Markov Logic: Combining Logic and Probability

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Overview

- Motivation & Background
- Markov logic
- Inference & Learning
- Abductive Plan Recognition
Social Network and Smoking Behavior

Smoking

Cancer
Social Network and Smoking Behavior

Smoking leads to Cancer
Social Network and Smoking Behavior

Smoking leads to Cancer

Friendship

Similar Smoking Habits
Social Network and Smoking Behavior

Smoking leads to Cancer

Friendship leads to Similar Smoking Habits
Statistical Relational AI

- Real world problems characterized by
  - Entities and Relationships
  - Uncertain Behavior
- Relational Models
  - Horn clauses, SQL queries, first-order logic
- Statistical Models
  - Markov networks, Bayesian networks
- How to combine the two?
- Markov Logic
  - Markov Networks + First Order Logic
Statistical Relational AI

- Probabilistic logic [Nilsson, 1986]
- Statistics and beliefs [Halpern, 1990]
- Knowledge-based model construction [Wellman et al., 1992]
- Stochastic logic programs [Muggleton, 1996]
- Probabilistic relational models [Friedman et al., 1999]
- Bayesian Logic Programs [Kersting and De Raedt 2001]
- Relational Markov networks [Taskar et al., 2002]
- BLOG [Milch et al., 2005]
- **Markov logic** [Richardson & Domingos, 2006]
First-Order Logic

- Constants, variables, functions, predicates
  - Anil, x, MotherOf(x), Friends(x,y)
- Grounding: Replace all variables by constants
  - Friends (Anna, Bob)
- Formula: Predicates connected by operators
  - Smokes(x) ⇒ Cancer(x)
- Knowledge Base (KB): A set of formulas
  - Can be equivalently converted into a clausal form
- World: Assignment of truth values to all ground atoms
First-Order Logic

- Deal with finite first-order logic
- Assumptions
  - Unique Names
  - Domain Closure
  - Known Functions
Markov Networks

- **Undirected** graphical models

Markov Networks

- Log-linear model:
  
  \[
  P(x) = \frac{1}{Z} \exp \left( \sum_i w_i f_i(x) \right)
  \]

  \[
  f_1(\text{Smoking, Cancer}) = \begin{cases} 
  1 & \text{if } \text{Smoking } \Rightarrow \text{Cancer} \\
  0 & \text{otherwise}
  \end{cases}
  \]

  \[
  w_1 = 1.5
  \]
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A logical KB is a set of hard constraints on the set of possible worlds.

Let’s make them soft constraints: When a world violates a formula, it becomes less probable, not impossible.

Give each formula a weight (Higher weight ⇒ Stronger constraint)

\[
P(\text{world}) \propto \exp\left(\sum \text{weights of formulas it satisfies}\right)
\]
A Markov Logic Network (MLN) is a set of pairs \((F, w)\) where
- \(F\) is a formula in first-order logic
- \(w\) is a real number

Together with a finite set of constants, it defines a Markov network with
- One node for each grounding of each predicate in the MLN
- One feature for each grounding of each formula \(F\) in the MLN, with the corresponding weight \(w\)
Example: Friends & Smokers

- Smoking causes cancer.
- Friends have similar smoking habits.
Example: Friends & Smokers

\[ \forall x \text{ Smokes}(x) \implies \text{Cancer}(x) \]
\[ \forall x, y \text{ Friends}(x, y) \land \text{Smokes}(x) \implies \text{Smokes}(y) \]
# Example: Friends & Smokers

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Two constants: **Anil** (A) and **Bunty** (B)
Example: Friends & Smokers

Two constants: Anil (A) and Bunty (B)

1.5 \( \forall x \ Smokes(x) \implies Cancer(x) \)

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**Example: Friends & Smokers**

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Two constants: **Anil** (A) and **Bunty** (B)

![Diagram showing relationships between Friends, Smokes, and Cancer for Anil and Bunty]
Example: Friends & Smokers

Two constants: Anil (A) and Bunty (B)

