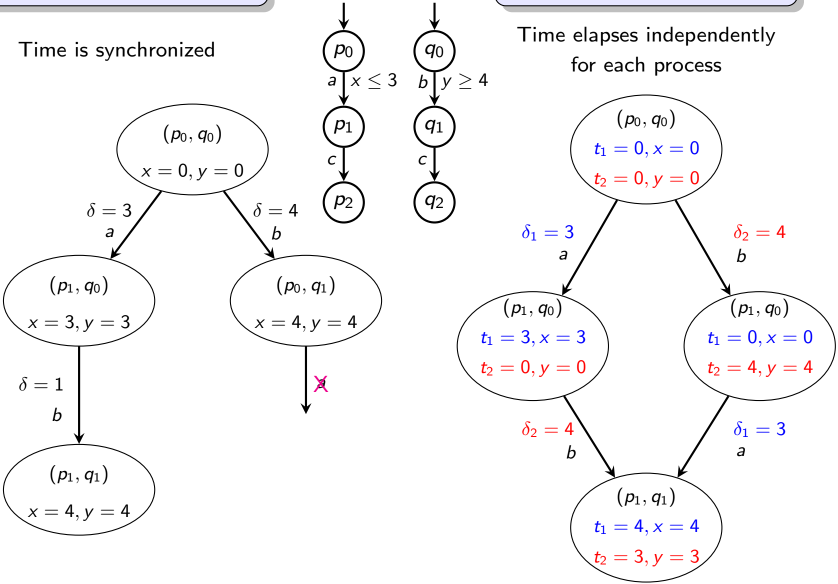
Time is synchronized



Global time semantics (Standard)

Local time semantics

[Bengtsson et al. '98]



Time is synchronized

Time elapses independently for each process

Local time semantics

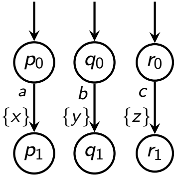
Independence [Bengtsson et al. '98]

If $(q_0, v_0) \xrightarrow{\sigma_1}_{lt} (q, v)$ and $\sigma_1 \sim \sigma_2$, then $(q_0, v_0) \xrightarrow{\sigma_2}_{lt} (q, v)$.

Correctness [Bengtsson et al. '98]

- Every global run is a local time run.
- If there is a local time run to a (q, v), where v is a synchronized valuation, then there is a global run to (q, v).

Local Zone Graph



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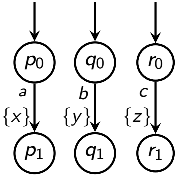

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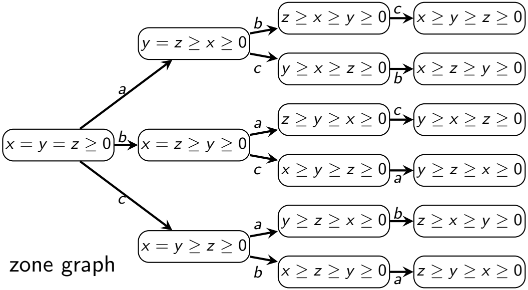

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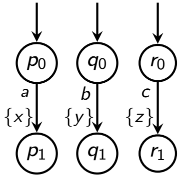

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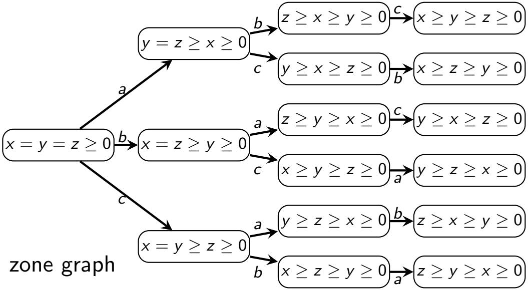

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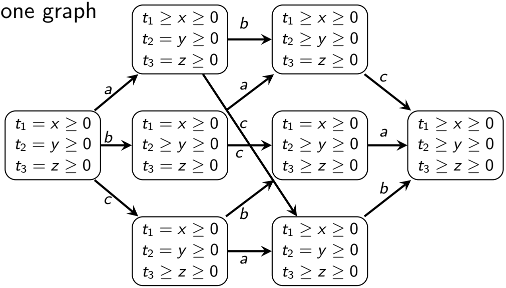

Local zone graphs

Challenge: Local zone graphs are not finite in general!

