Streaming String Transducers
Towards a Theory of Regular Transformations

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Theory of Regular Languages

Regular Languages of Finite Strings
\[ \varphi : \Sigma^* \rightarrow \{0, 1\} \]

- Logically: MSO logic
- Computational model: Finite state automata
Finite State Automata

Automaton accepting **strings of even length**:

```
Start  E  O
   0,1  0,1
```

Automaton accepting **strings with an even number of 1’s**:

```
Start  E  O
   0    1  0
   1    1
```

Automaton accepting **even strings (multiple of 2)**:

```
Start  E  O
   0    1  1
   1    0
```
Monadic Second Order Logic (MSO) over Graphs

- The structure is

\[ (N, E, L_a, L_b, \ldots, L_k) \]

The domain (set of nodes)

Edge relation \( E \subseteq N \times N \)

Some unary predicates \( L_a, L_b, \ldots, L_k \subseteq N \) partitioning \( N \)
Monadic Second Order Logic (MSO) over Graphs

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\[(\mathbb{N}, E, L_a, L_b, \ldots, L_k)\]

- The domain (set of nodes)

- Edge relation \(E \subseteq \mathbb{N} \times \mathbb{N}\)

- Some unary predicates \(L_a, L_b, \ldots, L_k \subseteq \mathbb{N}\) partitioning \(\mathbb{N}\)

- Strings are interpreted structures: e.g. \((\{1, \ldots, 10\}, E, L_a, L_b, L_c)\)

\[
s = \quad a \quad b \quad b \quad a \quad b \quad c \quad a \quad b \quad c \quad c
\]
\[
L_a = \{ 1, \quad 4, \quad 7\}
\]
\[
L_b = \{ \quad 2, \quad 3, \quad 5, \quad 8\}
\]
\[
L_c = \{ \quad 6, \quad 9, \quad 10\}
\]
Monadic Second Order Logic (MSO) over Graphs

- The structure is

\((N, E, L_a, L_b, \ldots, L_k)\)

- The domain (set of nodes)
- Edge relation \(E \subseteq N \times N\)
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- Strings are interpreted structures: e.g. \((\{1, \ldots, 10\}, E, L_a, L_b, L_c)\)

\[ s = a \ b \ b \ a \ b \ c \ a \ b \ c \ c \]
\[ L_a = \{1, 4, 7\} \]
\[ L_b = \{2, 3, 5, 8\} \]
\[ L_c = \{6, 9, 10\} \]

- Formulas are defined inductively:
  - first-order variables: \(x, y, z\) ranging over nodes
  - second-order variables: \(X, Y, Z\) ranging over node sets
  - Atomic formulas: \(E(x, y), L_a(x), x = y\) and \(x \in X\)
  - Boolean connectives: \(\varphi_1 \land \varphi_2, \lnot \varphi_3\)
  - First-order quantification: \(\exists x. \varphi\)
  - Second-order quantification: \(\exists X. \varphi\)
Examples

Set of strings with an even number of letters:

- Consider two sets of positions Even and Odd.
- Both sets are disjoint.
- First position is in Odd and the last position is in Even.
- For each position in Even the next position (if exists) is in Odd and vice-versa.
Examples

Set of strings with an even number of letters:

- Consider two sets of positions **Even** and **Odd**.
- Both sets are disjoint.
- First position is in **Odd** and the last position is in **Even**.
- For each position in **Even** the next position (if exists) is in **Odd** and vice-versa.

\[
\exists \text{Odd}. \exists \text{Even}.
\]

\[
(\forall x.((x \in \text{Odd}) \rightarrow \neg(x \in \text{Even}) \land ((x \in \text{Even}) \rightarrow \neg(x \in \text{Odd})))
\]

\[
\land \text{First}(x) \rightarrow (x \in \text{Odd})
\]

\[
\land \text{Last}(x) \rightarrow (x \in \text{Even})
\]

\[
\forall x \forall y((x \in \text{Odd}) \land E(x, y)) \rightarrow y \in \text{Even}
\]

\[
\forall x \forall y((x \in \text{Even}) \land E(x, y)) \rightarrow y \in \text{Odd}).
\]
Theory of Regular Languages

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\[ \varphi : \Sigma^* \rightarrow \{0, 1\} \]

MSO logic → Logically

Finite state automata → Computational model


Theorem ([Büchi, 1960, Elgot, 1961, Trakhtenbrot, 1962])

A language of finite strings is accepted by a finite state automaton iff it is MSO-definable.

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A language of finite strings is accepted by a finite state automaton iff it is MSO-definable.

Why bother?
- new tools to solve problems in logic
- revolutionized the field of automata theory as Büchi initiated the study of equivalent finite state models for MSO over infinite strings.
Theorem ([Büchi, 1962, McNaughton, 1966])

An language of infinite strings is accepted by a Muller automaton iff it is MSO-definable.
Theory of Regular Languages

Regular Languages of $\omega$-Strings

$\varphi : \Sigma^\omega \rightarrow \{0, 1\}$

Logically

MSO logic

Equi-expressiveness [Büchi, 1962, McNaughton, 1966]

Computationally

Büchi, Muller automata

Theorem ([Büchi, 1962, McNaughton, 1966])

An language of infinite strings is accepted by a Muller automaton iff it is MSO-definable.

