Discovering Relational Specifications

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What are we interested in?

Formal specifications of library functions

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Formal specifications of library functions

- Problems:
  - code unavailable
  - large code
- partial behavior of these functions
- discover a rich class of specifications

Problem

- Given a function f and a data set D, a partial picture of i/o behavior of f, perhaps collected through some random testing

What can we learn about the function f by simply analyzing the dataset D?

Example 1

f		
i_1	i_2	**r**
1	2	3
3	4	7
5	6	11
4	3	7
⋮	⋮	⋮

f is commutative

Specification

Example 2

f

i_1	**r**
1	1
7	7
-10	10

$$f(x) = |x|$$

Specification

Example 2

f

i_1	**r**
1	1
7	7
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$$f(x) = x$$

Specification

D-restricted assignment (σ_D)

f

i_1	**r**
1	1
7	7
-10	10

$$f(x) = x$$

Specification

assign each variable of specification to a constant that appears in the dataset

- $\sigma_D = \{x \to 1\}$ is a D-restricted assignment to $f(x) = x$
- but $\sigma_D = \{x \to 2\}$ is not a D-restricted assignment because f is not defined for 2 in the given dataset

Example 2

f

i_1	**r**
1	1
7	7
-10	10

$$f(x) = x$$

Specification

positive evidence

- D-restricted assignments that satisfies the specification
- positive evidence is $\{x \to 1, x \to 7\}$

Example 2

f

i_1	**r**
1	1
7	7
-10	10

$$f(x) = x$$

Specification

negative evidence

- D-restricted assignments that does not satisfy the specification
- negative evidence is $\{x \rightarrow -10\}$

What does it mean for a specification to explain a data-set?

- if there exists a *negative evidence* - the specification is considered inconsistent with the data
- otherwise the specification is considered *more likely* to be true depending on a measure of the *positive evidence* that is available for it

Want to learn specifications

commutativity

$$f(x, y) = z \Leftrightarrow f(y, x) = z$$

transitivity

$$g(x, y) = t \wedge g(y, z) = t \Rightarrow g(x, z) = t$$

`sin` is periodic by 2π

$$\exists k.x = 2\pi k + y \Rightarrow sin(x) = z \Leftrightarrow sin(y) = z$$

rotating a shape by a multiple of 2π does not change the shape

$$\exists k.x = 2\pi k \Rightarrow rotate(y, x) = y$$

Example 3

f		
i_1	i_2	**r**
1	2	3
3	4	7
5	6	11
4	3	7
⋮	⋮	⋮

$$f(x, y) = z \Leftrightarrow f(y, x) = z$$

Specification

Example 3

f

i_1	i_2	**r**
1	2	3
3	4	7
5	6	11
4	3	7
⋮	⋮	⋮

$$f(x, y) = z \Leftrightarrow f(y, x) = z$$

Specification

positive and negative evidence

- positive evidence is $\{\{x \to 3, y \to 4\}, \{x \to 4, y \to 3\}\}$
- no negative evidence

Example 4

concat

i_1	i_2	**r**
a	b	ab
a	ϵ	a
ϵ	a	a
b	ϵ	b
⋮	⋮	⋮

len

i_1	**r**
a	1
ϵ	0
b	1
ab	2
⋮	⋮

Specification: $len(concat(x, y)) = z \Leftrightarrow len(x) = z$

Example 4

concat

i_1	i_2	**r**
a	b	ab
a	ϵ	a
ϵ	a	a
b	ϵ	b
⋮	⋮	⋮

len

i_1	**r**
a	1
ϵ	0
b	1
ab	2
⋮	⋮

Specification: $len(concat(x, y)) = z \Leftrightarrow len(x) = z$

positive and negative evidence

- positive evidence is $\{\{x \to a, y \to \epsilon\}, \{x \to b, y \to \epsilon\}\}$
- negative evidence is $\{\{x \to a, y \to b\}, \{x \to \epsilon, y \to a\}\}$

Example 4

add **constraint** to *weaken* the specification by finding a formula G s.t.