Overview of the talk

✓ Partial order reduction

✓ Timed automata and our problem

✓ Local time semantics	✓ Local zone graph
Solution 1 **Local time semantics and aggregated zones**	Sync-subsumption Interleavings in local sync graphs
Solution 2 **Local time semantics and POR**	Local sync graphs + POR ? No finite subsumption for LZG that allows POR POR for spread-bounded systems

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Current attempts at finiteness for local zone graphs

- Catchup equivalence
  [Bengtsson et al. '98]
- Extrapolation operator
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Not efficiently computable.

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all reference clocks are equal

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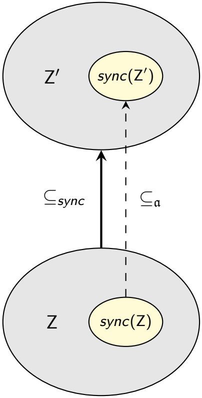

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$Z \subseteq_{sync} Z'$ if
$sync(Z) \subseteq \mathfrak{a}_{LU}(sync(Z'))$

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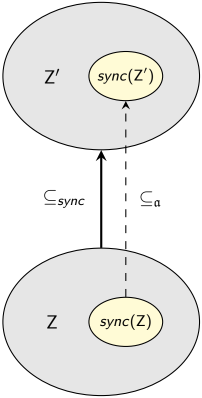

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Allows use of subsumption techniques for standard zones.

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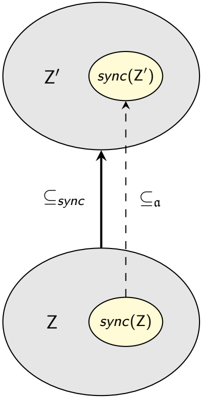

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Local sync graph - local zone graph with sync-subsumption

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✓ Local time semantics	Local zone graph
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Aggregated zones in local zone graph

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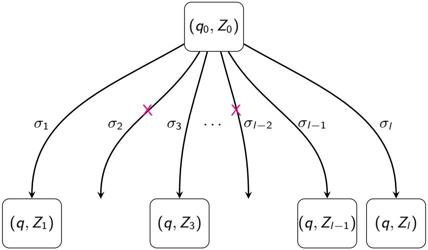
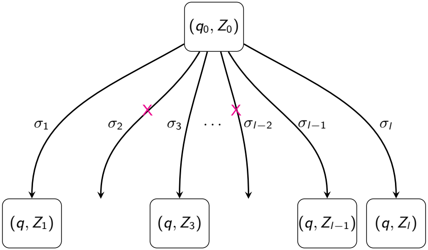


Global zone graph

Aggregated zones in local zone graph



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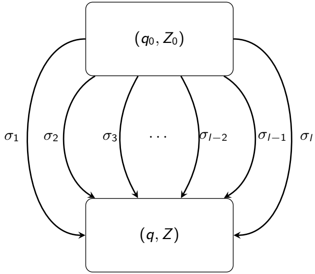

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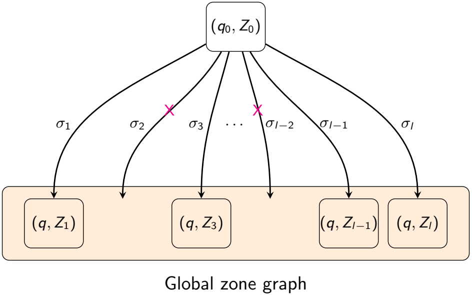

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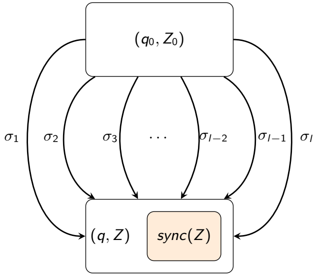