Since then the theory of regular languages has been lifted to languages of Trees [Rabin, 1969], partial-orders [Thomas, 1995], and more.
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Can we go beyond Languages!
Theory of Regular Transformations

Regular Transformations of Finite Strings
\[ \varphi : \Sigma^* \rightarrow \Gamma^* \]

- Logically: MSO logic [Courcelle, 1994]
- Computational model: Finite state transducers
Theory of Regular Transformations

Regular Transformations of Finite Strings
\[ \varphi : \Sigma^* \rightarrow \Gamma^* \]

Logically
MSO logic [Courcelle, 1994]

Equi-expressiveness [Engelfriet and Hoogeboom, 2001]

Computationally
(two-way) Finite state transducers

Unfortunately, two-way finite state transducers cannot be naturally generalized with such ease.
It would be nice to have a one-way (streaming) transducer precisely capturing the class of MSO-definable transformations.

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Theory of Regular Transformations

Regular Transformations of Finite Strings
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- Unfortunately, two-way finite state transducers cannot naturally be generalized with such ease
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Logically
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Computational model
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Equi-expressiveness [Engelfriet and Hoogeboom, 2001]

- MSO-definable transformations can be naturally extended to define transformations for more general structures
- Unfortunately, two-way finite state transducers can not naturally be generalized with such ease
- Also, it would be nice to have a one-way (streaming) transducer precisely capturing the class of MSO-definable transformations
Alur and Černý introduced streaming string transducers (SSTs) to model and analyze single-pass list processing programs [Alur and Černý, 2010], e.g.
- imperative programs manipulating heap-allocated lists
- functional programs using tail recursion
- commonly used routines include insert, delete, and reverse.
Streaming String Transducers

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- decidable (PSPACE) functional equivalence and verification (pre/post condition) problem
Streaming String Transducers

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  - imperative programs manipulating heap-allocated lists
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  - commonly used routines include *insert*, *delete*, and *reverse*.

- decidable (PSPACE) **functional equivalence** and verification (pre/post condition) problem

- first one-way (streaming) transducer model that precisely captures the MSO-definable transformations

Theory of regular transformations
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- SSTs naturally generalize to model transformation of more general structures
  - string-to-tree [Alur and D’Antoni, 2012],
  - tree-to-tree [Alur and D’Antoni, 2012],
  - \(\omega\)-string to \(\omega\)-strings [Alur et al., 2012],
  - \(\omega\)-string to \(\omega\)-trees [Alur et al., 2013b].
  - strings to costs [Alur et al., 2013a]
Streaming String Transducers

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  - strings to costs [Alur et al., 2013a]

Theory of regular transformations
Regular Transformations of Finite Strings

Regular Transformations of Infinite Strings

Conclusion
A transformation from $\Sigma$ to $\Gamma$ is a (partial) function $f : \Sigma^* \rightarrow \Gamma^*$.

Generalizes the concept of a language $f : \Sigma^* \rightarrow \{0, 1\}$.

Example:

- $a^n \mapsto a^n b^n$
- $a^n b^m \mapsto a^{2n-1} b^m$

- local transformations, e.g., delete each $a$, repeat every $b$
- reverse transformation, i.e. $a_1 a_2 \ldots a_n \mapsto a_n a_{n-1} \ldots a_1$,
- swapping transformation, e.g. $\alpha \# \beta \mapsto \beta \# \alpha$,
- look-ahead based transformations, e.g.
  - replace each $a$ with $b$ if the string contains a $\#$.
  - replace each $a$ with $b$ if the string contains a prime number of $\#$. 

A transducer is an abstract machine defining a transformation.

Transducers generalize the concept of automata

Similar to languages, a transformation can also be defined using logic, most notably Monadic second-order logic (MSO) over finite strings.
Transformations of Finite Strings

- A **transformation** from $\Sigma$ to $\Gamma$ is a (partial) function $f : \Sigma^* \rightarrow \Gamma^*$.
- Generalizes the concept of a language $f : \Sigma^* \rightarrow \{0, 1\}$.
- Example:
  - $a^n \mapsto a^n b^n$
  - $a^n b^m \mapsto a^{2^n - 1} b^m$
- **Local** transformations, e.g., delete each $a$, repeat every $b$
- **Reverse** transformation, i.e. $a_1 a_2 \ldots a_n \mapsto a_n a_{n-1} \ldots a_1$
- **Swapping** transformation, e.g. $\alpha \# \beta \mapsto \beta \# \alpha$
- **Look-ahead** based transformations, e.g.
  - replace each $a$ with $b$ if the string contains a $\#$.
  - replace each $a$ with $b$ if the string contains a **prime number** of $\#$.
- A **transducer** is an abstract machine defining a transformation.
- Transducers generalize the concept of **automata**
Transformations of Finite Strings

- A **transformation** from $\Sigma$ to $\Gamma$ is a (partial) function $f : \Sigma^* \rightarrow \Gamma^*$.
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- A **transducer** is an abstract machine defining a transformation.
- Transducers generalize the concept of **automata**
- Similar to languages, a transformation can also be defined using logic, most notably **Monadic second-order logic** (MSO) over finite strings.
MSO-definable Transformations

**Definition (Defining Transformation using MSO)**

A transformation using MSO is specified by:
- **input** and **output** alphabets;
- an MSO formula specifying the **domain** of the transformation;
- output string is specified using a **finite number of copies** of nodes of input string graph;
- the **node labels** are specified using MSO formulas; and
- the **existence of edges** between nodes of various copies is specified using MSO formulas.

