Reachability in timed and probabilistic systems

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FM Update, Delhi

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Reachability in TA

Reachability in PTA

Conclusions

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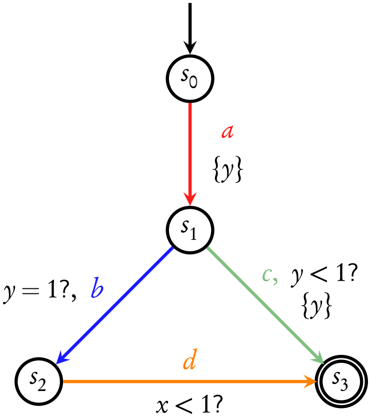
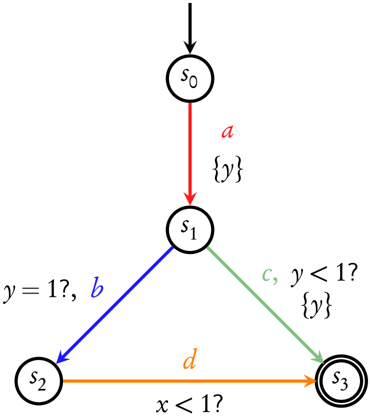
Conclusions

Timed Automata [AD90]



Timed Automata [AD90]



Run:

$(s_0, 0, 0)$

$\xrightarrow{0.4,\ a}$

$(s_1, 0.4, 0)$

$\xrightarrow{0.5,\ c}$

$(s_3, 0.9, 0.5)$

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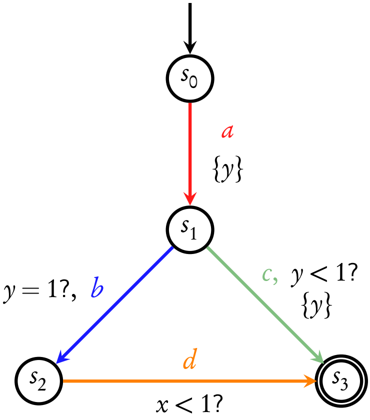

Run:

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$\xrightarrow{0.4,\ a}$

$(s_1, 0.4, 0)$

$\xrightarrow{0.5,\ c}$

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Reachability Problem

Does there **exist** a run to the final state?

Reachability

- PSPACE-complete [Alur, Dill '90]

Reachability

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- PSPACE-complete with **3 clocks** [Courcoubetis, Yannakakis '92]

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- 2 clock TA $\leftarrow$ LOGSPACE $\rightarrow$ **Bounded 1-counter automata**

  [Hasse, Ouaknine, Worrell '12]

Reachability

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- 2 clock TA $\leftarrow$ LOGSPACE $\rightarrow$ **Bounded 1-counter automata** [Hasse, Ouaknine, Worrell '12]
- PSPACE-complete for bounded 1-counter automata [Fearnley, Jurdziǹski '13]

Reachability in 2-clock TA is PSPACE-complete

Tools

- UPPAAL [Aalborg (Denmark)]
- KRONOS [Verimag (France)]
- RED [Wang]

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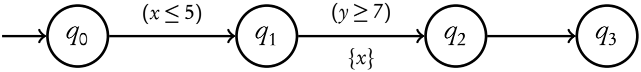
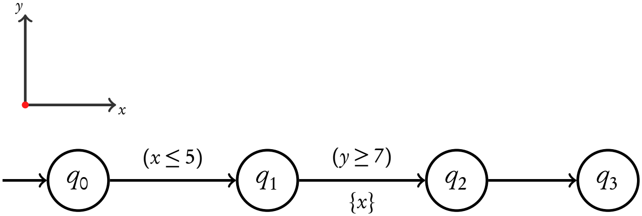
Coming next: zone-based approach of UPPAAL, KRONOS

Forward analysis



Forward analysis



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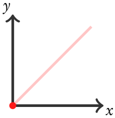

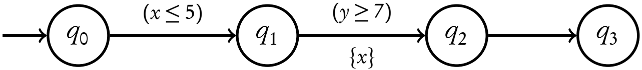

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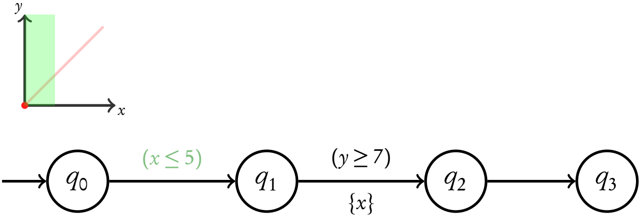

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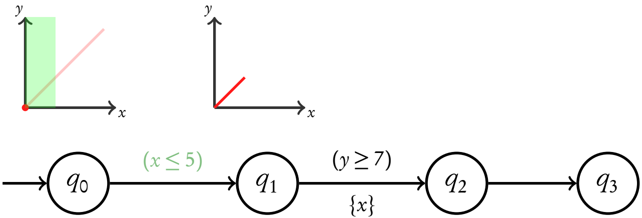