Local zone graph

$$Z_1 \cup Z_2 \cup \cdots \cup Z_l = sync(Z)$$

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Experimental Results

Models	UPPAAL			Global ZG			Local ZG		
(# processes)	visited	stored	sec.	visited	stored	sec.	visited	stored	sec.
CorSSO 3	64378	61948	1.48	64378	61948	1.41	1962	1962	0.05
CorSSO 4	timeout			timeout			23784	23784	0.69
CorSSO 5	timeout			timeout			281982	281982	16.71
Critical reg. 4	78049	53697	1.45	75804	53697	2.27	44490	28400	2.40
Critical reg. 5	timeout			timeout			709908	389614	75.55
Dining Phi. 7	38179	38179	34.61	38179	38179	7.28	2627	2627	0.32
Dining Phi. 8	timeout			timeout			8090	8090	1.65
Dining Phi. 9	timeout			timeout			24914	24914	7.10
Dining Phi. 10	timeout			timeout			76725	76725	30.20
Parallel 6	11743	11743	4.82	11743	11743	1.09	256	256	0.02
Parallel 7	timeout			timeout			576	576	0.04
Parallel 8	timeout			timeout			1280	1280	0.11
CSMACD 4	258	258	0.03	258	258	0.04	258	258	0.04
CSMACD 5	850	850	0.04s	850	850	0.07	850	850	0.11

Timeout is set to 90 seconds.

TChecker: an open-source model-checker for timed systems.
Available at `https://github.com/ticktac-project/tchecker`

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Results

Local-sync graph implementation

Results

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Reachability algorithm based on local time semantics.

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Local-sync graph implementation

Reachability algorithm based on local time semantics.

Merges interleavings into a single node.

Results

Local-sync graph implementation

Reachability algorithm based on local time semantics.

Merges interleavings into a single node.

Good experimental results

- Very good performance on some examples.
- At least as good as the standard algorithm on every example.

Overview of the talk

✓ Partial order reduction

✓ Timed automata and our problem

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Local zone graph

✓ Solution 1

Local time semantics and aggregated zones

✓ Sync-subsumption

✓ Interleavings in local sync graphs

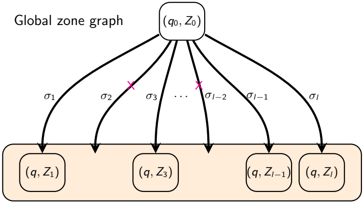
Solution 2

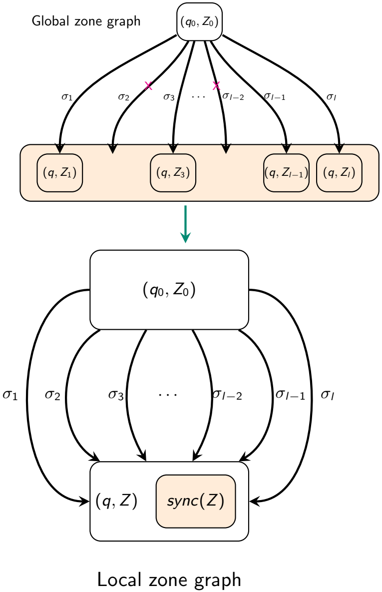
Local time semantics and POR

Local sync graphs + POR ?

No finite subsumption for LZG that allows POR

POR for spread-bounded systems


Global zone graph
(q_0, Z_0)
σ_1
σ_2
σ_3
$\cdots$
σ_{l-2}
σ_{l-1}
σ_l
(q, Z_1)
(q, Z_3)
(q, Z_{l-1})
(q, Z_l)



So far: All interleavings of a sequence of actions merged into a single node.

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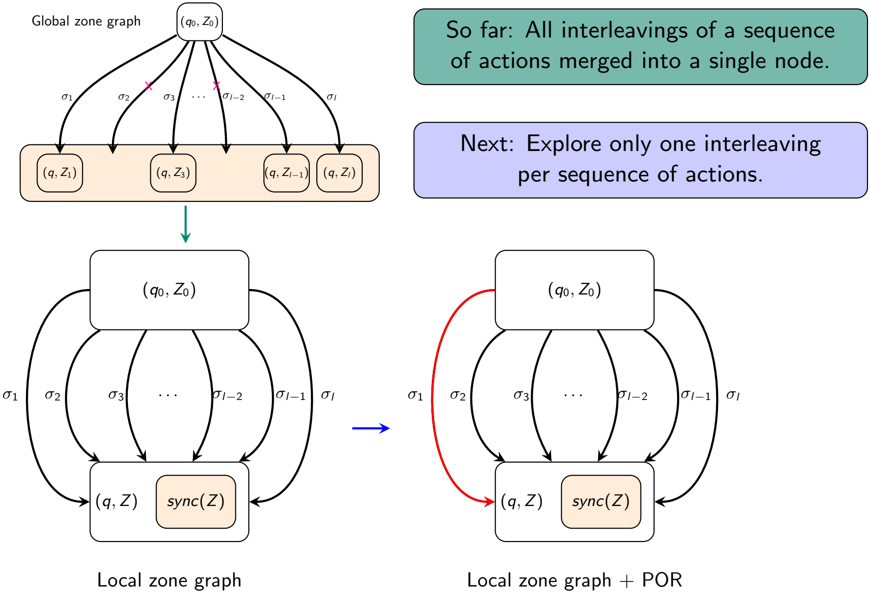
Next: Explore only one interleaving per sequence of actions.