- for all negative evidences, G is unsat.
- for some positive evidences, G is sat.
- $G \Rightarrow \mathit{len}(\mathit{concat}(x, y) = z) \Leftrightarrow \mathit{len}(x) = z$ has some positive evidences and has no negative evidence.

$$y = \epsilon \Rightarrow \mathit{len}(\mathit{concat}(x, y)) = z \Leftrightarrow \mathit{len}(x) = z$$

Bach

A technique for discovering *likely specifications* from data generated for a number of standard libraries.

Bach

A techn*ique for discovering likely specifications from data generated* for a number

Discovers rich array of specifications

by combining novel insights of program synthesis and databases.

Specification

Consider specification as a formula over an interpreted theory

Specification ($\mathcal{F}$):

$$\forall V.G \Rightarrow (\Psi \Leftrightarrow \Phi) \text{ or } \forall V.G \Rightarrow (\Psi \Rightarrow \Phi),$$

where $\Psi = \wedge_i \psi_i$ and $\Phi = \wedge_j \phi_j$

- V : set of variables
- G : a formula over interpreted set of predicates and function symbols
- each ψ_i is an atom of the form $t = o$ (analogously, ϕ_i)
- t is a nested function application over $V, \sum$
- $\sum$ is a finite set of uninterpreted functions $\{f_1, ..., f_n\}$
- $o \in V$

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E.g. $\forall x, y.x > 0 \Rightarrow (f(g(x)) = y \Leftrightarrow g(f(x)) = y)$

Searching of specifications: Specification Induction

- iteratively constructs specifications by traversing set of programs and connections between them
- in order from smallest to largest based on a set of rules

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Enumerative synthesis

Specification preference

- Given Ψ, Φ
  - learn $\Psi \Leftrightarrow \Phi$
  - if fail, learn either $\Psi \Rightarrow \Phi$ or $\Phi \Rightarrow \Psi$
  - If no implication can be learned, Bach resorts to abduction

Guard abduction

Bach solves a number of abduction problems to learn guard

- $G \Rightarrow (\Psi \Leftrightarrow \Phi)$, $G \Rightarrow (\Psi \Rightarrow \Phi)$, $G \Rightarrow (\Phi \Rightarrow \Psi)$
  - Each provided predicate is instantiated with every combination of variables
  - **E.g.** if $a > b$ is provided and $vars(\mathcal{F}) = \{x, y\}$, abduction will use $x > y$ and $y > x$

Specification Preference: Example

h_1	
i_1	**r**
1	true
2	false
3	true
⋮	⋮

h_2	
i_1	**r**
1	true
2	true
3	true
⋮	⋮

- $\Psi : h_1(x) = p, \Phi : h_2(x) = p$, where $p = \{true, false\}$

Specification: $h_1(x) = p \Leftrightarrow h_2(x) = p$

$(\Rightarrow)$

$$h_1(x) = p \Rightarrow h_2(x) = p$$

$(\Rightarrow)$

$$h_1(x) = p \Rightarrow h_2(x) = p$$

Negative evidence

$$\{x = 2, p = \mathit{false}\}$$

$(\Leftarrow)$

$$h_2(x) = p \Rightarrow h_1(x) = p$$

$(\Leftarrow)$

$$h_2(x) = p \Rightarrow h_1(x) = p$$

Negative evidence

$$\{x = 2, p = \mathit{true}\}$$

Guard abduction

- $G \Rightarrow (h_1(x) = p \Leftrightarrow h_2(x) = p)$
- $G \Rightarrow (h_1(x) = p \Rightarrow h_2(x) = p)$
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Guard abduction

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Learned specification

$$p = \mathit{true} \Rightarrow (h_1(x) = p \Rightarrow h_2(x) = p)$$

Specification Consistency Verification

How to efficiently verify the consistency of the specification with the dataset?

How to efficiently verify the consistency?