State of the World $\equiv \{0,1\}$ Assignment to the nodes
Markov Logic Networks

- MLN is template for ground Markov nets
- Probability of a world $x$:

$$P(x) = \frac{1}{Z} \exp \left( \sum_{k \in \text{ground formulas}} w_k f_k(x) \right)$$
Markov Logic Networks

- MLN is **template** for ground Markov nets
- Probability of a world $x$:

$$P(x) = \frac{1}{Z} \exp \left( \sum_{k \in \text{ground formulas}} w_k f_k(x) \right)$$

$$P(x) = \frac{1}{Z} \exp \left( \sum_{i \in \text{MLN formulas}} w_i n_i(x) \right)$$

- **Weight of formula $i$**
- **No. of true groundings of formula $i$ in $x$**
Relation to Statistical Models

- Special cases:
  - Markov networks
  - Markov random fields
  - Bayesian networks
  - Log-linear models
  - Exponential models
  - Logistic regression
  - Hidden Markov models
  - Conditional random fields

- Obtained by making all predicates zero-arity
Relation to First-Order Logic

- Infinite weights $\Rightarrow$ First-order logic
- Satisfiable KB, positive weights $\Rightarrow$
  Satisfying assignments = Modes of distribution
- Markov logic allows contradictions between formulas
- Relaxing Assumptions
  - Known Functions (Markov Logic in Infinite Domains)
    [Singla & Domingos 07]
  - Unique Names (Entity Resolution with Markov Logic)
    [Singla & Domingos 06]
Overview

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Inference

**blue ? – non-evidence (unknown)**

**green/orange – evidence (known)**
MPE Inference

- **Problem**: Find most likely state of world given evidence

\[
P(y \mid x) = \frac{1}{Z_x} \exp \left( \sum_i w_i n_i (x, y) \right)
\]
MPE Inference

Problem: Find most likely state of world given evidence

$$\arg \max_y \frac{1}{Z_x} \exp \left( \sum_i w_i n_i(x, y) \right)$$
MPE Inference

Problem: Find most likely state of world given evidence

$$\arg \max_y \sum_i w_i n_i(x, y)$$
**MPE Inference**

- **Problem**: Find most likely state of world given evidence

\[
\arg \max_y \sum_i w_i n_i(x, y)
\]

- This is just the weighted MaxSAT problem
- Use weighted SAT solver (e.g., MaxWalkSAT [Kautz et al. 97])

Lazy Grounding of Clauses: LazySAT [Singla & Domingos 06]
Marginal Inference

- **Problem**: Find the probability of query atoms given evidence

\[ P(y \mid x) = \frac{1}{Z_x} \exp\left( \sum_i w_i n_i(x, y) \right) \]
Marginal Inference

- **Problem:** Find the probability of query atoms given evidence

$$P(y \mid x) = \frac{1}{Z_x} \exp \left( \sum_i w_i n_i(x, y) \right)$$

Computing $Z_x$ takes exponential time!
Marginal Inference

- **Problem**: Find the probability of query atoms given evidence

\[
P(y | x) = \frac{1}{Z_x} \exp \left( \sum_i w_i n_i (x, y) \right)
\]

Approximate Inference: Gibbs Sampling, Message Passing

[Richardson & Domingos 06, Poon & Domingos 06, Singla & Domingos 08]
### Learning Parameters

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Three constants: **Anil, Bunty, Chaya**
Learning Parameters

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Closed World Assumption:
Anything not in the database is assumed false.
Learning Parameters

| \( w_1 \) | \( \forall x \, \text{Smokes}(x) \Rightarrow \text{Cancer}(x) \) |
| \( w_2 \) | \( \forall x, y \, \text{Friends}(x, y) \land \text{Smokes}(x) \Rightarrow \text{Smokes}(y) \) |

Three constants: Anil, Bunty, Chaya

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Maximize the Likelihood: Use Gradient Based Approaches [Singla & Domingos 05, Lowd & Domingos 07]
Learning Structure