Current object: Local sync graph

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Sync-subsumption does not preserve enabled actions
Local sync graph does not have diamonds!

Current object: Local sync graph

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Can we directly apply an existing POR
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NO!

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Challenge: To obtain a finite version of local zone graph that allows application of POR.

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What do we need?

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- Finiteness
- Soundness

Challenge: To obtain a finite version of local zone graph that allows application of POR.

What do we need?

- Finiteness
- Soundness
- Preservation of enabled actions

Our Results

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Spread-bounded systems:

Our Results

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A new subsumption $\mathfrak{a}^*_M$ that guarantees soundness and preservation of enabled actions, but not finiteness.

Subclass of networks where a modification of $\mathfrak{a}^*_M$ also guarantees finiteness.

Spread-bounded systems: Networks for which every run can be converted to a run with only a bounded divergence between reference clocks.

Finite subsumption for spread-bounded systems

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$\mathfrak{a}_D^M$ **subsumption**

Finite subsumption for spread-bounded systems

$\mathfrak{a}_D^M$ subsumption

Theorem

Local zone graph + $\mathfrak{a}_D^M$ subsumption is finite, sound and preserves enabled actions for a D-spread-bounded system.

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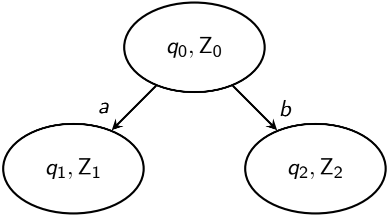

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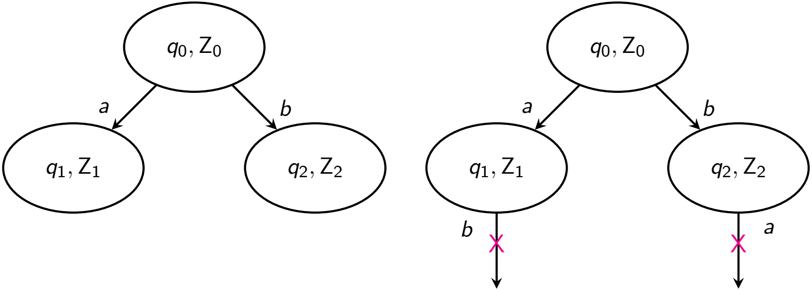

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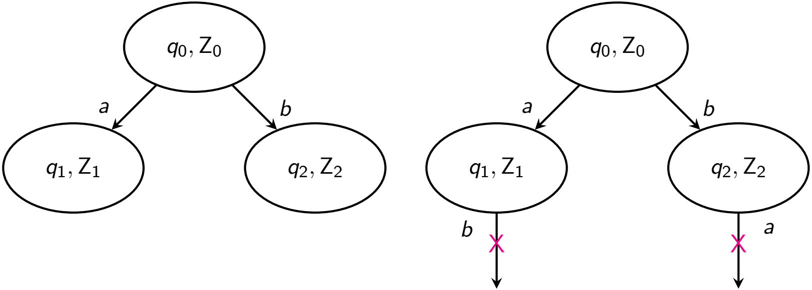

Problem: a and b are enabled, but neither ab nor ba are feasible

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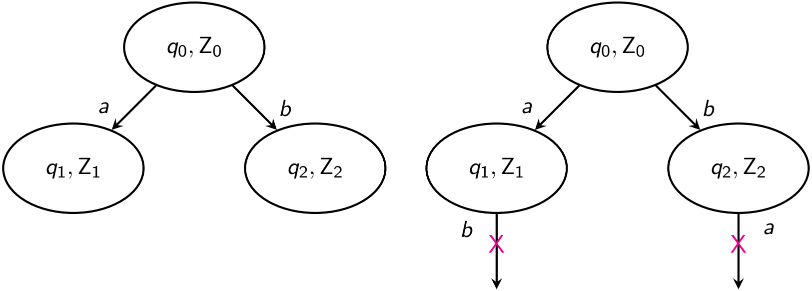

Problem: a and b are enabled, but neither ab nor ba are feasible

No forward diamonds!