- model positive and negative evidence of a formula $\mathcal{F}$ and data-set D as a union of conjunctive query (UCQ).
- the evaluation should return the positive and negative evidence
- formulation as a database query evaluation allow us to leverage efficient, highly engineered database engines and Datalog server
- query is typically small and data is large

Encoding Specifications

Specification($\mathcal{F}$):

$$\forall \bar{\mathbf{x}}.\Psi \Leftrightarrow \Phi,$$

where $\Psi = \wedge_i \psi_i$ and $\Phi = \wedge_j \phi_j$

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$$\forall \bar{\mathbf{x}}.\Psi \Leftrightarrow \Phi,$$

where $\Psi = \wedge_i \psi_i$ and $\Phi = \wedge_j \phi_j$

- for each var $x \in \bar{\mathbf{x}}$ in formula $\mathcal{F}$, create a Datalog variable $X_x \in \bar{\mathbf{X}}$, i.e.
  - $x \in \bar{\mathbf{x}} \Rightarrow X_x \in \bar{\mathbf{X}}$
- for each n-ary function f, create $(n+1)$-ary relation R_f

Encoding Specifications: Example

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Specification

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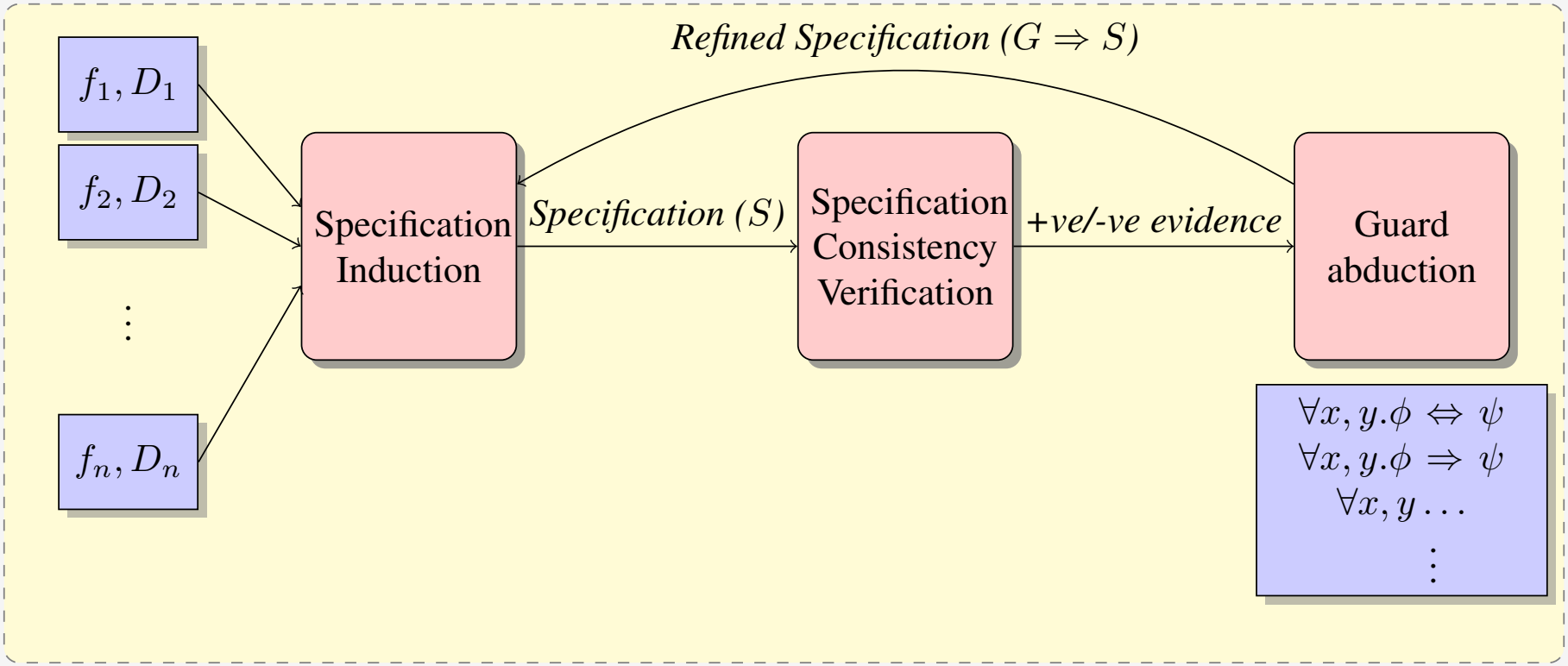
$$f(x, y) = z \Leftrightarrow f(y, x) = z$$