Three constants: Anil, Bunty, Chaya

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Can we learn the set of the formulas in the MLN?
Learning Structure

| $w_1$? | $\forall x \text{ Smokes}(x) \Rightarrow \text{Cancer}(x)$ |
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Can we refine the set of the formulas in the MLN?
Can we refine the set of the formulas in the MLN?
## Learning Structure

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### Three constants: Anil, Bunty, Chaya

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### ILP style search for formulas

[Kok & Domingos 05, 07, 09, 10]
Alchemy

Open-source software including:

- Full first-order logic syntax
- Inference algorithms
- Parameter & structure learning algorithms

alchemy.cs.washington.edu
Overview

- Motivation & Background
- Markov logic
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- Abductive Plan Recognition
Applications

- Web-mining
- Collective Classification
- Link Prediction
- Information retrieval
- Entity resolution
- Activity Recognition
- Image Segmentation & De-noising

- Social Network Analysis
- Computational Biology
- Natural Language Processing
- Robot mapping
- Abductive Plan Recognition
- More..
Abduction

- **Abduction**: Given the observations and the background, find the best explanation

- **Given:**
  - Background knowledge (B)
  - A set of observations (O)

- **To Find:**
  - A hypothesis, H, a set of assumptions

- \( B \cup H \not\models \bot, B \cup H \models O \)
Plan Recognition

- Given planning knowledge and a set of low-level actions, identify the top level plan
- Involves abductive reasoning

B: Planning Knowledge (Background)
O: Set of low-level Actions (Observations)
H: Top Level Plan (Hypothesis)

\[ B \cup H \not\equiv \bot, \quad B \cup H \models O \]
Plan Recognition Example

- Emergency Response Domain [Blaylock & Allen 05]
- Background Knowledge
  \[\text{heavy}\_\text{snow}(\text{loc}) \land \text{drive}\_\text{hazard}(\text{loc}) \Rightarrow \text{block}\_\text{road}(\text{loc})\]
  \[\text{accident}(\text{loc}) \land \text{clear}\_\text{wreck}(\text{crew,loc}) \Rightarrow \text{block}\_\text{road}(\text{loc})\]
- Observation
  \[\text{block}\_\text{road}(\text{Plaza})\]
- Possible Explanations
  - Heavy Snow?
  - Accident?
Abduction using Markov logic

Given

\[ \text{heavy_snow}(\text{loc}) \land \text{drive_hazard}(\text{loc}) \Rightarrow \text{block_road}(\text{loc}) \]
\[ \text{accident}(\text{loc}) \land \text{clear_wreck}(\text{crew, loc}) \Rightarrow \text{block_road}(\text{loc}) \]

Observation: \text{block_road}(\text{plaza})
Abduction using Markov logic

- Given
  
  \[
  \text{heavy\_snow(loc)} \land \text{drive\_hazard(loc)} \Rightarrow \text{block\_road(loc)} \\
  \text{accdent(loc)} \land \text{clear\_wreck(crew, loc)} \Rightarrow \text{block\_road(loc)} 
  \]

  Observation: block\_road(plaza)

  Does not work!

- Rules are true independent of antecedents
- Need to go from effect to cause
Introducing Hidden Cause

\[ \text{heavy_snow(loc)} \land \text{drive_hazard(loc)} \Rightarrow \text{block_road(loc)} \]

\[ \text{rb_C1(loc)} \rightarrow \text{Hidden Cause} \]

\[ \text{heavy_snow(loc)} \land \text{drive_hazard(loc)} \Leftrightarrow \text{rb_C1(loc)} \]
Introducing Hidden Cause

\[
\text{heavy\_snow}(\text{loc}) \land \text{drive\_hazard}(\text{loc}) \Rightarrow \text{block\_road}(\text{loc})
\]

\[
\Rightarrow
\]

\[
\text{rb\_C1}(\text{loc}) \Rightarrow \text{block\_road}(\text{loc})
\]