We cannot directly apply an off-the-shelf POR technique on finite zone graph.

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We give specialized partial order reduction algorithms for spread-bounded systems.

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We give specialized partial order reduction algorithms for spread-bounded systems.

- Global-local POR
- Client-server POR

Experiments for POR implementation

Global-local POR

Models	Global ZG			Local Sync Graph			POR-LZG		
(# processes)	visited	stored	time	visited	stored	time	visited	stored	time
GL Toy 4	257	257	1.745s	257	257	2.295s	14	14	0.062s
GL Toy 5	1,025	1,025	10.264s	1,025	1,025	10.159s	17	17	0.091s
GL Toy 8	65,537	65,537	14m51s	65,537	65,537	14m52s	26	26	0.113s
Fire-alarm 4	271	271	2.269s	271	271	0.114s	17	17	0.299s
Fire-alarm 5	1,055	1,055	0.102s	1,055	1,055	0.123s	21	21	0.085s
Fire-alarm 8	**65,791**	**65,791**	**7.752s**	**65,791**	**65,791**	**29.622s**	**33**	**33**	**1.740s**
Fire-alarm 10	1mil	1mil	> 1hr	1mil	1mil	> 1hr	41	41	1.8s
CSMA CD 3	70	70	0.03s	70	70	0.03s	76	76	0.04s
CSMA CD 4	258	258	0.07s	258	258	0.12s	306	306	0.14s
CSMA CD 5	850	850	0.24s	850	850	0.46s	1100	1100	0.6s

Good results for **Fire-alarm**

Implemented in **TChecker**.

Available at https://github.com/ticktac-project/tchecker

Experiments for POR implementation

Client-server POR

Models (# processes)	Global ZG			Local Sync Graph			POR-LZG		
	visited	stored	sec.	visited	stored	sec.	visited	stored	sec.
CS Toy 3	216	216	1.3s	216	216	1.3s	56	56	0.2s
CS Toy 4	1296	1296	9.8s	1296	1296	9.6s	144	144	0.4s
CS Toy 5	7776	7776	1m27s	7776	7776	1m15s	352	352	4.2s
CS Toy 8	1.6M	1,6M	98m	1,6M	1,6M	95m	4352	4352	25.2s
WCET 1	138	138	2.2s	138	138	2.3s	113	101	2.3s
WCET 2	9,379	9,379	1m5s	8,803	8,803	53s	6260	4520	1m5s
WCET 3	**647,338**	**647,338**	$>$ **1hr**	**524,650**	**524,650**	$>$ **1hr**	**168,463**	**168,463**	**11m**
WCET 4	47mil	47mil	$>$ 1,027m	29mil	29mil	$>$ 472m	3mil	2.2mil	200m

Good results for **WCET**

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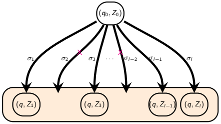

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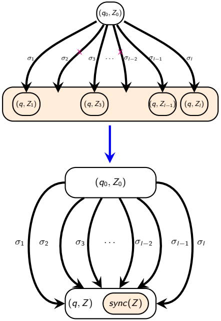

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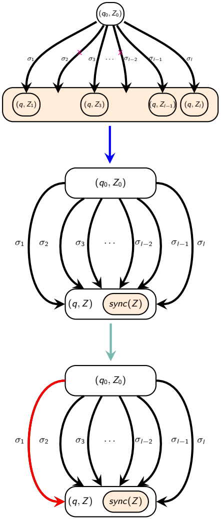

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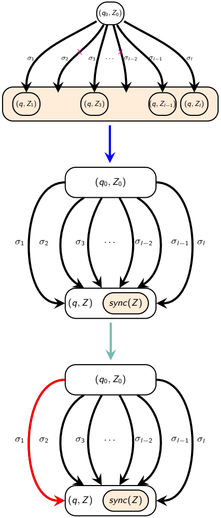

Evaluation of a prototype of the implementation of these methods using the tool TChecker.

Open problems and future directions

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Local sync graphs

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Extending our algorithm to other models

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Thank you!