Specification

positive and negative evidence

- positive evidence $P(X) \leftarrow R_f(X_x, X_y, O), R_f(X_y, X_x, O'), O = O'$
- negative evidence $N(X) \leftarrow R_f(X_x, X_y, O), R_f(X_y, X_x, O'), O \neq O'$

Components of Bach



Implementation

- Implemented in OCaml.
- **Input:**
  - a signature of simply typed functions
  - i/o data for each function
  - a set of predicates to compute guards
- Uses Souffle Datalog engine to compute +ve and -ve evidence

Exploratory Evaluation

Targeted 9 set of python libraries. Each benchmark consists of

- a finite set of signature
- a set of predicates
- a data-set of 1000 randomly samples executions for each function

Benchmark	Description
`list` (7)	standard list operations, including `hd`, `tl`, `cons`, etc.
`matrix` (7)	matrix operations from Python's `sympy` library.
`trig` (4)	trig. functions (`sin`, `cos`, etc.) in Python's `math` module.
`z3` (5)	API to Python's `z3` library, including `sat` and `valid`.
`geometry` (5)	manipulations of shapes from Python's `sympy` library.
`sets` (10)	functions from Python's `set` module.
`dict` (5)	functions from Python's `dict` (dictionary) module.
`fp199` (4)	arithmetic on $\mathbb{F}_{199}$, the finite field of order 199.
`strings` (9)	string operations from Python's `string` module.

Figure: List of benchmarks; number of functions is in parentheses

z3 specifications

The z3 benchmark contains functions from a subset of Python's z3 API.

- Learned specification for z3
  - $p = true \Rightarrow (valid(x) = p \Rightarrow sat(x) = p)$
  - $and(x, y) = z \Leftrightarrow and(y, x) = z$
  - $valid(x) = p \wedge valid(y) = p \Rightarrow valid(and(x, y)) = p$

strings specifications

The strings benchmark contains the typical set of funtions for manipulating strings.

- Learned specification for strings
  - $lstrip(x) = y \Rightarrow lstrip(y) = y$
  - $p = true \Rightarrow (prefix(x, x) = p)$
  - $concat(y, reverse(y)) = x \Rightarrow reverse(x) = x$

trig specifications

The trig benchmark contains trigonometric functions from Python's math module.

- Learned specification for trig
  - $\exists k.x = 2\pi k + y \Rightarrow (sin(x) = z \Leftrightarrow sin(y) = z)$
  - $arcsin(z) = x \Rightarrow sin(x) = z$

geometry specifications

The geometry benchmark contains functions from sympy's geometry module.

- Learned specification for geometry
  - $b = true \Rightarrow$
    $(encl(x, y) = b \wedge encl\_pt(y, p) = true \Rightarrow encl\_pt(x, p) = true)$
  - $\exists k.x = 2\pi k \Rightarrow rotate(y, x) = y$