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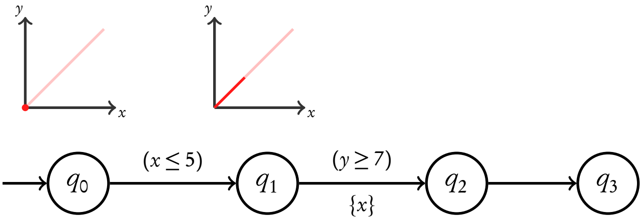

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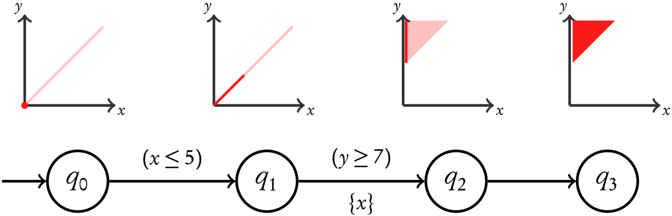

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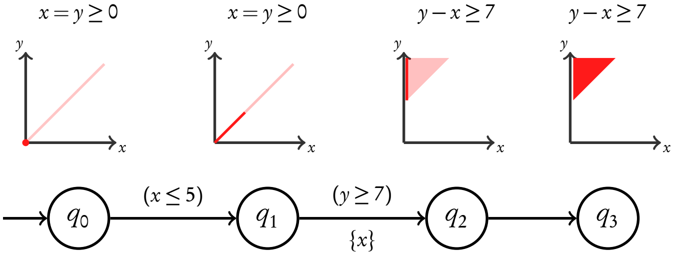
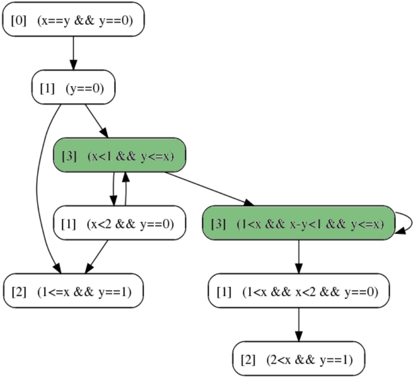


Zones and zone graph



- Zone: set of valuations defined by conjunctions of constraints:

$$x \sim c$$
$$x - y \sim c$$

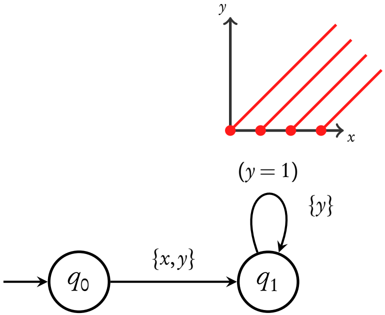
e.g. $(x - y \geq 1) \wedge (y < 2)$

- Representation: by DBM [Dil89]

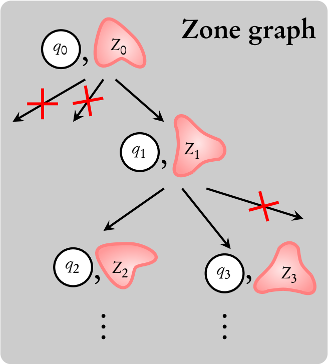
Sound and complete [DT98]

Zone graph preserves state **reachability**

Problem of non-termination

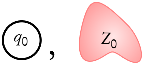


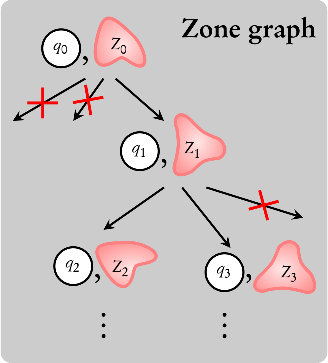
Abstractions



potentially infinite...

Abstractions





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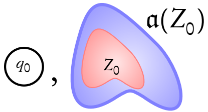

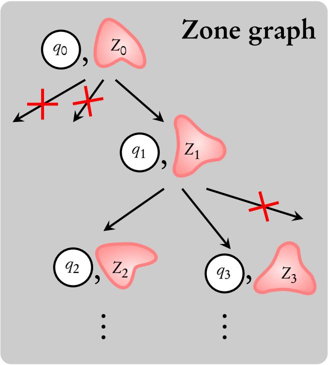

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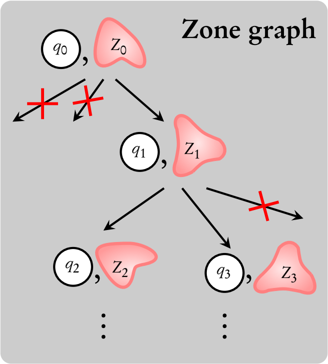