**Example**

Let $\Sigma = \{a, b, #\}$. Consider a transformation $f_1 : \Sigma^* \rightarrow \Sigma^*$ such that $u_1#u_2#\ldots#u_{n-1}#u_n \mapsto u_1u_1#\ldots#u_nu_n$. where $u$ is reverse of $u$. 
**MSO-definable Transformations**

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**Example**

Let $\Sigma = \{a, b, \#\}$. Consider a transformation $f_1 : \Sigma^* \rightarrow \Sigma^*$

$$u_1 \# u_2 \# \ldots u_{n-1} \# u_n \# v \mapsto \overline{u_1} u_1 \# \ldots \# \overline{u_n} u_n \# v.$$  

where $\overline{u}$ is reverse of $u$. 
MSO-definable Transformations

- $\Sigma = \Gamma = \{a, b, \#\}$, $C = \{1, 2\}$, and
- Node Label Formulas
  - $\text{Label}_{c_1}^\alpha(x) = \text{Label}^{\text{inp}}(x) \land \neg \text{Label}^{\text{inp}}(x) \land \text{reach}_\#(x)$
  - $\text{Label}_{c_2}^\alpha(x) = \text{Label}^{\text{inp}}(x)$
- Edge Label Formulas
  - $\text{Edge}_{c_1,c_1}^{c_1}(x, y) = \text{Edge}^{\text{inp}}(y, x) \land \text{Label}^{\text{inp}}(x) \land \text{Label}^{\text{inp}}(y)$.
  - $\text{Edge}_{c_2,c_2}^{c_2}(x, y) = \text{Edge}^{\text{inp}}(x, y) \land (\neg \text{Label}^{\text{inp}}(x) \lor (\text{Label}^{\text{inp}}(x) \land \neg \text{reach}_\#(x)))$
  - $\text{Edge}^{1,2}(x, y) = (x=y) \land (\text{first}(x) \lor \exists z (\text{Label}^{\text{inp}}(z) \land \text{Edge}^{\text{inp}}(z, x)))$
  - $\text{Edge}^{2,1}(x, y) = \text{Label}^{\text{inp}}(x) \land \text{reach}_\#(x) \land (\exists z (\text{Edge}^{\text{inp}}(y, z) \land \text{Label}^{\text{inp}}(z)) \land (\forall z ((\text{path}(x, z) \land \text{path}(z, y)) \rightarrow \neg \text{Label}^{\text{inp}}(z))))$
MSO-definable Transformations

- \( \Sigma = \Gamma = \{ a, b, \# \} \), \( C = \{1, 2\} \), and
- Node Label Formulas
  - \( \text{Label}_{c1}^{\alpha}(x) = \text{Label}_{\alpha}^{\text{inp}}(x) \land \neg \text{Label}_{\#}^{\text{inp}}(x) \land \text{reach}_{\#}(x) \)
  - \( \text{Label}_{c2}^{\alpha}(x) = \text{Label}_{\alpha}^{\text{inp}}(x) \)
- Edge Label Formulas
  - \( \text{Edge}^{c1,c1}_{1,2}(x, y) = \text{Edge}_{\alpha}^{\text{inp}}(y, x) \land \text{Label}_{\alpha}^{\text{inp}}(x) \land \text{Label}_{\alpha}^{\text{inp}}(y) \)
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  - \( \text{Edge}^{2,1}_{2,1}(x, y) = \text{Label}_{\alpha}^{\text{inp}}(x) \land \text{reach}_{\#}(x) \land (\exists z (\text{Edge}_{\alpha}^{\text{inp}}(y, z) \land \text{Label}_{\#}^{\text{inp}}(z))) \land (\forall z ((\text{path}(x, z) \land \text{path}(z, y)) \rightarrow \neg \text{Label}_{\#}^{\text{inp}}(z))) \)
MSO-definable Transformations

- $a^n \mapsto a^n b^n \checkmark$
- $a^n b^m \mapsto a^{2n-1} b^m$
- local transformations, e.g., delete each $a$, repeat every $b \checkmark$
- reverse transformation, i.e. $a_1 a_2 \ldots a_n \mapsto a_n a_{n-1} \ldots a_1$, $\checkmark$
- swapping transformation, e.g. $\alpha \# \beta \mapsto \beta \# \alpha$, $\checkmark$
- look-ahead based transformations, e.g.
  - replace each $a$ with $b$ if the string contains a $\#$, $\checkmark$
  - replace each $a$ with $b$ if the string contains a prime number of $\#$
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- **look-ahead based** transformations, e.g.
  - replace each $a$ with $b$ if the string contains a $\#$ $\checkmark$
  - replace each $a$ with $b$ if the string contains a prime number of $\#$

**Regular Transformations**

Which transducers accept same class of transformations?
Deterministic Generalized Sequential Machines

Example: For all strings containing a #, replace all a with b.

\[ \begin{array}{c}
\text{start} \\
1 \\
\alpha | \alpha \\
\end{array} \quad \begin{array}{c}
# | # \\
2 \\
\alpha | \alpha \\
\end{array} \]

\[ a \mid b \]

† Here \( \alpha \) stands for any symbol other than \( a \).

- Extend finite automata with output
- Can express local transformations
- Can not express reverse, swap, or regular look-ahead
- Non-deterministic variants can express regular look-ahead
2-Way Deterministic Finite State Transducers

Example: \( u \mapsto \text{if } u \text{ contains a } \# \text{ then } \overline{u} \text{ else } u \)

- Extend two-way finite automata with output
- Allowing transitions based on regular look-ahead do not increase expressiveness (Chytil and Jakl [1977])
- Two-way finite-state transducers capture the same class of MSO-definable transformations (Engelfriet and Hoogeboom [2001])
2-Way Deterministic Finite State Transducers