Empirical Evaluation: Scalability

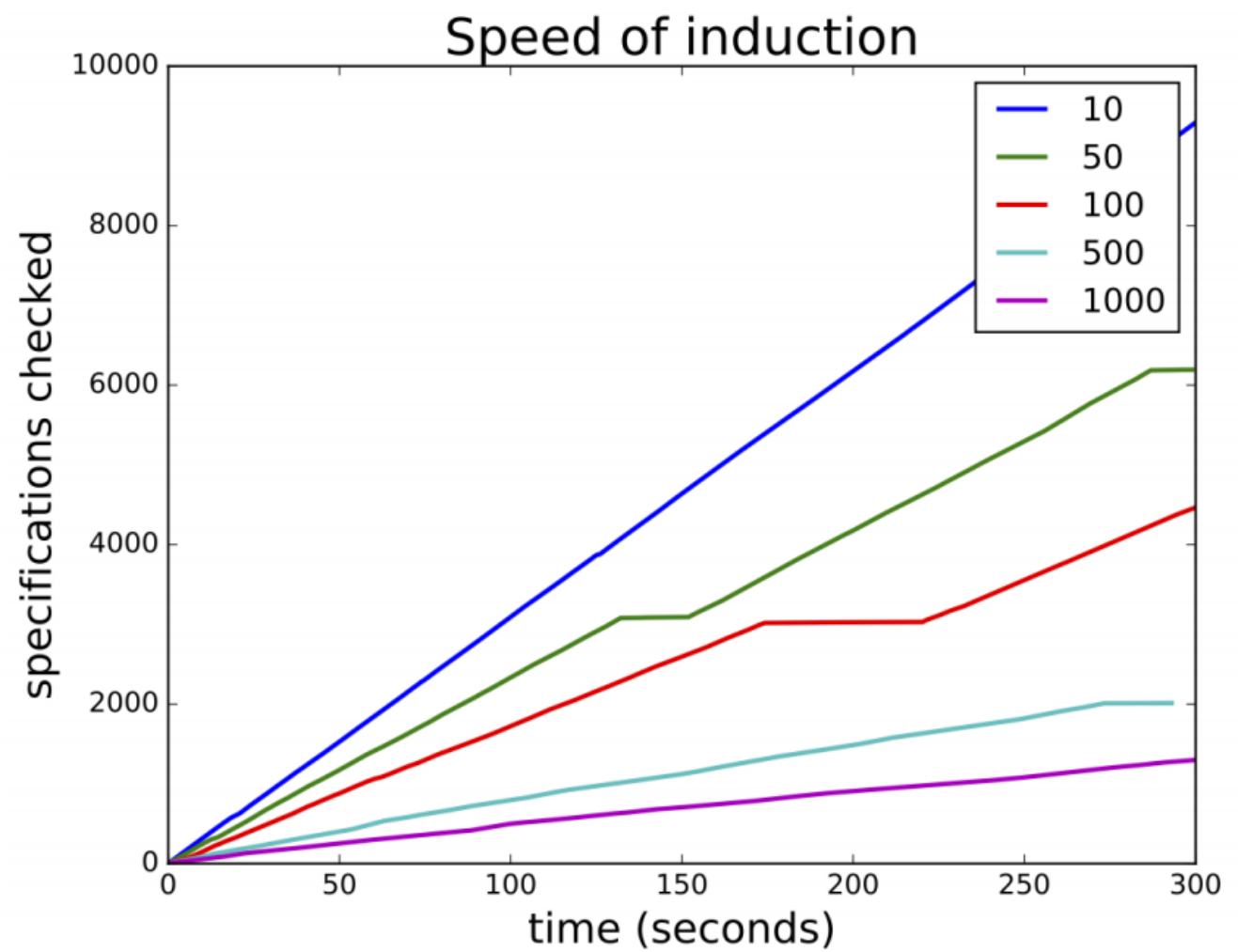


Figure: with more data, Bach checks less specification in same amount of time

Empirical Evaluation: Error Analysis

	10 observations			50 observations			100 observations			500 observations		
Benchmark	T_1	T_2	Size	T_1	T_2	Size	T_1	T_2	Size	T_1	T_2	Size
`ff199`	1.8	17.6	4.2	5.6	13.2	12.4	6.4	10.2	16.2	6.4	6.2	20.2
`trig`	2	19.2	10.8	2	0.8	29.2	0	0	28	0	0	28
`dict`	5.2	0.2	20	1.4	0	16.4	1	0	16	1	0	16
`geometry`	18	12	25	9	3.8	24.2	4	1	22	1	0	20
`lists`	40	17.6	52.4	4.8	0.4	34.4	1.4	0	31.4	0.4	0	30.4
`matrices`	18.2	11.2	25	15.4	3.2	30.2	7.8	0.6	25.2	5.2	0	19.4
`sets`	52.8	53.4	79.4	6.2	3.8	82.4	0.4	0	80.4	0	0	80
`strings`	159.6	234	215.6	85.6	50.8	324.8	22.2	1.6	310.6	0.2	0	290.2

Figure: Average correctness results

(T_1 is the type-1 error, T_2 is the type-2 error, Size is the number of specifications produced)