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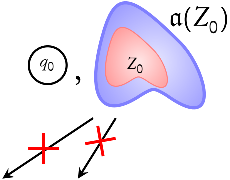

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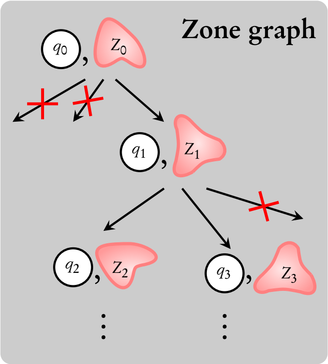

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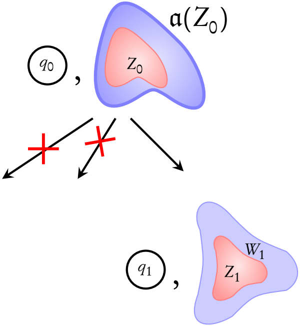

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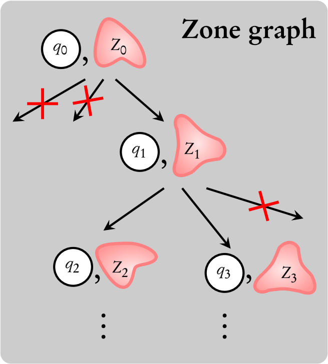

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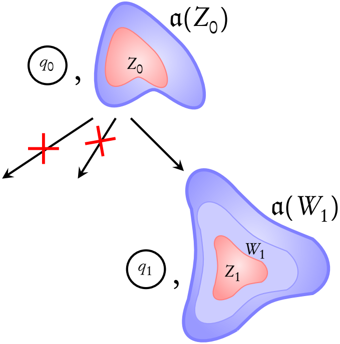

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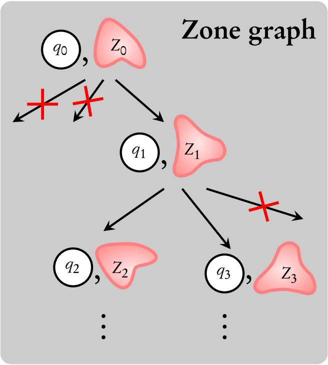

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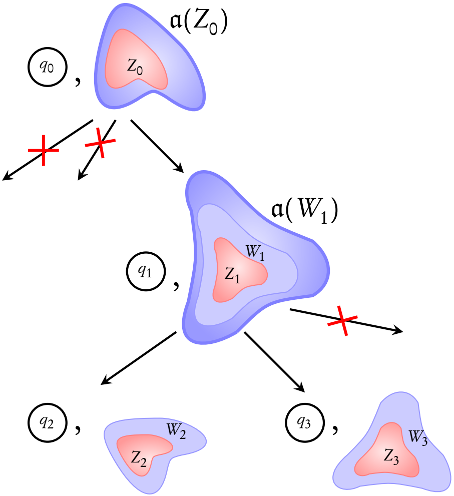

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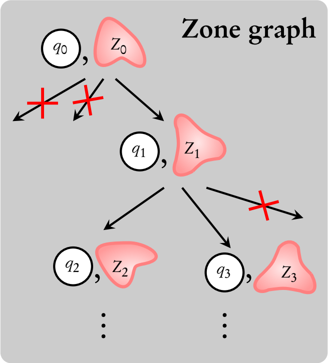

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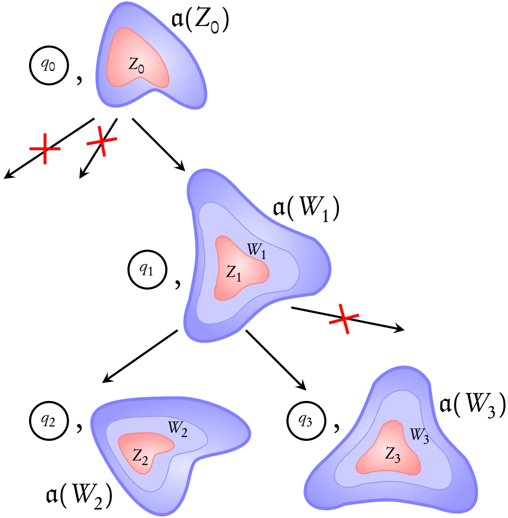

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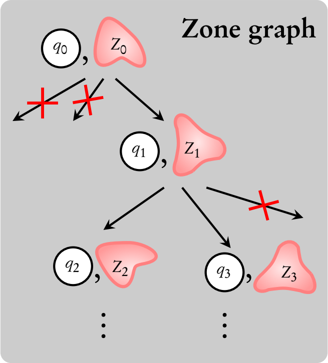