Example: \( u \mapsto \) if \( u \) contains \# then \( \overline{u} \) else \( u \)

```
1  a  b  b  #  b  a  a  1
\uparrow
head

a, b | \epsilon, R

\downarrow

# | \epsilon, R

\begin{align*}
\alpha | \epsilon, R & \Rightarrow 2 \\
\alpha | \alpha, L & \Rightarrow 3
\end{align*}

\begin{align*}
\alpha | \epsilon, L & \Rightarrow 4 \\
\alpha | \epsilon, L & \Rightarrow 5
\end{align*}

\begin{align*}
\alpha | \epsilon, R & \Rightarrow 6
\end{align*}

\begin{align*}
\alpha | \epsilon, L & \Rightarrow 6
\end{align*}

\begin{align*}
\alpha | \alpha, R & \Rightarrow 6
\end{align*}

\downarrow

head

\downarrow

head
```
Example: $u \mapsto \text{if } u \text{ contains } \# \text{ then } \overline{u} \text{ else } u$

\[ \begin{array}{cccccccc}
\bot & a & b & b & \# & b & a & a & \bot \\
\text{head} \\
\end{array} \]

\[ a, b \mid \epsilon, R \]

\[ \alpha \mid \epsilon, R \]

\[ \alpha \mid \alpha, L \]

\[ \begin{array}{cccccccc}
1 & 2 & 4 & 3 & 5 & 6 \\
\text{start} \\
\end{array} \]

\[ \begin{array}{cccccccc}
\bot & \# & \bot & \bot & \bot & \bot & \bot & \bot \\
\text{head} \\
\end{array} \]
Example: \( u \mapsto \) if \( u \) contains \( \# \) then \( \overline{u} \) else \( u \)

\[
\begin{array}{ccccccccc}
\vdash & a & b & b & \# & b & a & a & \vdash \\
\end{array}
\]

\[
\begin{array}{ccccccccc}
\vdash & a & b & b & \# & b & a & a & \vdash \\
\end{array}
\]

\[
\begin{array}{ccccccccc}
\vdash & a & b & b & \# & b & a & a & \vdash \\
\end{array}
\]

\[
\begin{array}{ccccccccc}
\vdash & a & b & b & \# & b & a & a & \vdash \\
\end{array}
\]

\[
\begin{array}{ccccccccc}
\vdash & a & b & b & \# & b & a & a & \vdash \\
\end{array}
\]

\[
\begin{array}{ccccccccc}
\vdash & a & b & b & \# & b & a & a & \vdash \\
\end{array}
\]

\[
\begin{array}{ccccccccc}
\vdash & a & b & b & \# & b & a & a & \vdash \\
\end{array}
\]
2-Way Deterministic Finite State Transducers

Example: $u \mapsto$ if $u$ contains $\#$ then $\bar{u}$ else $u$
2-Way Deterministic Finite State Transducers

Example: \( u \mapsto \) if \( u \) contains \( \# \) then \( \overline{u} \) else \( u \)

\[\begin{array}{cccccccc}
\top & a & b & b & \# & b & a & a & \bot \\
1 & 2 & 4 & 5 & 3 & 6 & 1 & 2 & 3 & 4 & 5 & 6 \end{array}\]

\[\begin{array}{ll}
a, b | \epsilon, R & \alpha | \epsilon, R \\
\# | \epsilon, R & - | \epsilon, L \\
- | \epsilon, L & \alpha | \epsilon, L \\
\alpha | \epsilon, R & \alpha | \alpha, L \\
\alpha | \alpha, L & \alpha | \alpha, R \\
\end{array}\]
2-Way Deterministic Finite State Transducers

Example: $u \mapsto$ if $u$ contains $\#$ then $\overline{u}$ else $u$

![Diagram of a 2-way deterministic finite state transducer](image-url)
Example: $u \mapsto$ if $u$ contains $\#$ then $\overline{u}$ else $u$

![Diagram of a 2-Way Deterministic Finite State Transducer](image-url)
Example: $u \mapsto \text{if } u \text{ contains } \# \text{ then } \overline{u} \text{ else } u$

\[
\begin{array}{cccccccc}
\vdash & a & b & b & \# & b & a & a & \vdash \\
\end{array}
\]
Example: \( u \mapsto \) if \( u \) contains \( \# \) then \( \overline{u} \) else \( u \)

![Diagram of a 2-Way Deterministic Finite State Transducer]

- States: 1, 2, 3, 4, 5, 6
- Transitions:
  - \( a, b | \epsilon, R \)
  - \( \# | \epsilon, R \)
  - \( \overline{\epsilon} | \epsilon, L \)
  - \( \alpha | \epsilon, R \)
  - \( \alpha | \epsilon, L \)
  - \( \alpha | \alpha, L \)
  - \( \alpha | \alpha, R \)
  - \( \overline{\epsilon}, L \)
  - \( \overline{\epsilon}, R \)

- Initial state: 1
- Final states: 6 and 5
- Head symbols:
  - Head a
  - Head b

- Alphabet: \( a, b, \# \)
- Empty string: \( \epsilon \)
2-Way Deterministic Finite State Transducers

Example: \( u \mapsto \) if \( u \) contains \# then \( \overline{u} \) else \( u \)

\[
\begin{array}{cccccccc}
\bot & a & b & b & \# & b & a & a & \bot \\
\end{array}
\]

\[
\text{head}
\]

\[
\begin{array}{cccccccc}
a, b | \epsilon, R \\
\end{array}
\]

\[
\begin{array}{cccccccc}
\alpha | \epsilon, R \\
\end{array}
\]

\[
\begin{array}{cccccccc}
\alpha | \alpha, L \\
\end{array}
\]

\[
\begin{array}{cccccccc}
\text{start} & \rightarrow & 1 & 2 & 3 & 4 & 5 & 6 \\
\end{array}
\]

\[
\begin{array}{cccccccc}
a, a \\
\text{head}
\end{array}
\]

\[
\begin{array}{cccccccc}
a | \epsilon, R \\
\end{array}
\]

\[
\begin{array}{cccccccc}
a | \alpha, L \\
\end{array}
\]

\[
\begin{array}{cccccccc}
\alpha | \epsilon, L \\
\end{array}
\]

\[
\begin{array}{cccccccc}
\alpha | \alpha, R \\
\end{array}
\]

\[
\begin{array}{cccccccc}
\rightarrow | \epsilon, L \\
\end{array}
\]

\[
\begin{array}{cccccccc}
\rightarrow | \epsilon, R \\
\end{array}
\]

\[
\begin{array}{cccccccc}
\rightarrow | \epsilon, L \\
\end{array}
\]
2-Way Deterministic Finite State Transducers