\[
\text{rb\_C1}(\text{loc}) \Leftrightarrow \text{heavy\_snow}(\text{loc}) \land \text{drive\_hazard}(\text{loc})
\]

\[
\Rightarrow
\]

\[
\text{rb\_C1}(\text{loc}) \Leftrightarrow \text{block\_road}(\text{loc})
\]
Introducing Hidden Cause

heavy_snow(loc) \land drive_hazard(loc) \implies block_road(loc)

rb_C1(loc) \implies Hidden Cause

heavy_snow(loc) \land drive_hazard(loc) \iff rb_C1(loc)

rb_C1(loc) \implies block_road(loc)

accident(loc) \land clear_wreck(loc, crew) \implies block_road(loc)

rb_C2(loc, crew)

accident(loc) \land clear_wreck(loc) \iff rb_C2(loc, crew)

rb_C2(loc, crew) \implies block_road(loc)
Introducing Reverse Implication

Explanation 1:\(\text{heavy} \_\text{snow}(\text{loc}) \land \text{clear} \_\text{wreck}(\text{loc}) \iff \text{rb} \_\text{C1}(\text{loc})\)

Explanation 2:\(\text{accident}(\text{loc}) \land \text{clear} \_\text{wreck}(\text{loc}) \iff \text{rb} \_\text{C2}(\text{loc}, \text{crew})\)

Multiple causes combined via reverse implication
Existential quantification

\(\text{block} \_\text{road}(\text{loc}) \Rightarrow \text{rb} \_\text{C1}(\text{loc}) \lor (\exists \text{crew } \text{rb} \_\text{C2}(\text{loc}, \text{crew}))\)
Low-Prior on Hidden Causes

Explanation 1: heavy_snow(loc) \land clear_wreck(loc) \iff rb_C1(loc)

Explanation 2: accident(loc) \land clear_wreck(loc) \iff rb_C2(loc, crew)

Multiple causes combined via reverse implication

block_road(loc) \Rightarrow rb_C1(loc) \lor (\exists \text{crew } rb_C2(loc, crew))

Existential quantification

-w1 \text{rb}_C1(loc)
-w2 \text{rb}_C2(loc, crew)
Hidden Causes: Avoiding Blow-up

Hidden Cause Model
Max clique size = 3

[Singla & Domingos 2011]
Hidden Causes: Avoiding Blow-up

[Singla & Domingos 2011]

Max clique size = 3

Hidden Cause Model

Max clique size = 5

Pair-wise Constraints

[Kate & Mooney 2009]
Second Issue: Ground Network Too Big!

- Grounding out the full network may be costly
- Many irrelevant nodes/clauses are created
- Complicates learning/inference
- Can focus the grounding (KBMC)
Abductive Model Construction

Observation: block_road(Plaza)

- heavy_snow (Plaza)
- drive_hazard (Plaza)
- block_road (Plaza)
Abductive Model Construction

Observation:
block_road(Plaza)

Constants:
..., Mall, City_Square, ...

heavy_snow
(Mall)
drive_hazard
(Mall)

heavy_snow
(City_Square)
drive_hazard
(City_Square)

block_road
(Mall)

block_road
(City_Square)
Abductive Model Construction

Observation:
block_road(Plaza)

Constants:
…, Mall, City_Square, ...

heavy_snow (Plaza)
drive_hazard (Plaza)

Not a part of abductive proof trees!
Abductive Model Construction

Observation: block_road(Plaza)

Constants:
…, Mall, City_Square, ...

Not a part of abductive proof trees!

Backward chaining to get proof trees [Stickel 1988]
Abductive Markov Logic [Singla & Domingos 11]

- Re-encode the MLN rules
  - Introduce reverse implications
- Construct ground Markov network
  - Use abductive model construction
- Perform learning and inference
Summary

- Real world applications
  - Entities and Relations
  - Uncertainty
- Unifying logical and statistical AI
- Markov Logic – simple and powerful model
- Need to do to efficient learning and inference
- Applications: Abductive Plan Recognition