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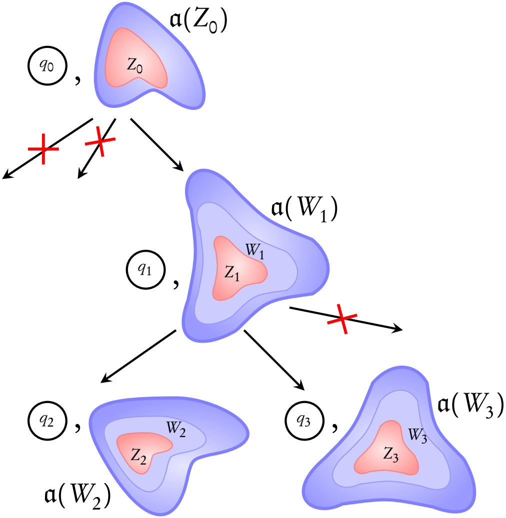
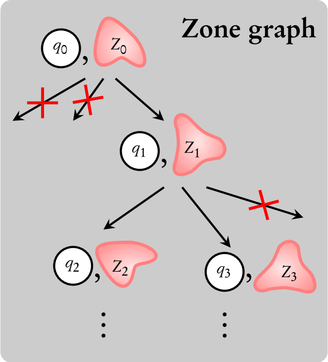


Find $\mathfrak{a}$ such that number of **abstracted** sets is **finite**

Abstractions



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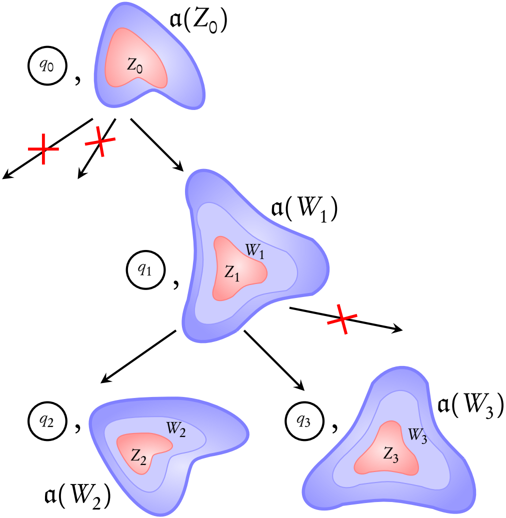
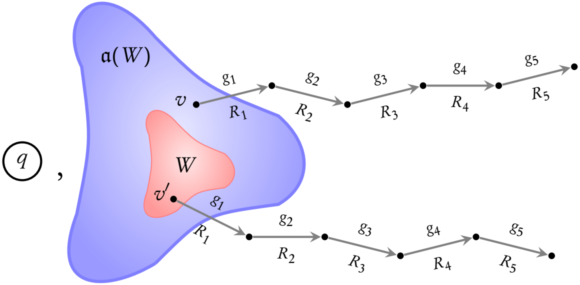


Coarser the abstraction, **smaller** the abstracted graph

Condition 1: Abstractions should have **finite range**

Condition 2: Abstractions should be sound $\Rightarrow$ $\mathfrak{a}(W)$ can contain only valuations **simulated** by W



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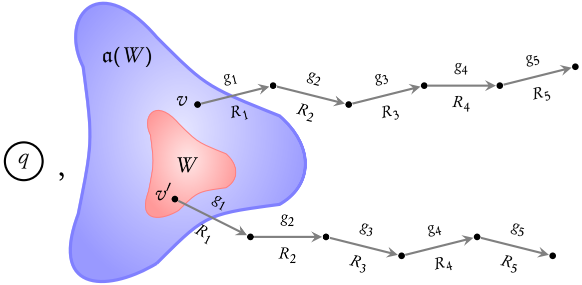
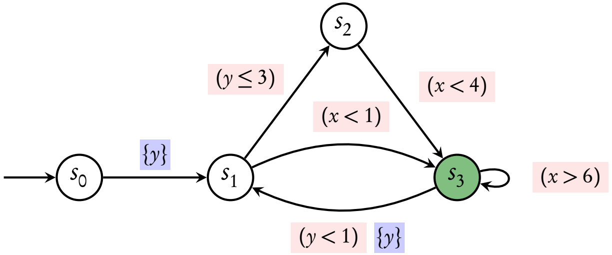


Question: Why not add **all** the valuations **simulated** by W?