Example: \( u \mapsto \text{if } u \text{ contains } \# \text{ then } \overline{u} \text{ else } u \)

![Diagram of 2-Way Deterministic Finite State Transducer]

\( a, b | \epsilon, R \)

\( \alpha | \epsilon, R \)

\( \alpha | \alpha, L \)

\( \alpha | \alpha, R \)

\( \alpha | \epsilon, L \)

\( \alpha | \epsilon, R \)

\( \epsilon, L \)

\( \epsilon, R \)

\( \overline{\epsilon}, L \)

\( \overline{\epsilon}, R \)
Example: \( u \mapsto \text{if } u \text{ contains } \# \text{ then } \overline{u} \text{ else } u \)
Example: $u \mapsto \text{if } u \text{ contains } \# \text{ then } \overline{u} \text{ else } u$

![Diagram of a 2-Way Deterministic Finite State Transducer](image-url)
Example: $u \mapsto$ if $u$ contains # then $\overline{u}$ else $u$

![Diagram](image.png)
2-Way Deterministic Finite State Transducers

Example: $u \mapsto$ if $u$ contains $\#$ then $\overline{u}$ else $u$

![Diagram of 2-Way Deterministic Finite State Transducer]

- Start state: 1
- Transition rules:
  - $a, b \mid \epsilon, R$
  - $\# \mid \epsilon, R$
  - $\mid \epsilon, L$
  - $\mid \epsilon, R$
  - $\alpha \mid \epsilon, L$
  - $\alpha \mid \alpha, L$
  - $\alpha \mid \alpha, R$

States:
1. Start state
2. State with $a$, $b$, $\#$
3. State with $b$
4. State with $a$
5. State with $a$, $b$, $\#$
6. Final state

Head positions:
- Head at state 1
- Head at state 6
Example: \( u \mapsto \) if \( u \) contains \# then \( \overline{u} \) else \( u \)

\[a, b \mid \epsilon, R\]  

\[\alpha \mid \epsilon, R\]  

\[\# \mid \epsilon, R\]  

\[-\mid \epsilon, L\]  

\[\alpha \mid \alpha, L\]  

\[\alpha \mid \alpha, R\]  

\[\alpha \mid \epsilon, R\]  

\[\alpha \mid \epsilon, L\]  

\[\alpha \mid \epsilon, R\]  

\[\alpha \mid \epsilon, L\]  

\[\alpha \mid \alpha, R\]  

\[\alpha \mid \alpha, L\]  

\[\alpha \mid \epsilon, R\]  

\[\alpha \mid \epsilon, L\]  

\[\alpha \mid \alpha, R\]  

\[\alpha \mid \alpha, L\]  

\[\alpha \mid \epsilon, R\]  

\[\alpha \mid \epsilon, L\]  

\[\alpha \mid \alpha, R\]  

\[\alpha \mid \alpha, L\]  

\[\alpha \mid \epsilon, R\]  

\[\alpha \mid \epsilon, L\]  

\[\alpha \mid \alpha, R\]  

\[\alpha \mid \alpha, L\]
2-Way Deterministic Finite State Transducers

Example: \( u \mapsto \) if \( u \) contains a \# then \( \overline{u} \) else \( u \)

\[\begin{align*}
\alpha | \varepsilon, R \\
\# | \varepsilon, R \\
\leftarrow | \varepsilon, L \\
\alpha | \varepsilon, L \\
\alpha | \alpha, L \\
\alpha | \alpha, R \\
\leftarrow \\
\leftarrow \\
\leftarrow \\
\leftarrow
\end{align*}\]

\( \dagger \) Here \( \alpha \) stands for any symbol except end markers.

- Extend two-way finite automata with output
- Allowing transitions based on regular look-ahead do not increase expressiveness (Chytil and Jakl [1977])
- Two-way finite-state transducers capture the same class of MSO-definable transformations (Engelfriet and Hoogeboom [2001])
### Transducers: Streaming String Transducers

**Example:** \( u \mapsto \) if \( u \) contains a \( \# \) then \( \overline{u} \) else \( u \)

\[
\begin{align*}
\alpha \mid (x, y) & := (\alpha x, y\alpha) \\
\alpha \mid (x, y) & := (\alpha x, \varepsilon)
\end{align*}
\]

\[
\begin{align*}
\alpha \mid (x, y) & := (\alpha x, \varepsilon) \\
\# \mid (x, y) & := (\# x, \varepsilon)
\end{align*}
\]

†Here \( \alpha \) stands for any symbol except end markers.

- Extend deterministic finite-state automata with **string variables**
- String variables are updated in a **copyless** fashion
- Output is given as a function of states to **copyless concatenation** of string variables
Theorem ([Alur and Černý, 2011])

A transformation of finite strings is accepted by a streaming string transducer iff it is MSO-definable.
Properties of Regular Transformations

- Characterized by
  - MSO,
  - (deterministic) two-way finite-state transducers, and
  - (deterministic) streaming string transducers.

- They are closed under **sequential composition**

- **Equivalence problem**, deciding the equivalence of two regular transformations, is decidable.