Empirical Evaluation: Error Analysis

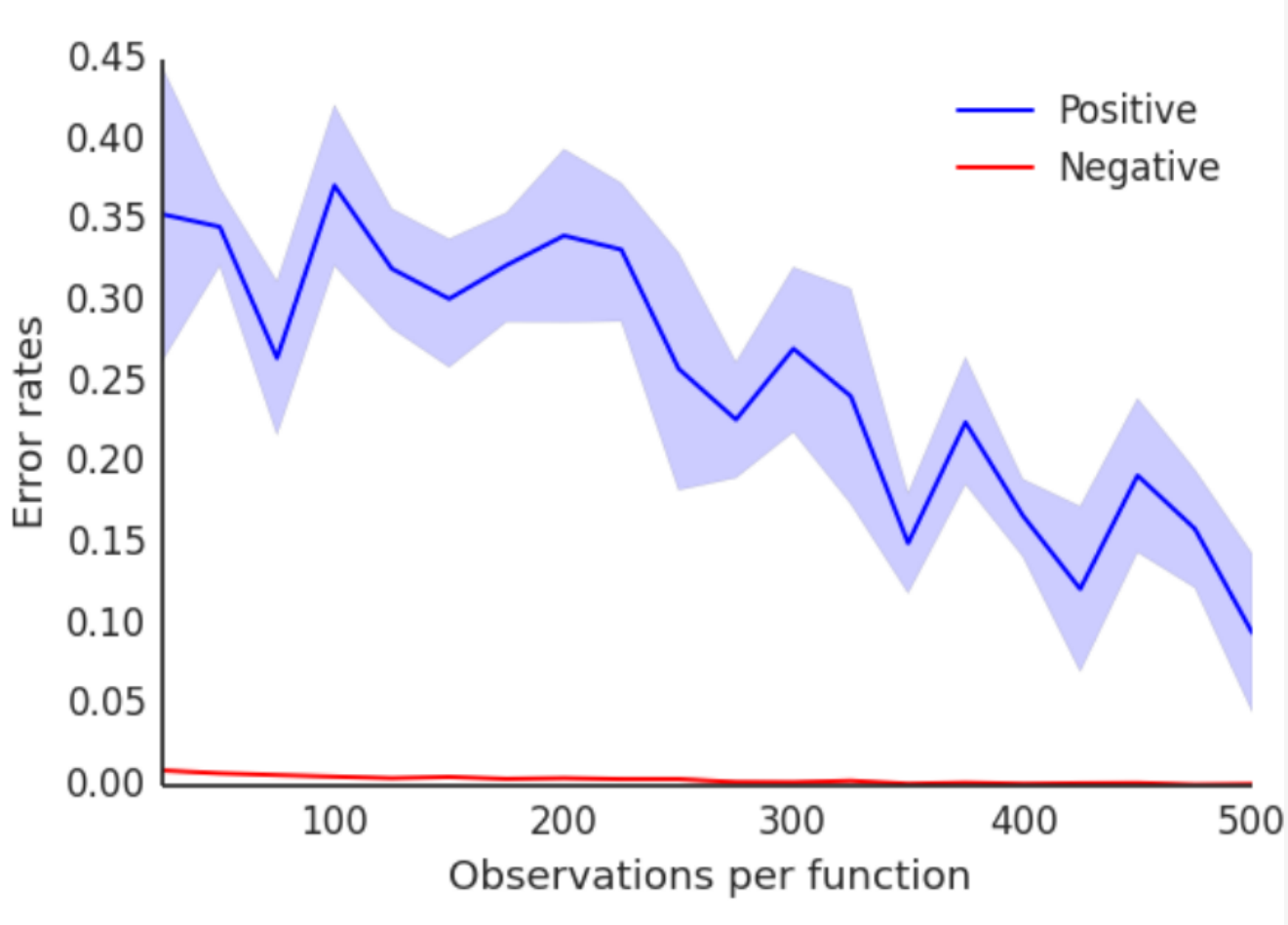


Figure: Worst-case benchmark's error rates with respect to number of observations