Theorem [LS00]

Coarsest simulation relation is **EXPTIME-hard**



Theorem [LS00]

Coarsest simulation relation is **EXPTIME-hard**

$(y \leq 3)$

$(x < 4)$

$(x < 1)$

$(x > 6)$

$(y < 1)$

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Coarsest simulation relation is **EXPTIME-hard**

$(y \leq 3)$

$(x < 4)$

$(x < 1)$

$(x > 6)$

$(y < 1)$

M-bounds [AD94]

$$M(x) = 6,\ M(y) = 3$$

$$v \preceq_M v'$$

Theorem [LS00]

Coarsest simulation relation is **EXPTIME-hard**

$(y \leq 3)$ $(x < 4)$

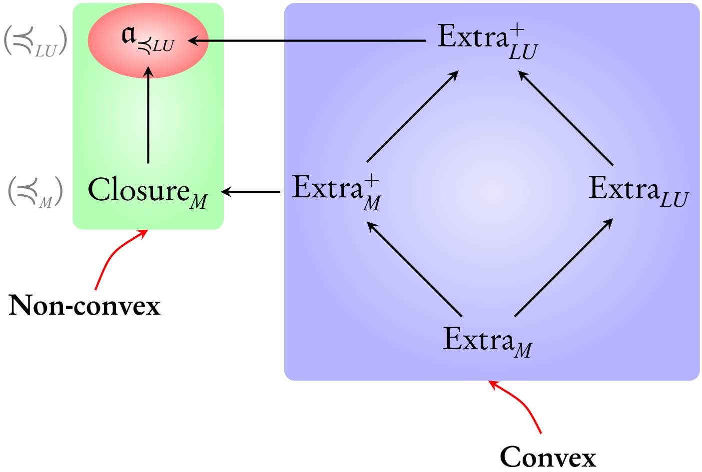
$(x < 1)$

$(x > 6)$

$(y < 1)$

M-bounds [AD94]	**LU-bounds** [BBLP06]
$M(x) = 6,\ M(y) = 3$	$L(x) = 6,\ L(y) = -\infty$ $U(x) = 4,\ U(y) = 3$
$v \preccurlyeq_M v'$	$v \preccurlyeq_{LU} v'$

Abstractions in literature [BBLP06, Bou04]



Recent results

- $\mathfrak{a}_{\preccurlyeq LU}$ can be **efficiently** used

  Given LU, $\mathfrak{a}_{\preccurlyeq LU}$ is **optimal**

  [Herbreteau, S., Walukiewicz '12]

- Better LU-bounds in a **lazy** way

  [Herbreteau, S., Walukiewicz '13]

- Multicore

  [Larsen et al. '12]

Reachability in TA

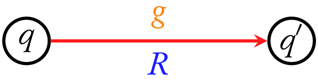
Reachability in PTA

Conclusions

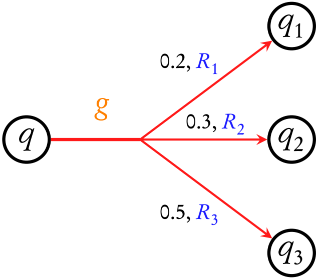
Transition:

$$q \xrightarrow[R]{g} q'$$

Transition:

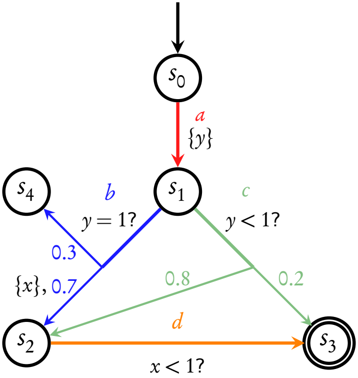


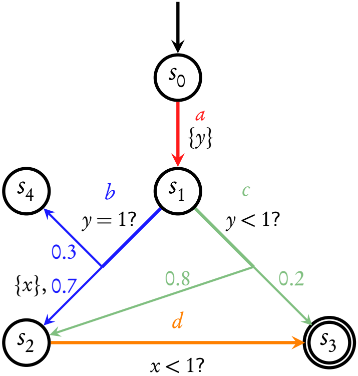
Probabilistic transition:



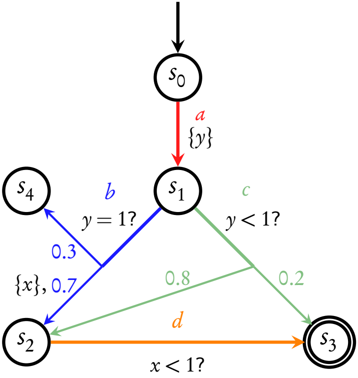
Probabilistic Timed Automata

[Jen96, KNSS02]

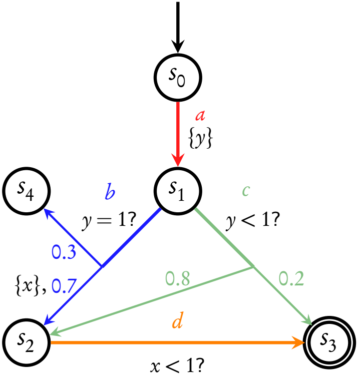




s_0
a
$\{y\}$
s_1
s_4
b
$y = 1?$
c
$y < 1?$
0.3
$\{x\}$, 0.7
0.8
0.2
d
s_2
s_3
$x < 1?$