- **Type checking problem**, deciding whether image of a given regular set $I$ under a regular transformation $T$ is contained in another given regular set $O$ i.e. $T(I) \subseteq O$, is decidable.

- Both problems are in **PSPACE** for streaming-string transducers [Alur and Černý, 2011]
Regular Transformations of Finite Strings

Regular Transformations of Infinite Strings

Conclusion
Transformations of Infinite Strings

- A transformation from $\Sigma$ to $\Gamma$ is a (partial) function $f : \Sigma^\omega \rightarrow \Gamma^\omega$.
- Generalizes the concept of an $\omega$-language $f : \Sigma^\omega \rightarrow \{0, 1\}$.
- Example:
  - $a^n \#^\omega \mapsto a^n b^n \#^\omega$
  - $a^n b^\omega \mapsto a^{2^n-1} b^\omega$
- local transformations, e.g., delete each $a$, repeat every $b$
- reverse transformation, i.e. $a_1 a_2 \ldots a_n \# u \mapsto a_n a_{n-1} \ldots a_1 \# u$,
- swapping transformation, e.g. $\alpha \# \beta \# u \mapsto \beta \# \alpha \# u$,
- look-ahead based transformations,
  - replace each $a$ with $b$ if the string contains a $\#$.
  - replace each $a$ with $b$ if the string contains a prime number of $\#$. 
Transformations of Infinite Strings

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- MSO on infinite strings can be used to define transformations on infinite strings [Courcelle, 1994]
Transformations of Infinite Strings

- A **transformation** from $\Sigma$ to $\Gamma$ is a (partial) function $f : \Sigma^\omega \rightarrow \Gamma^\omega$.
- Generalizes the concept of an $\omega$-language $f : \Sigma^\omega \rightarrow \{0, 1\}$.
- Example:
  - $a^n \#^\omega \mapsto a^n b^n \#^\omega$
  - $a^n b^\omega \mapsto a^{2^n - 1} b^\omega$
  - **local** transformations, e.g., delete each $a$, repeat every $b$
  - **reverse** transformation, i.e. $a_1 a_2 \ldots a_n \#^u \mapsto a_n a_{n-1} \ldots a_1 \#^u$,
  - **swapping** transformation, e.g. $\alpha \# \beta \#^u \mapsto \beta \# \alpha \#^u$,
  - **look-ahead** based transformations,
    - replace each $a$ with $b$ if the string contains a $\#$.
    - replace each $a$ with $b$ if the string contains a prime number of $\#$.

- MSO on infinite strings can be used to define transformations on infinite strings [Courcelle, 1994]
- What classes of **finite-state transducers** have equal expressive power?
- What **decision problems** about MSO-definable transformations of infinite strings can be solved?
### Definition (Defining Transformation using MSO)

A transformation using MSO is specified by:

- **input** and **output** alphabets;
- an MSO formula specifying the **domain** of the transformation;
- output string is specified using a **finite number of copies** of nodes of input string graph;
- the **node labels** are specified using MSO formulas; and
- the **existence of edges** between nodes of various copies is specified using MSO formulas.

**Example**

Let $\Sigma = \{a, b, \#\}$. Consider a transformation $f: \Sigma^\omega \rightarrow \Sigma^\omega$ where $u$ is the reverse of $u$ and $v \in \{a, b\}$.

$$u_1 \# u_2 \# \ldots \# u_{n-1} \# u_n \mapsto u_1 u_1 \# \ldots \# u_n u_n \# v.$$
**MSO-definable Transformations**

**Definition (Defining Transformation using MSO)**

A transformation using MSO is specified by:

- **input** and **output** alphabets;
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- the **existence of edges** between nodes of various copies is specified using MSO formulas

**Example**

Let $\Sigma = \{ a, b, \# \}$. Consider a transformation $f_2 : \Sigma^\omega \rightarrow \Sigma^\omega$

$$u_1 \# u_2 \# \ldots u_{n-1} \# u_n \# v \mapsto \overline{u_1} u_1 \# \ldots \# \overline{u_n} u_n \# v.$$ 

where $\overline{u}$ is **reverse** of $u$ and $v \in \{ a, b \}^\omega$.
MSO-definable Transformations

- \( \Sigma = \Gamma = \{a, b, \#\} \), \( C = \{1, 2\} \), and
- Node Label Formulas
  - \( \text{Label}^{c_1}_{\alpha}(x) = \text{Label}^{\text{inp}}_{\alpha}(x) \land \neg \text{Label}^{\#}_{\text{inp}}(x) \land \text{reach}_{\#}(x) \)
  - \( \text{Label}^{c_2}_{\alpha}(x) = \text{Label}^{\text{inp}}_{\alpha}(x) \)
- Edge Label Formulas
  - \( \text{Edge}^{c_1,c_1}_{c_1}(x, y) = \text{Edge}^{\text{inp}}_{c_1}(y, x) \land \text{Label}^{\text{inp}}_{\alpha}(x) \land \text{Label}^{\text{inp}}_{\alpha}(y) \)
  - \( \text{Edge}^{c_2,c_2}_{c_2}(x, y) = \text{Edge}^{\text{inp}}_{c_2}(x, y) \land (\neg \text{Label}^{\#}_{\text{inp}}(x) \lor (\text{Label}^{\#}_{\text{inp}}(x) \land \neg \text{reach}_{\#}(x))) \)
  - \( \text{Edge}^{1,2}_{c_1}(x, y) = (x=y) \land (\text{first}(x) \lor \exists z (\text{Label}^{\#}_{\text{inp}}(z) \land \text{Edge}^{\text{inp}}_{c_1}(z, x))) \)
  - \( \text{Edge}^{2,1}_{c_1}(x, y) = \text{Label}^{\text{inp}}_{\#}(x) \land \text{reach}_{\#}(x) \land (\exists z (\text{Edge}^{\text{inp}}_{\alpha}(y, z) \land \text{Label}^{\#}_{\text{inp}}(z)) \land (\forall z ((\text{path}(x, z) \land \text{path}(z, y)) \rightarrow \neg \text{Label}^{\#}_{\text{inp}}(z))) \)

\[ a, b, b, #, b, a, #, \ \omega \]

\[ a, b, b, b, b, b, b, b, a, #, a, #, \ \omega \]

\[ a, b, b, b, b, b, b, b, a, #, a, #, \ \omega \]
MSO-definable Transformations

input:

\[ a \quad b \quad b \quad b \quad \# \quad b \quad a \quad \# \quad a^\omega \]