Empirical Evaluation: Error Analysis

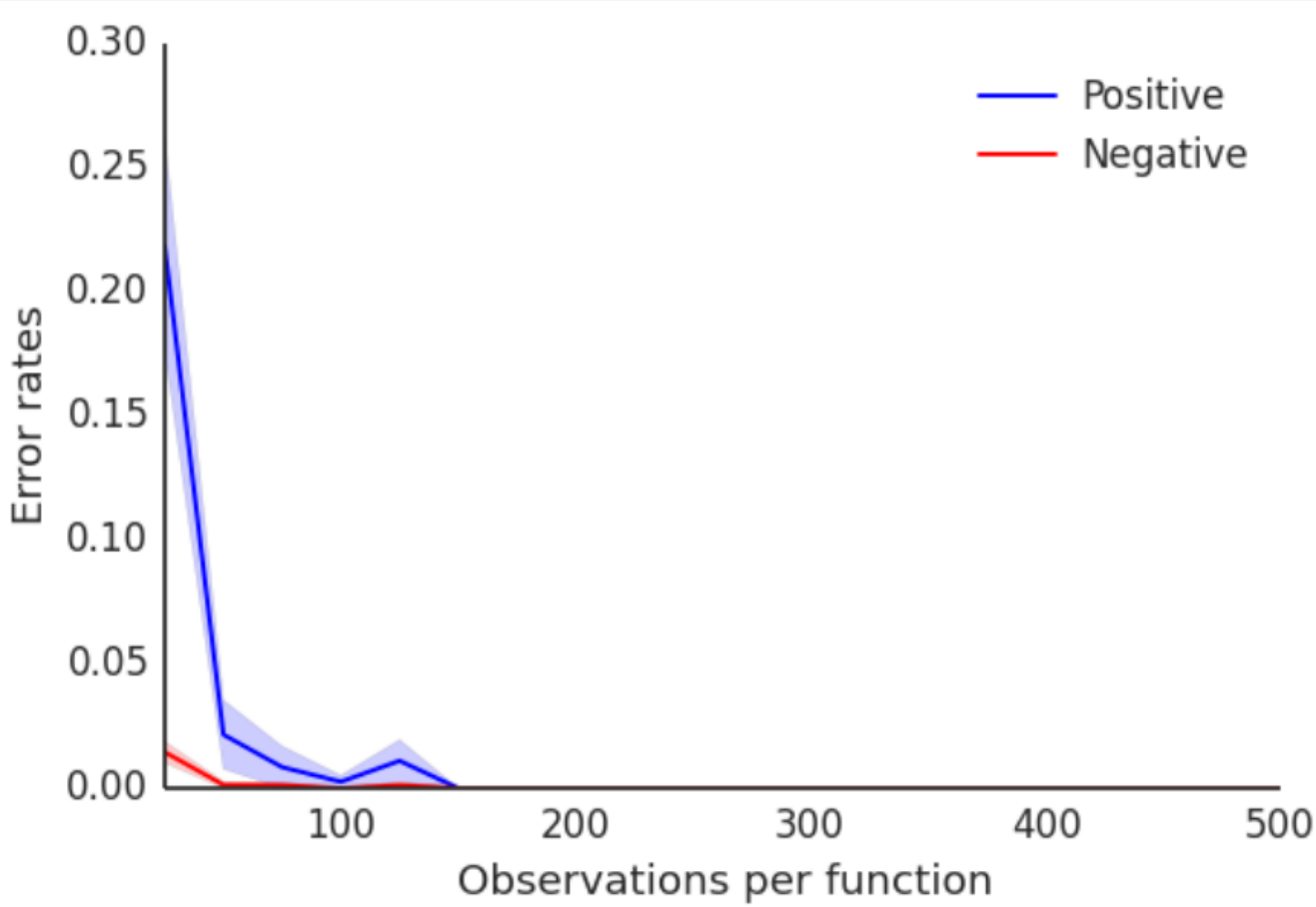


Figure: Best-case benchmark's error rates with respect to number of observations

Conclusion

- a technique for learning relational specification from i/o data
- learns specification that correlates different executions of multiple functions
- novel idea combining program synthesis and databases
- learns interesting specifications of real world libraries
- useful in program verification and development tasks

Questions?