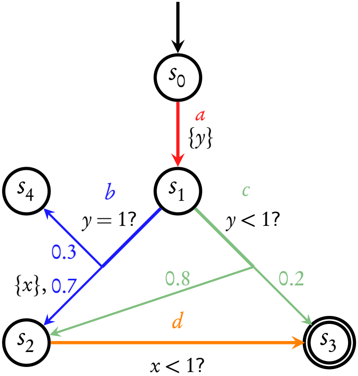
$n_0 : (s_0 , 0 , 0)$



Scheduler

$\sigma_1(n_0) = (0.4, a)$

$n_0 : (s_0 , 0 , 0)$



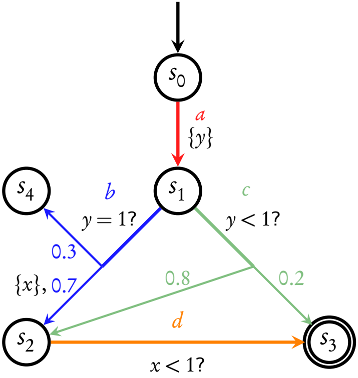
Scheduler

$\sigma_1(n_0) = (0.4, a)$

$n_0 : (\, s_0 \,,\, 0 \,,\, 0 \,)$

$\downarrow (0.4, a)$

$n_1 : (\, s_1 \,,\, 0.4 \,,\, 0 \,)$

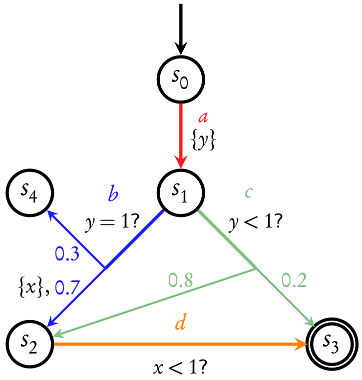


Scheduler

$$\sigma_1(n_0) = (0.4, a)$$

$$\sigma_1(n_0 n_1) = (1, b)$$

$$\begin{tikzcd} n_0 : (\, s_0 \,,\, 0 \,,\, 0 \,) \arrow[d, "{(0.4, a)}"] \\ n_1 : (\, s_1 \,,\, 0.4 \,,\, 0 \,) \end{tikzcd}$$



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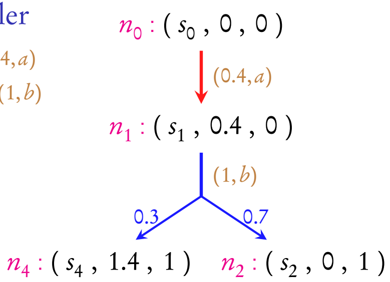
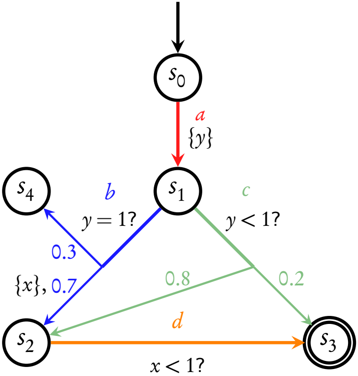


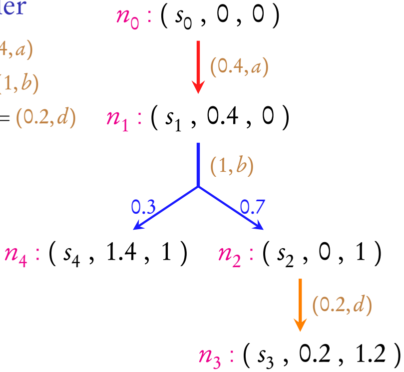


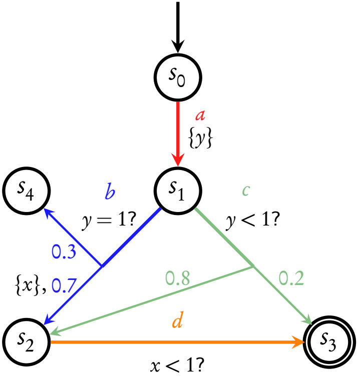
Scheduler

$\sigma_1(n_0) = (0.4, a)$

$\sigma_1(n_0 n_1) = (1, b)$

$\sigma_1(n_0 n_1 n_2) = (0.2, d)$





Scheduler

$\sigma_1(n_0) = (0.4, a)$

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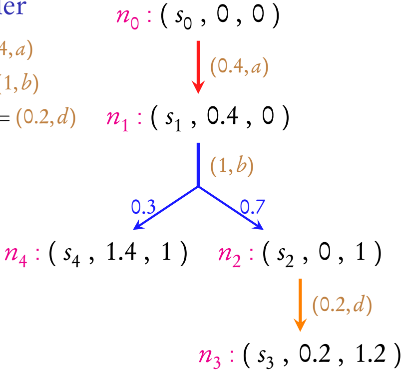
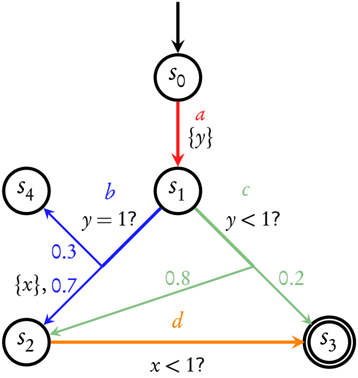
Markov chain