- \( \Sigma = \Gamma = \{a, b, \#\} \), \( C = \{1, 2\} \), and
- Node Label Formulas
  - \( \text{Label}_{c1}^\alpha(x) = \text{Label}_{\alpha}^{\text{inp}}(x) \land \neg \text{Label}_{\#}^{\text{inp}}(x) \land \text{reach}_{\#}(x) \)
  - \( \text{Label}_{c2}^\alpha(x) = \text{Label}_{\alpha}^{\text{inp}}(x) \)
- Edge Label Formulas
  - \( \text{Edge}_{c1,c1}^{c1}(x, y) = \text{Edge}_{\text{inp}}^{\text{c1}}(y, x) \land \text{Label}_{\#}^{\text{inp}}(x) \land \text{Label}_{\#}^{\text{inp}}(y) \).
  - \( \text{Edge}_{c2,c2}^{c2}(x, y) = \)
    \[
    \text{Edge}_{\text{inp}}^{\text{c2}}(x, y) \land (\neg \text{Label}_{\#}^{\text{inp}}(x) \lor (\text{Label}_{\#}^{\text{inp}}(x) \land \neg \text{reach}_{\#}(x))).
    \]
  - \( \text{Edge}^{1,2}(x, y) = (x=y) \land (\text{first}(x) \lor \exists z (\text{Label}_{\#}^{\text{inp}}(z) \land \text{Edge}_{\text{inp}}^{\text{c2}}(z, x))) \)
  - \( \text{Edge}^{2,1}(x, y) = \text{Label}_{\#}^{\text{inp}}(x) \land \text{reach}_{\#}(x) \land (\exists z (\text{Edge}_{\text{inp}}^{\text{c2}}(y, z) \land \text{Label}_{\#}^{\text{inp}}(z))) \land (\forall z ((\text{path}(x, z) \land \text{path}(z, y)) \rightarrow \neg \text{Label}_{\#}^{\text{inp}}(z))) \)
MSO-definable Transformations

- $a^n \#^\omega \mapsto a^n b^n \#^\omega \checkmark$
- $a^n b^\omega \mapsto a^{2^n-1} b^\omega$
- local transformations, e.g., delete each $a$, repeat every $b \checkmark$
- reverse transformation, i.e. $a_1 a_2 \ldots a_n \# u \mapsto a_n a_{n-1} \ldots a_1 \# u$, $\checkmark$
- swapping transformation, e.g. $\alpha \# \beta \# u \mapsto \beta \# \alpha \# u$, $\checkmark$
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  - replace each $a$ with $b$ if the string contains a $\#$ $\checkmark$
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- **look-ahead** based transformations, e.g.
  - replace each $a$ with $b$ if the string contains a $\#$ $\checkmark$
  - replace each $a$ with $b$ if the string contains a **prime number** of $\#$

Regular Transformations on Infinite Strings

Which transducers accept the same class of transformations?
Deterministic Generalized Sequential Machines

Example: For all strings containing a #, replace all a with b.

start \[\rightarrow\] 1
\[\alpha \mid \alpha\]

\[\rightarrow\] 2
\[\alpha \mid \alpha\]

\[\# \mid \#\]

† Here \(\alpha\) stands for any symbol other than a.

- Extend Muller automata with output
- Can express local transformations
- Can not express transformations such as reverse or swap
Example: Reverse the sub-string before the first #

- Extend two-way Muller automata with output
- Allowing ω-regular look-ahead increases expressiveness
- Two-way finite-state transducers with ω-regular look-ahead capture the same class of transformations as MSO.
Example: Reverse the sub-string before the first \# \\
\[ \alpha \mid (x, y) := (\alpha x, y\alpha) \]

\[ \beta \mid (x, y) := (x\beta, \varepsilon) \]

\[ \# \mid (x, y) := (#x, \varepsilon) \]

† Here \( \alpha \) is any symbol except \#, while \( \beta \) is any symbol.

- Extend Muller automata with string variables
- String variables are updated in a copyless fashion
- Output is given as a function of set of states to copyless concatenation of string variables
- We enforce syntactic restrictions that ascertain that output string is always an infinite string
Expressiveness of Streaming String Transducers

MSOT

EH01

Two-Way Transducers with Look-ahead

Shepherdson

Functional NSSTs with Look-ahead

Miyano and Hayashi’ 84

SST

SSTs with Bounded Copy

Functional NSSTs

[Alur and Černý, 2011]
Proof Sketch

**Theorem**

A transformation of infinite strings is accepted by a streaming string transducer iff it is MSO-definable.

- simulate all look-aheads in parallel
- look-ahead $\sim$ universal transitions in an alternating Muller automaton
- use Miyano-Hayashi like construction to remove universality
**Proof Sketch**

**Theorem**

A transformation of infinite strings is accepted by a streaming string transducer iff it is MSO-definable.