Scheduler

$\sigma_1(n_0) = (0.4, a)$

$\sigma_1(n_0 n_1) = (1, b)$

$\sigma_1(n_0 n_1 n_2) = (0.2, d)$

Markov chain

$n_0 : (\, s_0 \,,\, 0 \,,\, 0 \,)$

$\downarrow (0.4, a)$

$n_1 : (\, s_1 \,,\, 0.4 \,,\, 0 \,)$

$(1, b)$: with probability 0.3 to n_4, with probability 0.7 to n_2

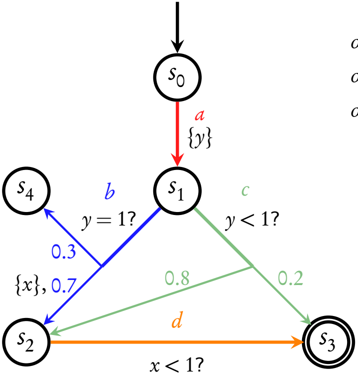
$n_4 : (\, s_4 \,,\, 1.4 \,,\, 1 \,)$ $\quad$ $n_2 : (\, s_2 \,,\, 0 \,,\, 1 \,)$

$\downarrow (0.2, d)$

$n_3 : (\, s_3 \,,\, 0.2 \,,\, 1.2 \,)$

Probability

$\mathbb{P}_{\sigma_1}(s_3) = 0.7$



$\sigma_2(n_0) = (0.4, a)$

$\sigma_2(n_0 n_1) = (0.5, c)$

$\sigma_2(n_0 n_1 n_2) = (0.05, d)$

$n_0 : (\, s_0 \,,\, 0 \,,\, 0 \,)$

$\xrightarrow{(0.4, a)}$

$n_1 : (\, s_1 \,,\, 0.4 \,,\, 0 \,)$

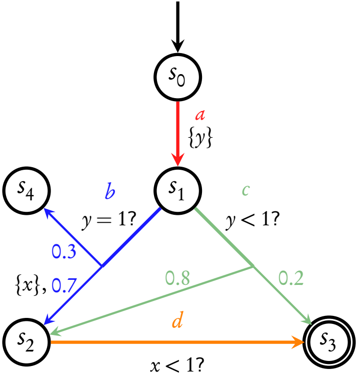
$(0.5, c)$: $0.8 \to n_2$, $0.2 \to n_{31}$

$n_2 : (\, s_2 \,,\, 0.9 \,,\, 0.5 \,)$ $\quad n_{31} : (\, s_3 \,,\, 0.9 \,,\, 0.5 \,)$

$\xrightarrow{(0.05, d)}$

$n_{32} : (\, s_3 \,,\, 0.95 \,,\, 0.55 \,)$

$$\mathbb{P}_{\sigma_2}(s_3) = 1$$



Reachability Problem for PTA

Given $\lambda \in [0, 1]$, does there **exist a scheduler** σ with

$$\mathbb{P}_\sigma(\text{final state}) \geq \lambda?$$

PTA reachability

- EXPTIME-complete

  [Kwiatkowska, Norman, Segala, Sproston '02]

  [Jurdziǹski, Sproston, Larrousinie '08]

PTA reachability

- EXPTIME-complete

  [Kwiatkowska, Norman, Segala, Sproston '02]

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- EXPTIME-complete for **2-clocks**

  PTIME-complete for **1-clock**

  [Jurdziǹski, Sproston, Larrousinie '08]

Tools

- PRISM [Oxford]

Tools

- PRISM [Oxford]

Coming next: Reachability algorithm of PRISM

$$
\begin{tikzcd}
\text{PTA} \arrow[d] \\
\text{Zone MDP} \arrow[d] \\
\exists\ \text{an } \textbf{(untimed) scheduler} \text{ in zone MDP with probability } \geq \lambda\text{?}
\end{tikzcd}
$$

PTA

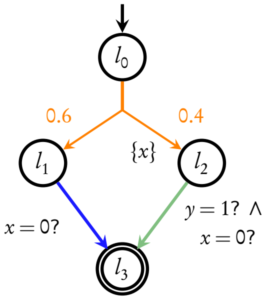
$\downarrow$

Zone MDP

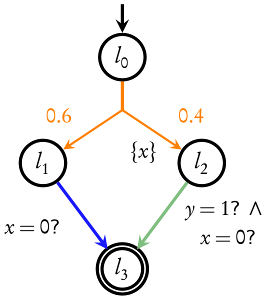
$\downarrow$

$\exists$ an **(untimed) scheduler** in zone MDP with probability $\geq \lambda$?

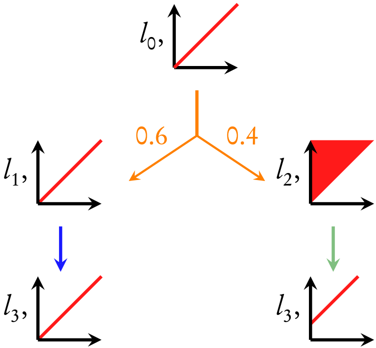
Coming next: Problem with forward analysis [KNSS02]

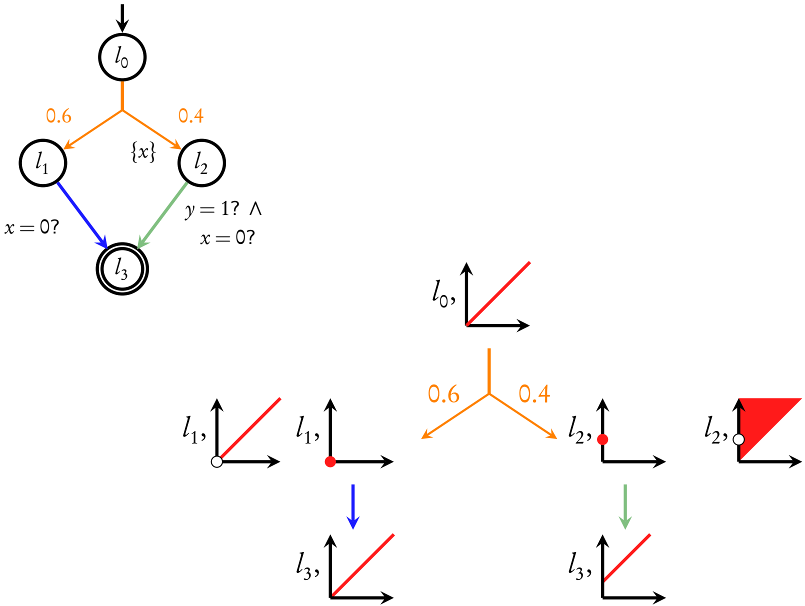


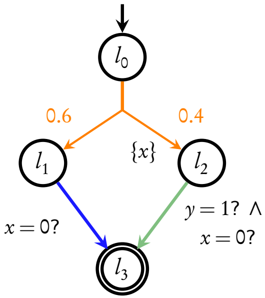
Max prob is 0.6



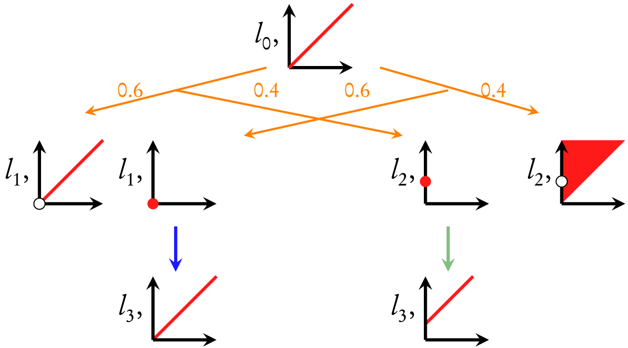
Zone MDP says 1




l_0
0.6
0.4
l_1
$\{x\}$
l_2
$y = 1? \wedge$
$x = 0?$
$x = 0?$
l_3
$l_0,$
0.6
0.4
$l_1,$
$l_1,$
$l_2,$
$l_2,$
$l_3,$
$l_3,$



Need an MDP like this



Two main approaches

- Game-based forward analysis

  [Kwiatkowska, Norman, Parker '09]

- Backward analysis

  [Kwiatkowska, Norman, Parker, Wang '07]

  [Berendsen, Jansen, Vaandrager]

Two main approaches

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  [Berendsen, Jansen, Vaandrager]

In many cases, **backward** performs better

Reachability in TA

Reachability in PTA

Conclusions

Summary

- Complexity:

  PSPACE-complete for TA, EXPTIME-complete for PTA

  (even for 2 clocks)

- Algorithm for TA:

  Forward analysis with abstraction

- Algorithm for PTA:

  Backward analysis, Game-based forward analysis

Perspectives

- Forward **versus** Backward
- Better abstractions for TA using **more semantics**
- Strategies for **order** of exploration
- Diagonal constraints
- Better methods/abstractions for PTA? Optimality?
- Rectangular hybrid automata

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