- simulate all runs in parallel
- functionality \(\Rightarrow\) at most \(|Q|\) runs have to be simulated in parallel
- use \(|Q|\) copies of each variable \(x \in X\)
- may introduce variable copy
Proof Sketch

Theorem

A transformation of infinite strings is accepted by a streaming string transducer iff it is MSO-definable.

- most technical result
- based on the notion of dependency graphs
- states are sufficient abstractions of dependency graphs
Proof Sketch

**Theorem**

A transformation of infinite strings is accepted by a streaming string transducer iff it is MSO-definable.

- simple extension of the finite string case
- uses two domain copies for each variable
Equivalence Problem

Theorem

*Equivalence problem is decidable in PSPACE for streaming-string transducers on infinite strings.*
**Equivalence Problem**

**Theorem**

*Equivalence problem is decidable in PSPACE for streaming-string transducers on infinite strings.*

$T_1$ and $T_2$ are inequivalent iff $\text{dom}(T_1) \neq \text{dom}(T_2)$ or

$\text{dom}(T_1) = \text{dom}(T_2)$ and $\exists u \in \text{dom}(T_1), \exists i \geq 0$ such that $T_1(u)[i] \neq T_2(u)[i]$  

1. **Domain equivalence** can be checked in PSPACE.
2. if domains are equivalent, then check existence of $u$
   - reduction to emptiness of reversal-bounded counter machines (NLogSpace, Ibarra)
   - **Product construction** to simulate runs of $T_1$ and $T_2$ on the same inputs
   - guess a position $i$ and check that there is a mismatch
   - as outputs are not produced synchronously, counters are used to retrieve the letters at position $i$ in both outputs
   - construction ensures that finite runs can be extended to infinite accepting runs that do not modify the letters at position $i$
Type-Checking Problem

Theorem

Type-checking, deciding whether image of a given regular set \( I \) under a regular transformation \( T \) is contained in another given regular set \( O \) i.e. \( T(I) \subseteq O \), is decidable in \( PSPACE \) for streaming-string transducers on infinite strings.

- Check whether \( T \) is defined for all strings of \( u \), i.e. \( \text{dom}(T) \subseteq I \).
- A Muller automaton recognizing the domain of \( T \) can be constructed in linear time, and therefore \( I \subseteq \text{dom}(T) \) can be checked in \( PSPACE \).
Type-Checking Problem

Theorem

*Type-checking*, deciding whether image of a given regular set $I$ under a regular transformation $T$ is contained in another given regular set $O$ i.e. $T(I) \subseteq O$, is decidable in PSPACE for streaming-string transducers on infinite strings.

- Check whether $T$ is defined for all strings of $u$, i.e. $\text{dom}(T) \subseteq I$.
- A Muller automaton recognizing the domain of $T$ can be constructed in linear time, and therefore $I \subseteq \text{dom}(T)$ can be checked in PSPACE.
- Next we check the language $L = \{u \in \Sigma^\omega \mid u \in I, T(u) \notin O\}$ for emptiness.
- The language $L$ can be defined by a Muller automaton $A_L$ that simulates automaton $A_I$ and $T$ on the input string, and $A_O$ on the output of $T$. 
Theorem

*Type-checking*, deciding whether image of a given regular set \( I \) under a regular transformation \( T \) is contained in another given regular set \( O \) i.e. \( T(I) \subseteq O \), is decidable in \( \text{PSPACE} \) for streaming-string transducers on infinite strings.

- Check whether \( T \) is defined for all strings of \( u \), i.e. \( \text{dom}(T) \subseteq I \).
- A Muller automaton recognizing the domain of \( T \) can be constructed in linear time, and therefore \( I \subseteq \text{dom}(T) \) can be checked in \( \text{PSPACE} \).
- Next we check the language \( L = \{ u \in \Sigma^\omega \mid u \in I, T(u) \notin O \} \) for emptiness.
- The language \( L \) can be defined by a Muller automaton \( A_L \) that simulates automaton \( A_I \) and \( T \) on the input string, and \( A_O \) on the output of \( T \).
- This can be done by computing functions \( \tau \) such that for all states \( q \) of \( A_O \) and all variables \( x \in X \), \( \tau(q, x) \) is the state of \( A_O \) after evaluating the current value of \( x \), starting from state \( q \).
- The size of \( A_L \) is exponential in \( A_I, A_O \) and \( T \), and its emptiness can be decided in \( \text{PSPACE} \).
Properties of Regular Transformations

- Characterized by
  - MSO,
  - two-way finite-state transducers with ω-regular look-ahead, and
  - streaming string transducers
- They are closed under sequential composition
Properties of Regular Transformations

- Characterized by
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**Theorem**

*Equivalence and type-checking problems are decidable in $\text{PSPACE}$ for streaming-string transducers on infinite strings.*
Properties of Regular Transformations

- Characterized by
  - MSO,
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- They are closed under sequential composition

Theorem

*Equivalence and type-checking problems are decidable in PSPACE for streaming-string transducers on infinite strings.*

Corollary

*Equivalence of MSO-transducers on infinite strings is decidable.*
Regular Transformations of Finite Strings

Regular Transformations of Infinite Strings

Conclusion
Summary

- Introduction of **streaming string transducers** renewed the interest in the study of **regular transformations**
- Streaming string transducers naturally extend from strings to more general structures, while conserving MSO equivalence.
- Streaming-string transducer models are **robust**: closed under bounded copy, functional nondeterminism, and regular look-ahead.
- Important verification problems like **functional equivalence and pre/post condition type-checking** are decidable for streaming string transducers.
Introduction of **streaming string transducers** renewed the interest in the study of **regular transformations**.

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A number of open problems!
Summary

- Introduction of streaming string transducers renewed the interest in the study of regular transformations.
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- A number of open problems!

Thank You!


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